

## THREE DIMENSIONAL CONVECTIVE FLOW AND HEAT TRANSFER IN A POROUS MEDIUM

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The effect of periodic variation of suction velocity on free convection flow and heat transfer through a porous medium is investigated. The problem becomes three dimensional due to variation of suction velocity in transverse direction on the wall. A series expansion method is used to get the solution of the governing equations and the expressions for velocity and temperature fields are obtained. The skin friction and the rate of heat transfer at the wall are analysed in detail.

### INTRODUCTION

The problem of laminar flow control has gained considerable importance in the field of Aeronautical Engineering in view of its application to reduce drag and hence the vehicle power requirement by a substantial amount. The development on this subject has been compiled by Lachmann<sup>1</sup>. Theoretical and experimental investigations have shown that the transition from laminar to turbulent flow which causes the drag coefficient to increase, may be prevented by the suction of fluid and heat transfer from boundary layer to the wall. Gersten and Gross<sup>2</sup> have studied the effect of transverse sinusoidal suction velocity on flow and heat transfer along an infinite vertical porous wall. The flow and heat transfer through a porous medium, however, has not received much attention despite its application in many branches of engineering. In this note, the effect of suction velocity variation on free convective flow and heat transfer in a porous medium is investigated.

### ANALYSIS

We consider free convection flow of a viscous incompressible fluid through a porous medium bounded by an infinite vertical porous wall. A coordinate system with the wall lying on  $x - z$  plane and  $y$ -axis perpendicular to it and directed into the fluid is introduced. The suction velocity distribution is taken in the form :

$$v_w(z) = v_0 \left[ 1 + \epsilon \cos \frac{\pi z}{L} \right] \quad \dots(1)$$

which consists of a basic steady distribution  $V_0 < 0$  with a superimposed weak transversally varying distribution  $\epsilon V_0 \cos \pi z/L$  of wave length  $L$ . Since the wall is infinite in  $x$ -direction, hence all physical quantities will be independent of  $x$ , however, the flow remains three dimensional due to variation of suction velocity. Let us denote velocity components  $u, v, w$  in  $x, y, z$  directions respectively and temperature by  $T$ . The governing equations are

$$v \frac{\partial v}{\partial y} + w \frac{\partial w}{\partial z} = 0 \quad \dots(2)$$

$$v \frac{\partial u}{\partial y} + w \frac{\partial u}{\partial z} = g\beta(T - T_\infty) + v \left( \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \right) - \frac{vu}{K} \quad \dots(3)$$

$$v \frac{\partial v}{\partial y} + w \frac{\partial v}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + v \left( \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial z^2} \right) - \frac{vv}{K} \quad \dots(4)$$

$$v \frac{\partial w}{\partial y} + w \frac{\partial w}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial z} + v \left( \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} \right) - \frac{vw}{K} \quad \dots(5)$$

$$v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} = \alpha \left( \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) \quad \dots(6)$$

In eqn. (3), variation in density is taken into account only in the derivation of the buoyancy force while other density variations are neglected within the frame-work of constant property fluid. The last terms in eqns. (3), (4) and (5) account for the pressure drop across the porous material. It is worthwhile to mention that the basic flow in the medium is entirely due to buoyancy force, caused by temperature difference between the wall and the medium.

The boundary conditions of the problem are :

$$\left. \begin{aligned} y = 0 : u = 0, v = v_w(z), w = 0, T = T_w \\ y \rightarrow \infty : u = 0, w = 0, p = p_\infty, T = T_\infty \end{aligned} \right\} \quad \dots(7)$$

