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SELECTION OF OPTIMAL SITE FOR NEW DEPOT OF SPECIFIED CAPACITY WITH TWO OBJECTIVES

SATYA PRAKASH

Department of Mathematics, Birla Institute of Technology & Science, Pilani (Rajasthan)

AND

VIVEK SAINI

Software Engineer, Tractor Division, Escorts, Faridabad (Haryana)

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The problem of selecting an optimal site for a new depot of specified capacity from several potential sites along with determining an optimal schedule to take buses from depots to the starting points of their routes and also determining the spare capacity available at each of the depots after the construction of a new depot is considered. Capacities of the respective depots and the number of buses required at the starting points of the respective routes are specified. The problem has two objectives—one primary and another secondary. The primary objective is to minimize the capital expenditure to be incurred in constructing a new depot plus the present value of the expenditure to be incurred in total dead mileage over a planning horizon. The secondary objective is to minimize the maximum of the dead mileage of individual buses. An algorithm is developed to obtain the solution of this two-objective location of problem.

1. INTRODUCTION

The problem of selection of an optimal site for a new depot from several potential sites is of current interest. For depots built in the past cannot provide parking facilities for all buses because of increase in their number with the passage of time and depots have reached upper limit of augmentation of their capacities. In the past, this problem has been tackled by applying certain rudimentary techniques based on commonsense and experience. But this has resulted at times into expenses which are avoidable. Owing to this and awareness that even a small percentage of savings in urban transportation schemes can result into substantial savings because of huge investments involved in them, there is need to minimize unnecessary expenditure by all means. One of the ways in which unnecessary expenditure can be minimized is to employ analytical tools to deal with the problems of urban bus transportation. Recently Sharma and Prakash[3] have applied analytical tools for optimizing dead mileage in urban bus routes. The present paper deals with the problem of selecting an optimal site for a new depot of specified capacity from several potential sites along with determining an optimal schedule to take buses from depots to the starting points of their routes and also determining the spare capacity available at each of the depots after the construction of a new depot. Capacities of the respective depots and the number of buses required at the starting points of the respective routes are specified. The problem has two objectives—one primary and another secondary. The primary objective is to minimize the capital expenditure to be incurred in constructing a new depot plus the present value of the expenditure to be incurred in total dead mileage over a planning horizon. The secondary objective is to minimize the maximum of the dead mileage of individual buses. The term dead mileage refers to the distance traversed by a vehicle when no service is provided. For instance, the distance traversed by a bus from a depot where it is parked over-night to the starting point of its route in the morning is dead mileage because this distance is traversed without severing any useful purpose. An algorithm is developed to obtain the solution of the two-objective location problem and is illustrated through a numerical example.

2. Formulation of the Problem

Suppose that there are m existing depots, s potential sites for a new depot and n starting points of routes. The total number of buses required at the starting points of the various routes is greater than the number of buses to be parked overnight at the depots necessitating the construction of a new depot. Capacity of the new depot is so specified that there is space at depots for parking a certain number of additional buses besides providing parking facilities for all buses operating on routes after its construction. For the convenience of notation, the proposed new depot at potential site l $(l = 1, \ldots, s)$ will be designated as depot $m + l$. Let a_i $(i = 1, \ldots, m + s)$ be the maximum number of buses that can be parked overnight at depot i, b_j $(j = 1, \ldots, n)$ be the number of buses required at the starting point of route j, b_{n+1} be the number of additional buses apart from buses operating on routes which can be parked overnight at depots after the construction of a new depot, c_{m+l} $(l = l, \ldots, s)$ units be the cost of creating parking facility for one bus at site l, d_{ij} $(i = 1, \ldots, m + s;\ j = 1, \ldots, n)$ units be the distance from depot i to the starting point of route j, k_{m+l} $(l = 1, \ldots, s)$ units be the initial setup cost of constructing depot at site l, N be the number of years over which the planning horizon is spread, p units be the expenditure incurred in traversing one unit of dead mileage by a bus, r be the rate of interest per annum, λ_{m+1} $(l = 1, \ldots, s)$ be an integer assuming value 0 or 1 according as site l is not selected or selected for new depot, x_{ij} $(i = 1, \ldots, m + s;\ j = 1, \ldots, n)$ be the number of buses to be sent from depot i to the starting point of route j, and $x_{i(n+1)}$ $(i = 1, \ldots, m + s)$ be the number of buses apart from buses operating on routes which can be parked overnight at depot i after the construction of a new depot. The capacity of each proposed new depot at potential site is identical and is equal to $\sum_{j=1}^{n+1} b_j - \sum_{i=1}^{m} a_i$. It is required to select an optimal site for a new depot from the potential sites along with to determine an optimal schedule to take buses from depots to the starting points of their routes and also to determine the spare capacity available at each of the depots after the construction of a new depot subject to the constraints of the problem. The primary objective is to minimize the capital expenditure to be incurred in constructing a new depot plus the present value of the expenditure to be incurred in total dead mileage over the entire planning horizon. The secondary objective is to minimize the maximum distance among the distances traversed by individual buses from depots to the starting points of their respective routes. The mathematical formulation of this two-objective location problem is as follows: Find $\lambda_{m+l} = 0$ or 1 $(l = 1, \ldots, s)$ and $x_{ij} \geqslant 0$ $(i = 1, \ldots, m + s; j = 1, \ldots, n + 1)$ which minimize

