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TRANSIENT MAGNETOTHERMOELASTIC WAVES IN A HALF-SPACE WITH THERMAL RELAXATIONS

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The distribution of temperature, deformation and magnetic field in a homogeneous isotropic, thermally and perfectly electrically conducting, elastic half-space, in contact with the vacuum, has been investigated by taking (i) a step in stress and (ii) a thermal shock at the plane boundary, in the context of Green-Lindsay theory of thermoelasticity. The Laplace transform on time has been used to obtain the solutions. Because the "second sound" effects are short-lived, so the small time approximations have been considered. The deformation and temperature are found to be continuous at the wavefronts whereas the magnetic field is found to be discontinuous in the case of normal load. But these quantities are discontinuous in the case of thermal shock.

1. INTRODUCTION

The magnetothermoelastic disturbances in a perfectly conducting elastic half-space, in contact with vacuum, due to an applied thermal disturbance on the plane boundary were studied by Kaliski and Nowacki[1] in the absence of coupling between temperature and strain fields. The problem[1] was also considered by Massalas and Dalmangas[2] by taking into account thermomechanical couplings. The problem[2] was then extended to generalized thermoelasticity theory developed by Green and Lindsay[3] involving two relaxation times by Chatterjee and Roychoudhuri[4].

In the present paper the distributions of deformation, temperature, and perturbed magnetic field, from (i) a normal load and (ii) a thermal shock acting on the boundary of an half-space, are obtained by employing generalised theory of thermoelasticity developed by Green and Lindsay[3]. The Laplace transform[5] technique is used to obtain the solutions. As the "second sound" effects are short lived, so small time approximations have been considered.

2. THE PROBLEM AND ITS SOLUTION

We consider a homogeneous isotropic thermally conducting elastic medium at uniform temperature T_0, in contact with vacuum. We suppose that an initial magnetic field is acting along x_3-axis in both the media. The simplified linear equations of electrodynamics of slowly moving bodies for a perfectly homogeneous conducting elastic medium are

$$\vec{\nabla} \times \vec{h} = \frac{4\pi}{c}\vec{J}, \quad \vec{\nabla} \times \vec{E} = -\frac{\mu_0}{c}\dot{\vec{h}}$$

$$\vec{\nabla}\cdot\vec{h} = 0, \quad \vec{E} = \frac{-\mu_0}{c}(\dot{\vec{u}} \times \vec{H}_0) \qquad \ldots(1)$$

where $\vec{h}$ denotes the perturbation of magnetic field, $\vec{J}$ is the elastic current density vector, $\vec{E}$ the electricfield, $\vec{H}_0$ the initial magnetic field, $\vec{u}$ the displacement vector, μ_0 is the magnetic permeability, and c the velocity light. The superposed dot represents the differentiation with respect to time.

The equations of motion and heat conduction in the context of Green and Lindsay theory[3] of thermoelasticity are

$$\mu\nabla^2\vec{u} + (\lambda + \mu)\vec{\nabla}\,\vec{\nabla}\cdot\vec{u} + \frac{\mu_0}{4\pi}[(\vec{\nabla}\times\vec{h})\times\vec{H}_0]$$

$$-\gamma(\vec{\nabla}\theta + \alpha\,\vec{\nabla}\dot{\theta}) = \rho\ddot{\vec{u}} \qquad \ldots(2)$$

and

$$\rho C_v(\dot{\theta} + \alpha^*\ddot{\theta}) + \gamma T_0\dot{\Delta} = K\theta_{,ii}\ (i, j = 1, 2, 3). \qquad \ldots(3)$$

where λ, μ are the Lame' constants, $\gamma = (3\lambda + 2\mu)\alpha_T$, α_T the coefficient of linear thermal expansion, $\theta = T - T_0$, T the absolute temperature, T_0 the uniform temperature of the body in its natural state, $K = \lambda_T C_\varepsilon$, λ_T represents the coefficient of heat conduction, C_ε the specific heat at constant strain, ρ the mass density, C_v the specific heat at constant volume, and α, α^* are thermal relaxation times.

