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P-WAVE SCATTERING AT A COASTAL REGION IN A SHALLOW OCEAN

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The present problem can be idealised to study diffraction of compressional waves at a coastal region in a shallow ocean. The method of solution is the Wiener-Hopf technique. An exact solution is obtained in terms of Fourier integrals. The evaluation of the integrals along an appropriate contour in a complex plane results in reflected Rayleigh and scattered waves. Near the scatterer, the scattered field has the behaviour of a cylindrical wave and at distant points, it is a plane wave. The scattered field excites the particle motion at right angles to the free surface of the ocean and the horizontal component of the displacement is absent in the surface. Numerical computations for the scattered field in terms of the distance from the scatterer and for the Rayleigh waves versus depth below the free surface reveal a sharp fall for the former and a slow decay in case of the latter.

1. INTRODUCTION

The problem of diffraction of elastic waves by a rigid plane barrier in a liquid medium has been investigated by various authors. The effect of a vertical plane barrier, fixed in an infinitely deep sea, on normally incident surface waves was first considered by Ursell[1] dealing with a two-dimensional case. Faulkner[2] has extended Ursell's study to a three-dimensional case to consider the effect of a fixed vertical plane barrier of finite depth on obliquely incident surface waves. He has used the Wiener-Hopf technique[3] and has obtained approximate expressions for the transmission and reflection co-efficients. The problem of diffraction of compressional waves which strike obliquely at a vertical barrier of finite depth fixed in a liquid halfspace or in a liquid layer over a solid half-space has been discussed by Deshwal[4-6]. Mann and Deshwal[7] have studied the problem of Rayleigh wave scattering at the edge of a finite plane barrier in the surface of a shallow ocean over a solid halfspace using the technique of Wiener and Hopf. The scattered waves have the behaviour of reflected plane waves near the scatterer and of cylindrical waves at distant points.

The present paper deals with the problem of diffraction of compressional waves at a coastal region in a shallow ocean. At a coastal region, there is shallow ocean over the solid mantle of the earth on one hand and there are hard rocks on the other hand. The shallow ocean is assumed to be a liquid layer and the hard rock to be a rigid plane boundary. The region $x \leqslant 0, z \geqslant 0$ is a rigid medium in which there is

no movement of waves. In the region $x \leqslant 0, z \geqslant 0$, there is a shallow ocean in the form of a liquid layer of depth h occupying the region $x \geqslant 0, z < h$ lying over a solid half space $x \geqslant 0, z \geqslant h$.

2. Statement of the Problem

The two media lie on the side of positive z-axis (pointing vertically downward). The liquid layer and the solid halfspace occupy the regions $x \geqslant 0, 0 \leqslant z \leqslant h$ and $x \geqslant 0, z \geqslant h$ respectively as in the Fig. 1. A two-dimensional compressional wave is

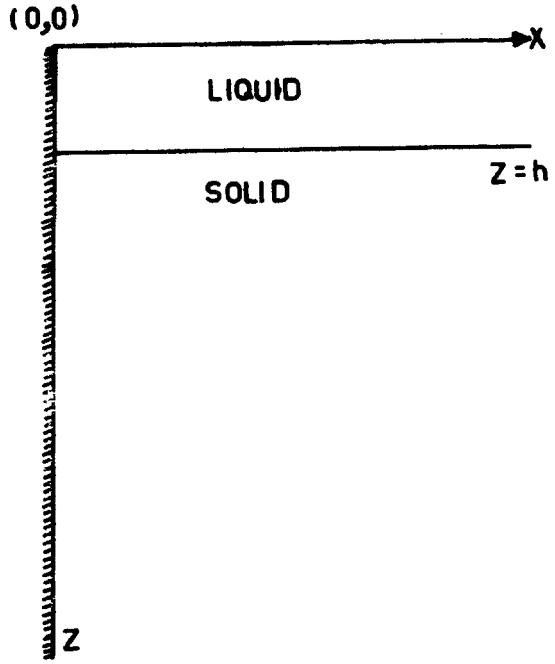

Fig. 1 Oceanic layer over solid mantle of the earth.

incident at the rigid plane boundary and the corner scatters the waves. The wave equations in the liquid and the solid are

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2} + k^2 \phi = 0, k = k_1 + ik_2 \qquad ...(1)$$

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi_1}{\partial z^2} + k'^2 \phi_1 = 0, k' = k'_1 + ik'_2 \qquad ...(2)$$

and a similar equation in ψ_1. k and k' are complex with their imaginary parts being small and positive. ϕ, ϕ_1 and ψ_1 are the displacement potentials in the liquid and the solid media respectively.