$$C = \left\{ \begin{array}{c} \sum_{l=1}^{s} \lambda_{m+l} \left[k_{m+l} + \left(\sum_{j=1}^{n+1} b_j - \sum_{i=1}^{m} a_i\right) c_{m+l} + \sum_{j=1}^{n} c_{(m+l)j}\, x_{(m+l)j}\right] \\ + \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}\, x_{ij} \end{array} \right\} \quad \ldots(1)$$

where

$$c_{ij} = 2\,(365)\, p\, d_{ij} \sum_{t=1}^{N} (1 + r/100)^{-t} \quad (i = 1, \ldots, m + s; j = 1, \ldots, n) \quad \ldots(2)$$

and

$$D = \max \{d_{ij} : x_{ij} > 0 \; (i = 1, \ldots m + s; j = 1, \ldots, n)\} \quad \ldots(3)$$

according to priorties in the order of their occurrence subject to the constraints

$$\sum_{l=1}^{s} \lambda_{m+l} = 1 \quad \ldots(4)$$

$$\sum_{j=1}^{n+1} x_{ij} = a_i \quad (i = 1, \ldots, m) \quad \ldots(5)$$

$$\sum_{l=1}^{s} \sum_{j=1}^{n+1} \lambda_{m+l}\, x_{(m+l)j} = \sum_{j=1}^{n+1} b_j - \sum_{i=1}^{m} a_i \quad \ldots(6)$$

$$\sum_{i=1}^{m} x_{ij} + \sum_{l=1}^{s} \lambda_{m+l}\, x_{(m+l)j} = b_j \; (j = 1, \ldots, n + 1). \quad \ldots(7)$$

3. Solution Procedure

The two-objective location problem formulated above is a mixed integer non-linear problem. A procedure is outlined to obtain the solution of this problem. As

λ_{m+l}'s are integers assuming values 0 or 1 and are subjected to the constraint (4), it follows that all elements except one in each distinct set of values assumed by λ_{m+1}, ..., λ_{m+s} are zero and that the nonzero element is 1; so the total number of distinct sets of values assumed by $\lambda_{m+1}, \ldots, \lambda_{m+s}$ is only s. Setting $\lambda_{m+1} = 1$ and $\lambda_{m+l} = 0$ ($l = 2, \ldots, s$), the two-objective location problem given by equations (1) through (7) yields the following problem. Find $x_{ij} \geqslant 0$ ($i = 1, \ldots, m, m+1; j = 1, \ldots, n+1$) which minimize

$$C_1 = k_{m+l} + \left(\sum_{j=1}^{n+1} b_j - \sum_{i=1}^{m} a_i \right) c_{m+1} \sum_{i=1}^{m,m+1} \sum_{j=1}^{n} c_{ij} x_{ij} \qquad \ldots(8)$$