For $H_0 = (0, 0, \vec{H}_3)$, eqns. (1) become

$$\vec{E} = \frac{\mu_0 H_3}{c}(0, \dot{u}_1, 0),\ \vec{h} = -\frac{c}{\mu_0}\left(0, 0, \frac{\partial E_2}{\partial x_1}\right),$$

$$\vec{J} = \frac{c}{4\pi}\left(0, -\frac{\partial h_3}{\partial x_1}, 0\right), \qquad \ldots(4)$$

and eqns. (2) and (3) become

$$(\lambda + 2\mu + a_0^2\rho)\frac{\partial^2 u_1}{\partial x_1^2} - \gamma\left(\frac{\partial\theta}{\partial x_1} + \alpha\frac{\partial^2\theta}{\partial x_1\,\partial t}\right) = \rho\ddot{u}_1 \qquad \ldots(5.1)$$

$$\rho C_v\left(\frac{\partial\theta}{\partial t} + \alpha^*\frac{\partial^2\theta}{\partial t^2}\right) + \gamma T_0\frac{\partial^2 u_1}{\partial x_1\,\partial t} = K\frac{\partial^2\theta}{\partial x_1^2} \qquad \ldots(5.2)$$

where $a_0 = \sqrt{(\mu_0 H_3^2/4\pi\rho)}$ is the Alfven wave velocity.

For convenience, we shall use notations $u_1 = u$, $x_1 = x$. In vacuum, the system of equations of electrodynamics is expressed as

$$\left(\frac{\partial^2}{\partial x'^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right) h_3^0 = 0, \left(\frac{\partial^2}{\partial x'^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right) E_2^0 = 0 \quad \text{...(5.3)}$$

where $x' = -x$.

The components of Maxwell's stress tensors in elastic medium T_{11}, in vacuum T_{11}^0

$$T_{11} = T_{ij}\,\Big|_{i=j=1} = \frac{\mu_0}{4\pi}[h_i H_j + h_j H_i - \delta_{ij}(\vec{h}.\vec{H})]_{i=j=1}$$

$$= -\frac{\mu_0}{4\pi} h_3 H_3 \quad \text{...(6.1)}$$

$$T_{11}^0 = T_{ij}^0\,\Big|_{i=j=1} = \frac{1}{4\pi}\quad[h_i^0 H_j + h_j^0 H_i - \delta_{ij}(\vec{h}.\vec{H})]_{i=j=1}$$

$$= -\frac{h_3^0 H_3}{4\pi} \quad \text{...(6.2)}$$

$$\sigma_{11} = (\lambda + 2\mu)\frac{\partial u}{\partial x} - \gamma(\theta + \alpha\dot{\theta}). \quad \text{...(7)}$$

Normal load at the Boundary

The boundary conditions in the case of normal load acting at the plane boundary are given by

$$\sigma_{11} + T_{11} - T_{11}^0 = \sigma_0 H(t), \text{ at } x = x' = 0 \quad \text{...(8)}$$

$$E_2 = E_2^0, \text{ at } x = x' = 0 \quad \text{...(9)}$$

$$\theta(0, t) = 0, \text{ at } x = x' = 0 \quad \text{...(10)}$$

where $H(t)$ is Heaviside function.

Introducing the dimensionless quantities

$$\eta = c_0 x \mid k,\ \tau = c_0^2 t/k,\ U = (\lambda + 2\mu + a_0^2\rho)u/\gamma T_0 k,$$

$$Z = \theta/T_0,\ \epsilon = \gamma^2 T_0/C_\epsilon(\lambda + 2\mu + a_0^2\rho),\ \alpha' = \alpha\omega^*,\ \alpha^{*\prime} = \alpha^*\omega^*$$

where

$$c_1^2 = (\lambda + 2\mu)/\rho,\ c_0^2 = a_0^2 + c_1^2,\ k = K/\rho C_v,\ \omega^* = \rho C_v c_0^2/K$$

$$C_\epsilon = \rho C_v.$$

Equations (5.1), (5.2) and (5.3) become

$$\frac{\partial^2 U}{\partial\eta^2} - \frac{\partial Z}{\partial\eta} - \alpha'\frac{\partial^2 Z}{\partial\eta\,\partial\tau} - \frac{\partial^2 U}{\partial\tau^2} = 0 \text{ for } \eta > 0 \quad \text{...(11)}$$

$$\frac{\partial^2 Z}{\partial\eta^2} - \frac{\partial Z}{\partial\tau} - \alpha^{*\prime}\frac{\partial^2 Z}{\partial\tau^2} - \epsilon\frac{\partial^2 u}{\partial\eta\,\partial\tau} = 0 \text{ for } \eta > 0 \quad \text{...(12)}$$