The incident potential satisfying the stress-free conditions is

$$\phi_I = D \sin (kz \sin \theta) \exp (- ikx \cos \theta) \qquad ...(3)$$

such that the total potential in the liquid layer is

$$\phi_l = \phi_l + \phi\ (x, z). \qquad \ldots(4)$$

The displacement components (u_1, w_1) in the solid medium in terms of potentials are given by

$$u_1 = \frac{\partial \phi_1}{\partial x} - \frac{\partial \psi_1}{\partial z}, \quad w_1 = \frac{\partial \phi_1}{\partial z} + \frac{\partial \psi_1}{\partial x}. \qquad \ldots(5)$$

Since there is no transverse wave in the liquid, the ψ-potential is absent and the displacement components (u, w) in the liquid layer are in terms of the derivatives of ϕ only.

3. Boundary Conditions

The conditions on the boundaries are :

(i) $\phi_l\ (x, z) = 0$, on $z = 0$, for all x ...(6)

(ii) $\frac{\partial \phi_l}{\partial x} = 0$, on $x = 0$, for $0 \leqslant z \leqslant h$ ...(7)

(iii) $\frac{\partial \phi_1}{\partial x} - \frac{\partial \psi_1}{\partial z} = 0$, on $x = 0$, for $z \geqslant h$ ...(8)

(iv) $\frac{\partial \phi_1}{\partial z} + \frac{\partial \psi_1}{\partial x} = 0$, on $x = 0$, for $z \geqslant h$ ...(9)

(v) $w = w_1, p_{zz} = (p_{zz})_1, (p_{zx})_1 = 0$, on $z = h$ ...(10)

for $x \geqslant 0$, where the subscript 1 denotes the entities belonging to the solid and p_{zz}, p_{zx} are the normal and shear stresses.

We define the Fourier transform

$$\bar{\phi}_+\ (p, z) = \int_0^\infty \phi\ (x, z)\ e^{ipx}\, dx, p = \alpha + i\beta \qquad \ldots(11)$$

and assume that, for given z,

$$| \phi | \backsim D_1 \exp(-k_2 | x |) \text{ as } | x | \to \infty \qquad \ldots(12)$$

where D_1 is a constant, then

$$| \bar{\phi}_+\ (p, z) | \leqslant \int_0^\infty | \phi\ (x, z) |\ e^{-\beta x}\, dx. \qquad \ldots(1\ )$$

On account of the assumption, $\bar{\phi}_+\ (p, z)$ is bounded at infinity only when $\beta > -k_2$. Therefore $\bar{\phi}_+\ (p, z)$ is regular in the region $\beta > -k_2$ of the complex p-plane. $\bar{\phi}_{1+}\ (p, z)$ and $\psi_{1+}\ (p, z)$ obey the same behaviour.

4. Solution of the Problem

Fourier transformation of (1) leads to

$$\frac{d^2}{dz^2}\ \bar{\phi}_+ (p, z) - Y^2\ \bar{\phi}_+ (p, z) = \left(\frac{\partial \phi}{\partial x}\right)_0 - ip\,(\phi)_0 \qquad \ldots(14)$$

where

$$Y = \pm\,(p^2 - k^2)^{1/2}. \qquad \ldots(15)$$

The sign before the radical is such that Re $(Y) \geqslant 0$ for all p. The subscript 0 denotes the value at $x = 0$. Let us change p to $-p$ in (14) and add the new equation to it to find

$$\frac{d^2}{dz^2}\,[\bar{\phi}_+ (p, z) + \bar{\phi}_+ (-p, z)] - Y^2\,[\bar{\phi}_+ (p, z) + \bar{\phi}_+ (-p, z)]$$

$$= D\,(2ik\cos\theta)\sin(kz\sin\theta). \qquad \ldots(16)$$

The right hand side is the value of $(\partial\phi/\partial x)_0$ from (7) and the complete solution of this differential equation is

$$\bar{\phi}_+ (p, z) + \bar{\phi}_+ (-p, z) = c_1 (p)\, e^{-Yz} + c_2 (p)\, e^{Yz}$$

$$- \frac{(2ik\ D\cos\theta)\sin(kz\sin\theta)}{p^2 - k^2\cos^2\theta}. \qquad \ldots(17)$$