where

$$c_{ij} = 2\ (365)\ p\ d_{ij} \sum_{t=1}^{N} (1 + r/100)^{-t} \ (i = 1, \ldots, m, m+1;\ j = 1, \ldots, n) \qquad \ldots(9)$$

and

$$D_1 = \max \{d_{ij} : x_{ij} > 0 \ (i = 1, \ldots, m, m+1;\ j = 1, \ldots, n)\} \qquad \ldots(10)$$

according to priorities in the order of their occurence subject to the constraints (5),

$$\sum_{j=1}^{n+1} x_{(m+l)j} = \sum_{j=1}^{n+1} b_j - \sum_{i=1}^{m} a_i \qquad \ldots(11)$$

$$\sum_{i=1}^{m,m+1} x_{ij} = b_j \ (j = 1, \ldots, n+1). \qquad \ldots(12)$$

The problem described by equations (8), (9), (10), (5), (11), (12) is designated as the 1st two-objective problem of the given two-objective location problem. The problem obtained from the given two-objective location problem by setting $\lambda_{m+2} = 1$ and $\lambda_{m+l} = 0$ ($l = 1, 3, \ldots s$) is designated as the two-objective problem of the given two-objective location problem, and so on. Finally, the problem obtained from the given two-objective location problem by setting $\lambda_{m+s} = 1$ and $\lambda_{m+l} = 0$ ($l=1, \ldots, s-1$) is designated as the sth two-objective problem of the given two-objective location problem. These two-objective problems of the given two-objective location problem are of similar type and are reduced to equivalent single objective transportation-type problems following the procedure developed by Prakash[2]. The single-objective transportation-type problem equivalent to the lth two-objective problem of the given two-objective location problem is designated as the l-th single objective transportation-type problem of the given two-objective location problem. Among all the single-objective transportation-type problems of the given two-objective location problem, the one whose objective function has minimum value provides complete solution of the given two-objective location problem and the optimal site for a new depot is the site corresponding to it.

The procedure to reduce the Ist two-objective problem of the given two-objective location problem to the 1st single-objective transportation-type problem of the given two-objective location problem is briefly described below. First, the set $\{d_{ij} : i = 1, ...m, m + 1; j = 1, ... n\}$ is partitioned into subsets L_k $(k = 1,...,q)$ in the following way. Each of the subsets L_k consists of the d_{ij}'s having the same numerical value, L_1 consists of the d_{ij}'s having the largest numerical value, L_2 consists of the d_{ij}'s having the next largest numerical value. and so on Finally, L_q consists of the d_{ij}'s having the smallest numerical vaue. After this, priority factors. M_0, M_1, ..., M_q are assigned to

$$C_1, \sum_{L_1} x_{ij}, \ldots, \sum_{L_q} x_{ij}$$

respectively. Here $\sum_{L_k} x_{ij}$ is the sum of the x_{ij}'s corresponding to the d_{ij}'s belonging to L_k. The priority factors M_k's are all positive and are such that the expression $\sum_{k=0}^{q} \alpha_k M_k$ has the same sign as the nonzero α_k with the smallest subscript present in it irrespective of the values of other L_k's. Having done all this, the 1st two-objective problem of the given two-objective location problem is reduced to the equivalent 1st single-objective transportation-type problem of the given two-objective location problem seeking to determine $x_{ij} \geqslant 0$ $(i = 1, .. , m, m + 1; j = 1, ..., n + 1)$ which minimize

$$Z_1 = M_0 [k_{m+l} + (\sum_{j=1}^{n+1} b_j - \sum_{i=1}^{m} a_i) c_{m+l} + \sum_{i=1}^{m,m+1} \sum_{j=1}^{n} c_{ij} x_{ij}]$$

$$+ \sum_{k=1}^{q} M_k \sum_{L_k} x_{ij} \qquad ...(13)$$

subject to the equations (9), (5), (11), (12). This 1st single-objective transportation-type problem is amenable to solution by the standard transportation method discussed by Hadley[1]. The minimum value of the total cost C_1 comprising the capital expenditure to be incurred in constructing a new depot at potential site 1 plus the present value of the expenditure to be incurred in total dead mileage over the planning horizon is obtained by the coefficient of M_0 in the expression on the right-hand side of eqn. (13) after it has been minimized. And if M_u is the priority factor with the smallest subscript among all the priority factors assignel to the nonzero sums $\sum_{L_k} x_{ij}$ in the expression on the right-hand side of eqn. (13) after it has been minimized, the the minimum of the maximum distance D_1 among the distanccs traversed by individual buses from the depots to the starting points of their respective routes is givee by the value of the d_{ij}'s belonging to L_u.

