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TRANSIENT FORCED AND FREE CONVECTION FLOW PAST AN INFINITE VERTICAL PLATE

M. D. JAHAGIRDAR

Department of Mathematics, Maulana Azad College, Aurangabad 431001

AND

R. M. LAHURIKAR

Department of Mathematics, Institute of Science, Aurangabad 431001

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An exact solution to the unsteady free and forced convection flow of an incompressible viscous fluid past an infinite vertical plate is presented and the expressions for the velocity, the penetration distance and the skin-friction are derived. It is observed that an increase in the Prandtl number leads to a decrease in the penetration distance and the skin-friction when the time t is constant.

1. INTRODUCTION

Siegel[1], Schetz and Eichhorn[2], Menold and Yang[3], Chung and Anderson[4], Goldstein and Briggs[5] and Sugawava and Michiyoshi[6] studied the unsteady free convection flow under different condition past an infinite vertical plate. Goldstein and Eckert[7] confirmed experimentally some of these theoretical predictions. In all these studies, the infinite plate was assumed to be stationary and the fluid was supposed to move due to temperature difference only. If the fluid is stationary and the infinite plate, surrounded by the stationary fluid, is given an impulsive motion along with its temperature raised to T_w such that $T_w' > T_\infty$, where T_∞, is the temperature of the surrounding fluid, how the flow of the fluid takes its shape ? This was studied by Soundalgekar[8] in case of an isothermal plate. The effects of free convection currents on the flow and the skin-friction were studied in this paper. However, another physical situation which is often experienced in the industrial application is the unsteady free and forced convective flow past an infinite vertical isothermal plate of an incompressible fluid. This has not been studied in the literature. Hence the motivation to undertake this study. In section 2, the mathematical analysis is presented and in section 3, the conclusions are setout.

2. Mathematical Analysis

Here we consider the unsteady free and forced convection flow of a viscous incompressible fluid past an infinite vertical isothermal plate in the upward direction. The x'-axis is taken along the plate in the vertically upward direction and the y'-axis is taken normal to the plate. Then the physical variables are functions of y' and t' only. Then under usual Boussinesq's approximation, the flow is governed by the system of equations :

$$\rho \frac{\partial u'}{\partial t'} = g\,\beta\rho\,(T' - T'_\infty) + \mu \frac{\partial^2 u'}{\partial y'^2} \qquad \ldots(1)$$

$$\rho\, C_p \frac{\partial T'}{\partial t'} = k \frac{\partial^2 T'}{\partial y'^2} \qquad \ldots(2)$$

The initial and boundary conditions are

$$\left.\begin{array}{l} t' \leqslant 0,\ u' = 0,\ T' \to T'_\infty,\ \text{for all } y' \\ t' > 0,\ u' = 0,\ T \to T'_w \text{ at } y' = 0. \\ u' \to U_0,\ T' \to T'_\infty \text{ as } y' \to \infty. \end{array}\right\} \qquad \ldots(3)$$

Here u' is the velocity of the fluid in the x'-direction, ρ' the density, g the acceleration due to gravity, β the coefficient of volume expansion, T' the temperature of the fluid near the plate, T'_∞ the temperature of the fluid in the free-stream, μ the coefficient of viscosity, C_p the specific heat at constant pressure and k is the thermal conductivity. Initially, the plate temperature and the free stream temperature are the same everywhere. At $t' > 0$, the plate temperature T'_∞ is raised to T'_w .

On introducing the following non-dimensional quantities

$$\left.\begin{array}{l} y = y'\,U_0\,Gr^{1/2}/\nu,\ t = t'\ U_0^2\ Gr/\nu,\ u = u'/U_0 \\ Pr = \mu C_p/k,\ \theta = (T' - T'_\infty)/(T'_w - T'_\infty), \\ Gr = \nu g\beta\,(T'_w - T'_\infty)/U_0^3 \text{ (the Grashof number)} \end{array}\right\} \qquad \ldots(4)$$

in eqns. (1) – (3), we have

$$\frac{\partial u}{\partial t} = \theta + \frac{\partial^2 u}{\partial y^2} \qquad \ldots(5)$$

$$Pr \frac{\partial \theta}{\partial t} = \frac{\partial^2 \theta}{\partial y^2} \qquad \ldots(6)$$

and the initial and boundary conditions are

$$\left.\begin{array}{l} t \leqslant 0,\ u = 0,\ \theta = 0, \text{ for all } y \\ t \geqslant 0,\ u = 0,\ \theta = 1, \text{ at } y = 0 \\ u = 1,\ \theta = 0 \text{ as } y \to \infty. \end{array}\right\} \qquad ...(7)$$

Equations (5) – (6) subject to the conditions (7) are solved by the usual Laplace-transform technique and the solutions are as follows :

$$u = 1 - \text{erfc}\ (\eta) - \frac{t}{Pr - 1}\ [\text{erfc}\ (\eta \sqrt{Pr}) - \text{erfc}\ (\eta)]$$

$$+ 2\eta^2\ (Pr\ \text{erfc}\ (\eta \sqrt{Pr}) - \text{erfc}\ (\eta))$$

$$+ \frac{2\eta}{\sqrt{\pi}}\ (\exp\ (-\eta^2) - \sqrt{Pr}\ \exp\ (-\ Pr.\eta^2) \qquad ...(8)$$

$$\theta = \text{erfc}\ (\eta \sqrt{Pr}) \qquad ...(9)$$

knowing the velocity field, it is more interesting to study the leading edge effect. The penetration distance is derived by integrating u with respect to t and the maximum penetration distance $x_{p\max}$ at any time can be determined by differentiating x_p with respect to y holding t constant and then by setting the derivative equal to zero. Thus the penetration distance is given by