Let us utilize the condition (6) to find $c_2 (p)$ in terms of $c_1 (p)$. The condition results in

$$\bar{\phi}_+ (p, 0) + \bar{\phi}_+ (-p, 0) = 0. \qquad \ldots(18)$$

Using (18) in (17) to have

$$\bar{\phi}_+ (p, z) + \bar{\phi}_+ (-p, z) = 2c_1 (p)\sin hYz - \frac{(2ik\cos\theta)\ D\sin(kz\sin\theta)}{(p^2 - k^2\cos^2\theta)}. \qquad \ldots(19)$$

Putting $z = h$ in (19) and in its derivative with respect to z and eliminating $c_1 (p)$ between the resulting equations, we find

$$\bar{\phi}_+ (p) + \bar{\phi}_+ (-p) = \frac{1}{Y}\tanh Yh\ [\bar{\phi}'_+ (p) + \bar{\phi}'_+ (-p)]$$

$$- \frac{(2ik\cos\theta)\ D\sin(kh\sin\theta)}{(p^2 - k^2\cos^2\theta)}$$

$$+ \frac{(2ik^2\sin\theta\cos\theta)\ D\cos(kh\sin\theta)}{(p^2 - k^2\cos^2\theta)} \cdot \frac{\tanh Yh}{Y}. \qquad \ldots(20)$$

We have used the notations $\bar{\phi}_+(p)$, $\bar{\phi}'_+(p)$ for $\bar{\phi}_+(p, h)$ and $\bar{\phi}'_+(p, h)$. Similar notations will be used for the potentials in the solid medium.

Let us take the Fourier transform of the conditions (10) to get

$$\bar{\phi}'_+(p) + \frac{(ik \sin\theta) D \cos(kh \sin\theta)}{(p - k\cos\theta)} = \bar{\phi}'_{1+}(p) - ip\,\bar{\psi}_{1+}(p), \quad p \neq k\cos\theta \qquad ...(21)$$

$$-\lambda k^2\left[\bar{\phi}_+(p) + \frac{iD \sin(kh \sin\theta)}{(p - k\cos\theta)}\right] = -\lambda_1 k'^2\,\bar{\phi}_{1+}(p) + 2\mu_1(p^2 - k'^2)\,\bar{\phi}_{1+}(p) - 2\mu_1 ip\,\bar{\psi}'_{1+}(p) \qquad ...(22)$$

and

$$-2ip\,\bar{\phi}'_{1+}(p) + (k''^2 - 2p^2)\,\bar{\psi}_{1+}(p) = 0, \text{ on } z = h,\ x \geqslant 0. \qquad ...(23)$$

λ_1, μ_1 are Lame's constants of the solid and λ is that of the liquid, k'' being the wave number for transverse waves in the solid. Taking the Fourier transformation of (2) and using the boundary conditions (8) and (9), we find

$$\frac{d^2}{dz^2}[\bar{\phi}_{1+}(p, z) - \bar{\phi}_{1+}(-p, z)] - Y_1^2\,[\bar{\phi}_{1+}(p, z) - \bar{\phi}_{1+}(-p, z)] = 0,$$

$$Y_1 = \pm (p^2 - k'^2)^{1/2} \qquad ...(24)$$

whose solution is

$$\bar{\phi}_{1+}(p, z) - \bar{\phi}_{1+}(-p, z) = A(p)\,e^{-Y_1 z} + B(p)\,e^{Y_1 z}. \qquad ...(25)$$

Since $\bar{\phi}_{1+}(p, z) - \bar{\phi}_{1+}(-p, z)$ is bounded when z tends to infinity, it follows that $B(p) = 0$ and

$$\bar{\phi}_{1+}(p, z) - \bar{\phi}_{1+}(-p, z) = A(p)\,e^{-Y_1 z}. \qquad ...(26)$$

Similarly

$$\bar{\psi}_{1+}(p, z) - \bar{\psi}_{1+}(-p, z) = D(p)\,e^{-\delta_1 z},\ \delta_1 = \pm (p^2 - k''^2)^{1/2}. \qquad ...(27)$$