4. Numerical Example

Now the above procedure is applied to obtain the optimal solution of a numerical problem which is obtained by taking $b_{6+1} = 5$, $c_{2+1} = 2000$, $c_{2+2} = 2100$, k_{2+1}

$=200000$, $k_{2+2} = 250000, m = 2, n = 6. N = 20, p = 6, r = 10, s = 2$, and assigning numerical values to all other quantities in the location problem formulated in Section 2. The tableau representation of the numerical problem is shown in Table I.

In this Table, rows with the headings 'Depot 1' and 'Depot 2' refer to existing depots while entries in these rows in the column with the heading of 'Capacities of depots' refer to their capacities. And rows with the headings 'Depot (2 + 1)' and 'Depot (2 + 2)' refer to depots at potential sites while entries in these rows in the column with the heading "Capacities of depots" refer to their capacities which are

TABLE I

Capacities of depots aud buses required at starting points of routes and distances from depots to starting point of routes

	Starting points of route1	route2	route3	route4	route5	route6	Capacities of depots
Depot 1	2	1	.5	5	4	3	25
Depot 2	1	2	3	3	5	2	20
Depor (2 + 1)	3	.5	1	2	3	4	53+5—45=11
Depot (2 + 2)	2.5	2	1	3	2	3	51+5—45=1ì
Buses required	12	10	8	6	7	8	

TBALE II

Tableau providing optimal solution of 1st single-objective transportation-type problem of numerical problem

	Starting points of route 1	route 2	route 3	route 4	route 5	route 6	Spare capacity available	Capacity of depots
Depot 1	$74578.8M_0$ $+M_4$	$37289.4M_0$ $+M_5$ **10**	$18644.7M_0$ $+M_6$ **8**	$186447.0M_0$ $+M1$	$149157.6M_0$ $+M_2$ **2**	$111868.2M_0$ $+M_3$	0 **5**	25
Depot 2	$37289.4M_0$ $+M_5$ **12**	$74578\ 8M_0$ $+M_4$	$111868.2M_0$ $+M_3$	$111868.2M_4$ $+M_3$ **0**	$186447.0M_0$ $+M_1$	$74578.8M_0$ $+M_4$ **8**	0	20
Depot (2+1)	$111868.2M_0$ $+M_3$	$18644.7M_0$ $+M_6$	$37289.4M_0$ $+M_5$	$74578.8M_0$ $+M_4$ **6**	$111868\ 2M_0$ $+M_3$ **5**	$149157.6M_0$ $+M_2$	0	11
Buses required	12	10	8	6	7	8	5	

identical and equal to 51 + 5 − 45 = 11. For easy referenee. the term cell shall be used to denote the space in a column with the subheading of a route against a depot. The entry of all the cells (i, j) depicts the units of distance d_{ij} from depot i to the starting point of route j. The two objective functions of the numerical problem which are sought to be minimized are

$$C = \sum_{l=1}^{2} \lambda_{2+l} [k_{2+l} + 11\, c_2 + \sum_{j=1}^{6} c_{(2+l)j}\, x_{(2+l)j}) + \sum_{i=1}^{2} \sum_{j=1}^{6} c_{ij}\, x_{ij} \quad \ldots(14)$$

where

$$c_{ij} = 2\,(365)\,(6)\,d_{ij} \sum_{t=1}^{20} (1 + 10/100)^{-t} \; (i = 1, \ldots, 2 + 2;\; j = 1, \ldots, 6) \quad \ldots(15)$$

and

$$D = \max \{d_{ij} : x_{ij} > 0\; (i = 1, \ldots, 2+2; j=1, \ldots, 6)\}. \quad \ldots(16)$$