$$x_p = \int_0^t u\ (y, t)\ dt. \qquad ...(10)$$

This can be expressed in terms of the Laplace transform and inverse transform with respect to the variable t as

$$x_p = L^{-1} \left\{ \frac{1}{s}\ L\ (u\ (y,\ t)) \right\}$$

$$= L^{-1} \left\{ \frac{1}{s}\ \bar{u}\ (y, s) \right\}. \qquad ...(11)$$

Substituting for $\bar{u}\ (y, s)$ in (11), and taking the inverse, we have

$$X_p = t \left[(1 - \text{erfc}\ (\eta)) + \eta \left(\frac{2 \exp\ (-\eta^2)}{\sqrt{\pi}} - 2\eta\ \text{erfc}\ (\eta) \right) \right]$$

$$- \frac{t^2}{6\ (Pr - 1)} \left[4\eta^2\ \{(\eta^2\ P_r^2 + 3\ Pr)\ \text{erfc}\ (\eta\sqrt{Pr}) \right.$$

$$- (\eta^2 + 3)\ \text{erfc}\ (\eta)\} + 3\ (\text{erfc}\ (\eta \sqrt{Pr}) - \text{erfc}\ (\eta))$$

$$\left. + \frac{2\eta}{\sqrt{\pi}} ((2\eta^2 + 5)\ \exp\ (-\ \eta^2) - (2Pr\ \eta^2 + 5)\ \sqrt{Pr}\ \exp\ (-Pr\ \eta^2) \right]. \qquad ...(12)$$

To understand the physical meaning of the problem, we have calculated the numerical values of x_p for different values of Pr and these are plotted in Fig. 1. We observe from this Fig. 1 that the penetration distance x_p decreases with increasing the Prandtl number of the fluid. But it increases with t, the time.

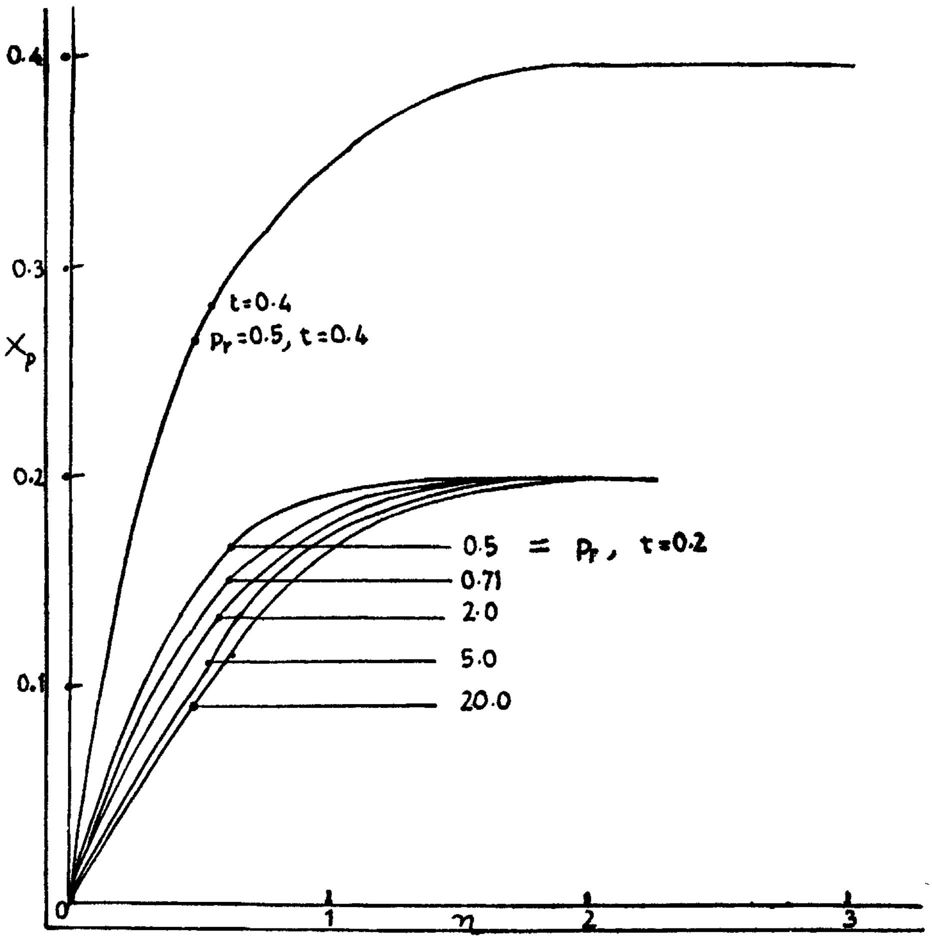

FIG. 1 Penetration distance

We now study the skin-friction. It is given by

$$\tau' = -\mu \left. \frac{\partial u'}{\partial y'} \right|_{y'=0} \qquad \text{...(13)}$$

and in view of (4), equation (13) reduces to

$$\tau = -\tau'/\rho U_0^2 \; Gr^{1/2} = \left. \frac{du}{d\eta} \right|_{\eta=0}. \qquad \text{...(14)}$$

Hence substituting for u from (12) in equation (14) and simplifying, we get,

$$\tau = \frac{1}{\sqrt{\pi t}} + \frac{2\sqrt{t}}{\sqrt{\pi}} \cdot \frac{1}{1+Pr} . \qquad ...(15)$$

From (15), we conclude that the skin-friction decreases with increasing Pr when t is constant.

3. Conclusion

The penetration distance and the skin-friction decrease with increasing the Prandtl number.

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