When the amplitude  $\epsilon$  of oscillations in suction velocity is small, we assume the main flow velocity  $u$  in the following form

$$u = u_0 + \epsilon u_1 + \epsilon^2 u_2 + \dots \quad \dots(8)$$

The similar equations hold for other variables  $v, w, p$  and  $\theta = \frac{T - T_\infty}{T_w - T_\infty}$ . When  $\epsilon = 0$ , the problem reduces to two-dimensional free convection flow in an infinite porous medium with constant suction velocity at the wall. In this case eqns. (2) to (6) reduce to

$$\frac{\partial v_0}{\partial y} = 0 \quad \dots(9)$$

$$v_0 \frac{du_0}{dy} = g\beta (T_w - T_\infty) \theta_0 + v \frac{d^2u_0}{dy^2} - \frac{vu_0}{K} \quad \dots(10)$$

$$v_0 \frac{d\theta_0}{dy} = \alpha \frac{d^2\theta_0}{dy^2} \quad \dots(11)$$

The solution of these equations is

$$u_0 = \frac{Gv}{Lp_1} (e^{-m\bar{y}} - e^{-PR\bar{y}}), v_0 = v_0, w_0 = 0 \quad \dots(12)$$

$$\theta_0 = e^{-PR\bar{y}}, p_0 = p_\infty \quad \dots(13)$$

where

$$\bar{Y} = y/L, P = v/\alpha, G = \frac{g\beta (T_w - T_\infty) L^3}{v^2}, P_1 = R^2 (P^2 - P) - K_1$$

$$R = -\frac{LV_0}{v}, K_1 = \frac{L^2}{K}, m = \frac{R}{2} + (R^2/4 + K_1)^{1/2}.$$

When  $\epsilon \neq 0$ , the series expansion (8) is substituted in equations (2) to (6) and like powers of  $\epsilon$  are equated to get the perturbation equations of various order in  $\epsilon$ . For small values of  $\epsilon$ , it is sufficient to consider the perturbation equations only of  $o(\epsilon)$ , which are

$$\frac{\partial v_1}{\partial y} + \frac{\partial w_1}{\partial z} = 0 \quad \dots(14)$$

$$v_0 \frac{\partial u_1}{\partial y} + v_1 \frac{\partial u_0}{\partial y} = \frac{Gv^2}{L^3} \theta_1 + v \left( \frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^2 u_1}{\partial z^2} \right) - \frac{v}{K} u_1 \quad \dots(15)$$

$$v_0 \frac{\partial y_1}{\partial y} = -\frac{1}{\rho} \frac{\partial P_1}{\partial y} + v \left( \frac{\partial^2 y_1}{\partial y^2} + \frac{\partial^2 v_1}{\partial z^2} \right) - \frac{v}{K} v_1 \quad \dots(16)$$

$$v_0 \frac{\partial w_1}{\partial y} = -\frac{1}{\rho} \frac{\partial p_1}{\partial z} + v \left( \frac{\partial^2 w_1}{\partial y^2} + \frac{\partial^2 w_1}{\partial z^2} \right) - \frac{v}{K} w_1 \quad \dots(17)$$

$$v_0 \frac{\partial \theta_1}{\partial y} + v_1 \frac{\partial \theta_0}{\partial y} = \alpha \left( \frac{\partial^2 \theta_1}{\partial y^2} + \frac{\partial^2 \theta_1}{\partial z^2} \right) \quad \dots(18)$$

with boundary conditions

$$\left. \begin{aligned} y = 0 : u_1 = 0, v_1 = V_0 \cos \pi \frac{z}{L}, w_1 = 0, \theta_1 = 0 \\ y \rightarrow \infty : u_1 = 0, w_1 = 0, p_1 = 0, \theta_1 = 0. \end{aligned} \right\} \quad \dots(19)$$

This is a set of linear partial differential equations which describe the three-dimensional cross flow.

First of all we shall consider the eqns. (14), (16) and (17). The solutions for  $v_1(y, z)$ ,  $w_1(y, z)$  and  $p_1(y, z)$  are independent of main flow component  $u_1$  and temperature field  $\theta_1$ . We now assume  $v_1$ ,  $w_1$  and  $p_1$  as

$$v_1(\mathcal{Y}, \bar{z}) = V_0 \pi v_{11}(\mathcal{Y}) \cos \pi \bar{z} \quad \dots(20)$$

$$w_1(\mathcal{Y}, \bar{z}) = -v_0 v_{11}(\mathcal{Y}) \sin \pi \bar{z} \quad \dots(21)$$

$$p_1(\mathcal{Y}, \bar{z}) = \rho V_0^2 p_{11}(\mathcal{Y}) \cos \pi \bar{z} \quad \dots(22)$$

where  $\bar{z} = z/L$  and prime denotes differentiation with respect to  $\mathcal{Y}$ .