The criteria for signs of Y_1 and δ_1 are the same as for Y. Putting $z = h$ in (26) and in its derivative with respect to z and eliminating $A(p)$, we have

$$\bar{\phi}_{1+}(p) - \bar{\phi}_{1+}(-p) = \frac{-1}{Y_1}[\bar{\phi}'_{1+}(p) - \bar{\phi}'_{1+}(-p)]. \qquad ...(28)$$

5. Solution of the Wiener-Hopf Type Equation

The functional equation (28) is solved by involving the Wiener-Hopf technique. We write (28) as

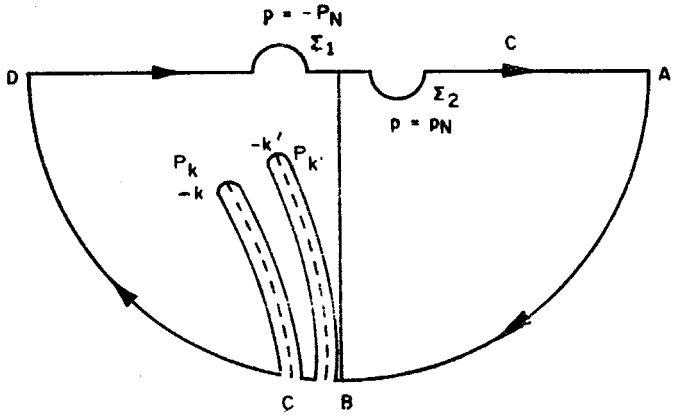

FIG. 2 Contour of integration in lower half of the complex plane.

$$\bar{\phi}_{1+}(p) + \frac{1}{(p-k')^{1/2}}\left[\frac{\bar{\phi}'_{1+}(p)}{(p+k')^{1/2}} - \frac{\bar{\phi}'_{1+}(k')}{(2k')^{1/2}}\right] - \frac{\bar{\phi}_{1+}(k')}{(-2k'(p+k')^{1/2})}$$

$$= \bar{\phi}_{1+}(-p) + \frac{1}{(p+k')^{1/2}}\left[\frac{\bar{\phi}'_{1+}(-p)}{(p-k')^{1/2}} - \frac{\bar{\phi}'_{1+}(k')}{(-2k')^{1/2}}\right]$$

$$- \frac{\bar{\phi}'_{1+}(k')}{(2k'(p-k'))^{1/2}}. \qquad \ldots(29)$$

The left hand member of (29) is analytic in the region $\beta > -k_2$ and the right hand member in $\beta < k_2$. By analytic continuation, they represent an entire function. Each member tends to zero in its respective region of analyticity as $|p| \to \infty$. By an extension of Liouville theorem (Sec. 2.61, Titchmarsh[8]) the entire function is identically zero. Thus we have

$$\bar{\phi}_{1+}(p) = -\frac{1}{Y_1}\bar{\phi}'_{1+}(p) + \bar{\phi}'_{1+}(k')\,g(p) \qquad \ldots(30)$$

where

$$g(p) = \frac{1}{[2k'(p-k')]^{1/2}} + \frac{1}{[(-2k')(p+k')]^{1/2}}. \qquad \ldots(31)$$

Similarly

$$\bar{\psi}_{1+}(p) = \frac{-1}{\delta_1}\bar{\psi}'_{1+}(p) + \bar{\psi}'_{1+}(k'')\,h(p) \qquad \ldots(32)$$

where $h(p)$ is obtained from $g(p)$ by replacing k' by k''.

Using (30) and (32) in the conditions (21-23), eliminating $\bar{\phi}'_{1+}(p)$ and $\bar{\psi}'_{1+}(p)$ between the resulting equations, $\bar{\phi}_+(p) + \bar{\phi}_+(-p)$ is obtained in terms $\bar{\phi}'_+(p) + \bar{\phi}'_+(-p)$. The result is used in (20) to obtain

$$\bar{\phi}'_+(p) + \bar{\phi}'_+(-p) = -\frac{(2ik \sin\theta \cos\theta)\, D \cos(kh \sin\theta)}{p^2 - k^2 \cos^2\theta} + \frac{2g(p)(-\lambda_1 k_1^2 + \mu_1 Y_1^2)\,\bar{\phi}'_{1+}(k')}{\lambda k^2 f(p)} \quad \ldots(33)$$

where

$$f(p) = \frac{\tanh Yh}{Y} + \frac{(2p^2 - k''^2)(-\lambda_1 k'^2 + 2\mu_1 Y_1^2)}{\lambda k^2 Y_1 k''^2} - \frac{4p^2 \mu_1 \delta_1}{\lambda k^2 k''^2} \quad \ldots(34)$$