For the 1st two-objective problem of the numerical problem, we find $q = 6$. The subsets forming partition of the set $\{d_{ij} : i = 1, 2. 2 + 1;\; j = 1, \ldots, 6\}$ are as follows:

$$L_1 = \{d_{14}, d_{25}\},\; L_2 = \{d_{15}, d_{36}\},\; L_3 = \{d_{16}, d_{23}, d_{24}, d_{31}, d_{35}\}$$

$$L_4 = \{d_{11}, d_{22}. d_{26}, d_{34}\},\; L_5 = (d_{12}, d_{21}, d_{33}\},\; L_6 = \{d_{13}, d_{32}\}$$

The d_{ij}'s belonging to L_1, L_2, L_3, L_4, L_5, L_6 have numerical values 5, 4, 3, 2, 1, .5 respectively. The objective function of the 1st single-objective transportation-type problem of the numerical probiem is given by

$$Z_1 = \left\{ \begin{array}{l} M_0\,(222000 + 74578.8x_{11} + 37289.4x_{12} + 18644.7x_{13} \\ +186447.0x_{14} + 149157.6x_{15} + 111868.2x_{16} + 37289.4x_{21} \\ + 74578.8x_{22} + 111868.2x_{23} + 111868.2x_{24} \\ + 186447.0x_{25} + 74578.8x_{26} + 111868.2x_{31} + 18644.7x_{32} \\ + 37289.4x_{33} + 74578.8x_{34} + 111868.2x_{35}\; 149157.6x_{36} \\ + M_1\,(x_{14} + x_{25}) + M_2\,(x_{15} + x_{36})\; M_3\,(x_{16} + x_{24} \\ + x_{31} + x_{35}) + M_4\,(x_{11} + x_{22} + x_{26} + x_{34}) + M_5\,(x_{12} \\ + x_{21} + x_{33}) + M_6\,(x_{13} + x_{32}) \end{array} \right\} \quad \ldots(17)$$

Applying the standard transportation method and remembering that the priority factors M_k's are such that the expreesion $\sum_{k=0}^{6} L_k M_k$ has the same sign as the nonzero L_k with the smallest subscript present in it irrespective of the values of other L_k's; an optimal basic feasible of the 1st single-objective transportation-type problem is obtained and is shown in Table II. Entry in the upper half of the cell in Table II depicts the cost associated with the variables. Values of the basic variables of the optimal solution are encircled in the tableau. The optimal value of the objective function Z_1 is given by

$$Z_1 = 3093283.8M_0 + 2M_2 + 5M_3 + 14M_4 + 22M_5 + 8M_6 \quad ...(18)$$

An optimal basic feasible solution of the 2nd single-objective transportation-type problem of the numerical problem is obtained proceeding exactly in the same way as done to obtain the optimal solution of the 1st single-objective transportation-type problem of the numerical problem. The optimal value of the objective function Z_2 of the 2nd single-objective transportation-type problem is given by

$$Z_2 = 3107094.4M_0 + 6M_3 + 17M_5 + 20M_6 + 8M_7 \quad ...(19)$$

Now among the two single-objective transportation-type problems of the numerical problem, the objective function of the 1st one has minimum value. So the site 1 is selected for construction of a new depot and the optimal solution of the numerical problem is provided by the optimal solution of its 1st single-objective transportation-type problem. For ready reference, the optimal solution of the numerical problem is shown in Table III.

TABLE III

Optimal solution of numerical problem

Optimal site for new depot	Values of basic variables of optimal solution	Minimal value of expenditure in construction plus present value of expenditure in total dead mileage	Minimum of maximum of dead mileage of individual buses
$l = 1$	$x_{12} = 10$, $x_{13} = 8$, $x_{15} = 2$, $x_{17} = 5$, $x_{21} = 12$, $x_{24} = 0$. $x_{26} = 8$, $x_{34} = 6$, $x_{35} = 5$	$C = 3093283.8$	$D = 4$

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