Equations (20) and (21) have been chosen so that the continuity equation (14) is satisfied. Substituting these expressions into (16) and (17) and applying the corresponding transformed boundary conditions, we get the values of  $v_1$ ,  $w_1$  and  $p_1$  as :

$$v_1(\mathcal{Y}, \bar{z}) = \frac{V_0}{\pi - \lambda} (\pi e^{-\lambda \bar{y}} - \lambda e^{-\pi \bar{y}}) \cos \pi \bar{z} \quad \dots(23)$$

$$w_1(\mathcal{Y}, \bar{z}) = \frac{\lambda V_0}{\pi - \lambda} (e^{\lambda \bar{y}} - e^{-\pi \bar{y}}) \sin \pi \bar{z} \quad \dots(24)$$

$$p_1(\mathcal{Y}, \bar{z}) = N_2 \rho V_0^2 \left( \frac{\lambda}{\pi - \lambda} \right) e^{-\pi \bar{y}} \cos \pi \bar{z} \quad \dots(25)$$

where

$$N_2 = 1 + \frac{K_1}{R\pi} \text{ and } \lambda = \frac{R}{2} (R^2/4 + \pi^2 K_1)^{1/2}.$$

Assuming  $u_1$  and  $\theta_1$  of the form

$$u_1(\mathcal{Y}, \bar{z}) = u_{11}(\mathcal{Y}) \cos \pi \bar{z} \quad \dots(26)$$

$$\theta_1(\mathcal{Y}, \bar{z}) = \theta_{11}(\mathcal{Y}) \cos \pi \bar{z} \quad \dots(27)$$

and substituting in eqns. (15) and (18), we get

$$\begin{aligned} u_1(\mathcal{Y}, \bar{z}) = & \frac{v}{L} \frac{G}{P_1} \frac{R}{\pi - \lambda} \left[ - \left( \frac{\pi}{2\lambda N_4} - \frac{\lambda}{\pi N_3} - A_1 + A_2 + A_3 \right) e^{-\lambda \bar{y}} \right. \\ & + \frac{\pi}{2\lambda N_4} e^{-(\lambda+m)\bar{y}} - \frac{\lambda}{\pi N_3} e^{-(\pi+m)\bar{y}} - A_1 e^{-(\lambda+PR)\bar{y}} \\ & \left. + A_2 e^{-(\pi+PR)\bar{y}} + A_3 e^{-\lambda \bar{y}} \right] \cos \pi \bar{z} \quad \dots(28) \end{aligned}$$

$$\theta_1(\mathcal{Y}, \bar{z}) = \frac{P^2 R}{\pi - \lambda} \left[ \frac{\pi}{P_2} e^{-(\lambda+PR)\bar{y}} - \frac{\lambda}{\pi P} e^{-(\pi+PR)\bar{y}} - \frac{A_3}{P_3} e^{-\lambda \bar{y}} \right] \cos \pi \bar{z} \quad \dots(29)$$

where

$$P_2 = \lambda(1 + P) + K_1/R, P_3 = \frac{P_1 P^2}{R \bar{\lambda}(P-1) - K_1}$$

$$N_3 = \frac{1}{m} (R^2 + 4K_1)^{1/2}$$

$$N_4 = 1 + K_1/2m\lambda, A_1 = \frac{\pi(1 + PP_1/RP_2)}{PR + 2\lambda - R}$$

$$A_2 = \frac{\lambda P(R + P_1/\pi)}{P^2 R^2 + 2\pi PR - R^2 P - \pi R - K_1}$$

$$A_3 = P_3(\pi/P_2 - \lambda/\pi P), \bar{\lambda} = \frac{PR}{2} + (\pi^2 + P^2 R^2/4)^{1/2}.$$