From (19) and (33), it follows that

$$\bar{\phi}_+(p, z) + \bar{\phi}_+(-p, z) = \frac{\sinh Yz}{Y \cosh Yh}\left[\frac{2g(p)(-\lambda_1 k'^2 + 2\mu_1 Y_1^2)\,\bar{\phi}'_{1+}(k')}{\lambda k^2 f(p)}\right] - \frac{(2ik\, D \cos\theta) \sin(kz \sin\theta)}{p^2 \quad k^2 \cos^2\theta}. \quad \ldots(35)$$

The potential $\phi(x, z)$ is given by the inverse Fourier transform, i.e.,

$$\phi(x, z) = \frac{1}{2\pi} \int_{-\infty + i\beta}^{\infty + i\beta} \bar{\phi}_+(p, z)\, e^{-ipx}\, dp, \quad x \geqslant 0 \quad \ldots(36)$$

where $-k_2 < \beta < k_2$.

6. Reflected Waves

The factor $\exp(-ipx) = \exp(-i\alpha x) \exp(\beta x)$ in (36) makes the integral vanish $\beta \to -\infty$ in the lower part of the complex plane if $x \geqslant 0$. For waves in region $x \geqslant 0$, the contour is in the lower half of the complex plane where $\bar{\phi}_+(-p, z)$ is analytic and hence

$$\phi(x, z) = \frac{1}{2\pi} \int_{-\infty + i\beta}^{\infty + i\beta} [\bar{\phi}_+(p, z) + \bar{\phi}_+(-p, z)]\, e^{-ipx}\, dp, \quad x \geqslant 0. \quad \ldots(37)$$

The contour of integration excludes the poles $p = \pm k \cos\theta$ and includes the poles $p = \pm p_n$, $(n = 1, 2, 3, \ldots)$ where $\pm p_n$ are the roots of the period equation $f(p) = 0$.

Schermann[9] has shown that the period equation has a finite number of real roots $p = \pm p_n$, n denoting the nth normal mode of the generalised Rayleigh waves. The conditions Re $(Y) = 0$, Re $(Y_1) = 0$ and Re $(\delta_1) = 0$ have been discussed by Ewing *et al.*[10] to show that the branch cuts are hyperbolic. Im (Y), Im (Y_1) and Im (δ_1) change signs along the two sides of the branch cuts as indicated in Fig. 2. The line integral (37) is taken along the real axis ($\beta = 0$) of the complex plane. Applying Cauchy residue theorem to the closed contour results in

$$\int_{-\infty}^{\infty} + \int_{A}^{B} + \int_{p_{k_l}} + \int_{p_{k_u}} + \int_{p_k} + \int_{C}^{D} = 2\pi i \text{ (sum of residues)} \qquad \ldots(38)$$

where

$$\int_{-\infty}^{\infty} = \lim_{\epsilon_1 \to 0} \int_{-\infty}^{-p_n - \epsilon_1} + \int_{\Sigma_1} + \lim_{\epsilon_1, \epsilon_2 \to 0} \int_{-p_n + \epsilon_1}^{-k\cos\theta - \epsilon_2} + \int_{\Sigma_2} + \lim_{\epsilon_2 \to 0} \int_{-k\cos\theta + \epsilon_2}^{k\cos\theta - \epsilon_2}$$

$$+ \int_{\Sigma_3} + \lim_{\epsilon_1, \epsilon_2 \to 0} \int_{k\cos\theta + \epsilon_2}^{p_n - \epsilon_1} + \int_{\Sigma_4} + \lim_{\epsilon_1 \to 0} \int_{p_n + \epsilon_1}^{\infty} \qquad \ldots(39)$$