### RESULTS AND DISCUSSIONS

We now discuss the important flow characteristics of the problem. The expression for shear stress along the direction of  $x$  is obtained as

$$\begin{aligned} C_{fx} &= \frac{\tau_x}{\rho v^2/L^2} = \frac{\mu}{\rho} \left( \frac{\partial u}{\partial y} \right)_{y=0} \\ &= \frac{G}{P_1} (PR - m) + \epsilon GF_1(P, R) \cos \pi \bar{z} \end{aligned} \quad \dots(30)$$

where

$$\begin{aligned} F_1(P, R) &= \frac{R}{P_1(\pi - \lambda)} \left[ -\frac{\pi m}{2\lambda N_4} + \frac{\lambda}{\pi N_3} (\pi + m - \lambda) \right. \\ &\quad \left. + A_1 PR + A_2 (\lambda - PR - \pi) + A_3 (\lambda - \bar{\lambda}) \right] \end{aligned} \quad \dots(31)$$

The skin friction factor  $F_1(P, R)$  in the limiting cases,  $R \rightarrow 0$  and  $R \rightarrow \infty$ , tends to zero. Equation (31) is numerically evaluated for different values of  $R$  and permeability parameter  $K_1$  and the results are shown in Fig. 1. From Fig. 1, it is obvious that  $F_1$  tends to its limiting values as  $R \rightarrow 0$  and  $R \rightarrow \infty$  for each  $K_1$ . It is also seen that  $F_1$  increases with  $R$  until it attains a maximum value, after which it decreases and approaches to zero. It has also been observed that, as the value of  $K_1$  is increased, the skin friction factor  $F_1$  decreases significantly.

The heat flux at the wall in terms of Nusselt number  $Nu$  is given by

$$\begin{aligned} Nu &= \frac{-q_w}{\rho V_0 c_p (T_w - T_\infty)} = \frac{k}{\rho V_0 c_p} \left( \frac{\partial \theta}{\partial y} \right)_{y=0} \\ &= 1 + \epsilon [1 - F_2(P, R)] \cos \pi \bar{z} \end{aligned} \quad \dots(32)$$

where

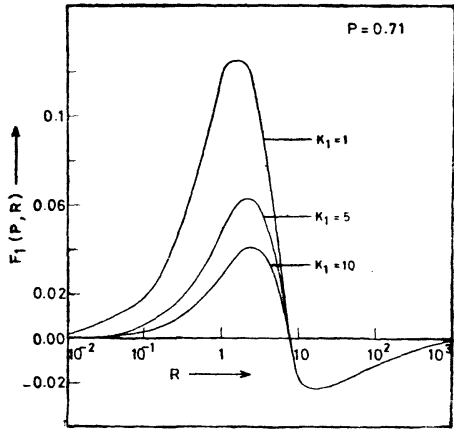


FIG. 1. Variation of skin friction factor  $F_1$  with  $R$  for different values of permeability parameter  $K_1$ .

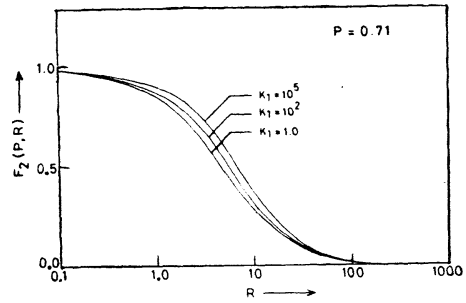


FIG. 2. Variation of heat transfer factor  $F_2$  with  $R$  for different values of permeability parameter  $K_1$ .

$$F_2(P, R) = \frac{1}{\lambda - \pi} \left[ \lambda \left( \frac{\lambda}{\pi} - \frac{\pi P}{P_2} \right) + \frac{\pi P}{P_2} (\lambda + PR) - \frac{\lambda PR}{\pi} - \pi \right] \dots(33)$$

It is found that  $F_2$  takes constant values for limiting values of  $R$ . When  $R \rightarrow 0$ ,  $F_2 \rightarrow 1$  which is obvious because there is no oscillatory flow. However, when  $R \rightarrow \infty$ ,  $F_2 \rightarrow 0$  which shows that heat transfer approaches to quasi-steady value. These results are shown in Fig. 2. It is interesting to note that  $F_2(P, R)$  increases with permeability parameter  $K_1$ .

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