ϵ_1 and ϵ_2 being the radii of circular indentations Σ_1, Σ_2, Σ_3 and Σ_4 around $p = -p_n$, $p = p_n$, $p = -k\cos\theta$ and $p = k\cos\theta$ respectively. The contribution due to indentations at $p = \pm k\cos\theta$ are

$$-\tfrac{1}{2} D \sin(kz\sin\theta) \exp(-ikx\cos\theta) \qquad \ldots(40)$$

and

$$\tfrac{1}{2} D \sin(kz\sin\theta) \exp(ikx\cos\theta). \qquad \ldots(41)$$

And the contribution due to indentations at $p = \pm p_n$ are

$$-\sum_n \frac{i(-\lambda_1 k'^2 + 2\mu_1 Y_{1n}^2)\, g(p_n)\, \bar{\phi}'_{1+}(k') \sin Y_n z\, e^{-ip_n x}}{Y_n \lambda k^2 f'(p_n) \cos Y_n h},$$

$$Y_{1n} = [(p_n^2 - k'^2)]^{1/2} \qquad \ldots(42)$$

and

$$\sum_n \frac{i(-\lambda_1 k'^2 + 2\mu_1 Y_{1n}^2)\, g(-p_n)\, \bar{\phi}_{1+}(k') \sin Y_n z\, e^{ip_n x}}{\lambda Y_n k^2 f'(-p_n) \cos Y_n h},$$

$$Y_n = [(k^2 - p_n^2)]^{1/2} \qquad \ldots(43)$$

In (40), the wave is half the incident wave whereas (41) gives the wave reflected from the rigid plane boundary and then from the free surface of the ocean. In (42) and (43), we have the generalized Rayleigh waves multiply reflected from the boundaries of the ocean.

7. Scattered Waves

Let us evaluate the integral along the branch cut P_k. On P_k, Re $(Y) = 0$ and Im (Y) changes signs along two sides of the branch cut. It is found that the integrals along two sides cancel each other.

We now consider

$$I(x, z) = \frac{1}{2\pi} \int_{P_{k'}} [\bar{\phi}_+ (p, z) + \bar{\phi}_+ (-p, z)] e^{-ipx} dp. \qquad ...(44)$$

On $p_{k'}$, we use $p = -k' - iu$ such that $Y_1 = \pm i (2k_2' u)^{1/2} = \pm i\bar{Y}_1$, $k_1' = 0$. Integrating (44) along two sides of $p_{k'}$, it follows that

$$I(x, z) = \frac{i}{2\pi} \int_{p_{k'}} [[\bar{\phi}_+ (p, z) + \bar{\phi}_+ (-p, z)]_{Y_1 = -i\bar{Y}_1} - [\bar{\phi}_+ (p, z)$$

$$+ \bar{\phi}_+ (-p, z)]_{Y_1 = -i\bar{Y}_1}] e^{-k_2' x} e^{-ux} du. \qquad ...(45)$$

This is evaluated by using the result (Ewing *et al.*[10], p. 52)

$$\int_0^\infty \sqrt{u}\, G(u) e^{-ux} du = \frac{G(0)\, \Gamma(3/2)}{x^{3/2}} + \frac{G'(0)\, \Gamma(5/2)}{x^{5/2}} + \ldots \qquad ...(46)$$

where $\Gamma(x)$ is the Gamma function. The first term of (45) is obtained to be

$$I(x, z) = Cx^{-3/2 - k_2' x} \sin((k^2 + k_2'^2)^{1/2} z) \qquad ...(47)$$

where

$$C = \frac{2k''^2 H_1(0)}{\pi H_2(0)} \left[- \frac{\lambda_1 k^2 e^{i\pi/4} (k''^2 - 2k'^2)(k^2 + k_2'^2)^{1/2}}{\sqrt{-k'}} \right.$$

$$+ \lambda k^2 k''^2 (2k_2')^{1/2} \tan((k^2 + k_2'^2)^{1/2} h)$$

$$\left. - 4i \mu_1 k'^2 (k_2'^2 + k''^2)^{1/2} (k^2 + k_2'^2)^{1/2} (2k_2')^{1/2} \right] \qquad ...(48)$$

$$H_1(0) = \lambda_1 k'^2 \Gamma(3/2) \bar{\phi}_{1+}'(k') \qquad ...(49)$$

$$H_2(0) = (\lambda_1^2 k'^4)(k^2 + k_2'^2)(k''^2 - 2k'^2)^2 \cos((k^2 + k_2'^2)^{1/2} h). \qquad ...(50)$$

The scattered wave in (47) has the behaviour of a cylindrical wave close to the scatterer and a plane wave at distant points.

8. CONCLUSIONS

The potential for the incident, reflected, Rayleigh and scattered waves in (3), (40)-(43) and (47) vanishes on the free surface ($z = 0$) of the ocean and the boundary condition (6) is satisfied. It can be further established that the condition (7) and all other boundary conditions are satisfied by the potentials of various waves. The horizontal component u of the displacement excited by various waves is absent on the free surface of the ocean. Thus the particle motion in the surface of the ocean is at right angles to it and the horizontal component of the displacement is absent. The waves reflected from the rigid plane boundary and then from the surface of the ocean are obtained in (41) having exactly half the amplitude of the incident wave. Multiply reflected generalised Rayeigh waves in (42) - (43) and the scattered waves in (47) are plane waves. When z is small, the scattered waves are of the form of exp $(-k'_2 r)/\sqrt{r}$,r being the distance from the corner of the plane boundary. Near the scatterered, the near-field behaves as a cylindrical wave. The scatted waves propagate with the speeds of the waves in the solid and not with the speed of the waves in the ocean.

Numerical computations have been worked out for the scattered field and the generalised Rayleigh waves in the oceanic layer. For a basaltic ocean bottom,

$$\rho_2 = 3\rho_1,\ \alpha_2 = \sqrt{3}\beta_2,\ \beta_2 = 3\alpha_1 \text{ and } h = 5.7\text{km}.$$

Amplitude of the scattered wave is plotted versus the distance from the scatterer just below the free surface. It falls steadily as the waves proceed away from the scatterer (Fig. 3). The displacement excited by the scattered waves in the free surface falls rapidly

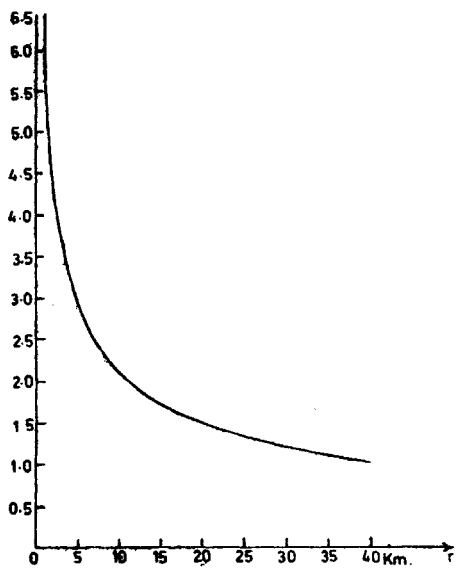

FIG. 3. Amplitude of the scattered wave just below the surface.

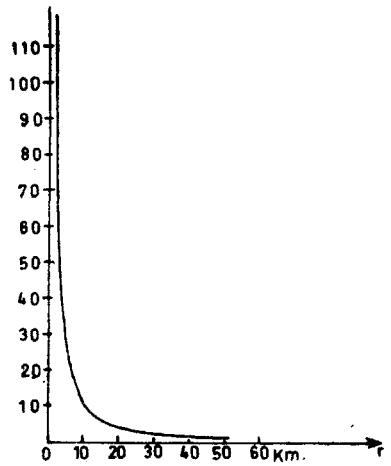

FIG. 4. Amplitude of the displacement excited by scattered waves in the free surface.

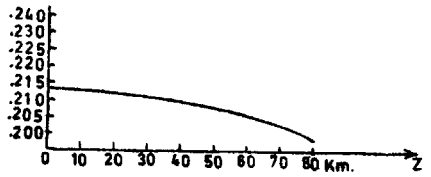

FIG. 5. Amplitude of the displacement excited by Rayleigh waves versus depth.

upto a distance of 10 km and then receds slowly when $\alpha_1 = 1.52$ km/sec, $\alpha_2 = 7.95$ km/sec. and $\beta_2 = 4.56$ km/sec. (Fig. 4). The amplitude of the displacement excited by Rayleigh waves is computed in terms of depth below the free surface and is found to fall down very slowly (Fig. 5).

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