$$\left(\frac{\partial^2 h_3^0}{\partial \eta'^2} - \beta^2 \frac{\partial^2 h_3^0}{\partial \tau^2}\right) = 0 \text{ for } \eta' > 0 \qquad \ldots(13)$$

where

$$\eta' = -\eta, \; \beta = c_0/c.$$

The initial conditions

$$u(x, 0) = 0, \; \theta(x, 0) = 0, \; \frac{\partial u}{\partial x}(x, 0) = 0,$$

in the new variables become

$$U(\eta, 0) = 0, \; Z(\eta, 0) = 0, \; \frac{\partial u}{\partial \eta}(\eta, 0) = 0. \qquad \ldots(14)$$

The boundary conditions (8), (9) and (10) become

$$\frac{\partial U}{\partial \eta} - Z - \alpha' \frac{\partial Z}{\partial \tau} + \beta_1 h_3^0 H(\tau)/\gamma T_0 = 0 \text{ at } \eta = \eta' = 0 \qquad \ldots(15)$$

$$\beta_2 \frac{\partial^2 U}{\partial \tau^2} - \frac{\partial h_3^0}{\partial \eta'} = 0, \text{ at } \eta = \eta' = 0 \qquad \ldots(16)$$

where

$$\left.\begin{aligned} h_3 &= -H_3 \frac{\partial u}{\partial x} \\ \beta_2 &= \mu_0 H_3 \gamma T_0/\rho c_0^2 \\ \beta_1 &= H_3/4\pi\gamma T_0 \\ Z(0, \tau) &= 0. \end{aligned}\right\} \qquad \ldots(17)$$

and

We consider the potential function ϕ defined by

$$U = \frac{\partial \phi}{\partial \eta}. \qquad \ldots(18)$$

Putting (18) into (11) and (12), we get

$$Z(\eta, \tau) + \alpha' \frac{\partial Z}{\alpha \tau} = \left(\frac{\partial^2}{\partial \eta^2} - \frac{\partial^2}{\partial \tau^2}\right) \phi \text{ for } \eta > 0 \qquad \ldots(19)$$

and

$$\frac{\partial^2 Z}{\partial \eta^2} - \frac{\partial Z}{\partial \tau} - \alpha^{*\prime} \frac{\partial^2 Z}{\partial \tau^2} - \epsilon \frac{\partial^3 \phi}{\partial \tau \, \partial \eta^2} = 0 \text{ for } \eta > 0. \qquad \ldots(20)$$

Applying Laplace transform to eqns. (19), (20) and (13) defined by

$$\bar{f}(s) = \int_0^\infty f(t) e^{-st} dt \qquad \ldots(21)$$

we obtain

$$(1 + \alpha' s)\, \bar{Z} = \left(\frac{\partial^2}{\partial \eta^2} - s^2 \right) \bar{\phi} \text{ for } \eta > 0 \qquad \ldots(22)$$

$$\left(\frac{\partial^2}{\partial \eta^2} - s - \alpha^{*\prime} s^2 \right) \bar{Z} = \epsilon\, s \, \frac{\partial^2 \bar{\phi}}{\partial \eta^2} \quad \text{for } \eta > 0 \qquad \ldots(23)$$

$$\bar{h}_3^0 = C_3\, e^{-\beta_3 \eta'} \text{ for } \eta' > 0. \qquad \ldots(24)$$

Using (18) into (14), (15) and (16), we obtain

$$\phi(\eta, 0) = 0, \; \frac{\partial \phi}{\partial \eta}(\eta, 0) = 0, \; \frac{\partial^2 \phi}{\partial \eta^2}(\eta, 0) = 0 \qquad \ldots(25)$$

$$\frac{\partial^2 \phi}{\partial \eta^2} - \left(Z + \alpha' \, \frac{\partial Z}{\partial \tau} \right) + \beta_1 h_3^0 - \sigma_0 H(\tau)/\gamma T_0 = 0 \text{ for } \eta = 0 \qquad \ldots(26)$$

$$\beta_2 \, \frac{\partial^3 \phi}{\partial \tau^2 \, \partial \eta} - \frac{\partial h_3^0}{\partial \eta'} = 0 \text{ for } \eta = \eta' = 0. \qquad \ldots(27)$$

Applying Laplace transform to eqns. (26), (27) and (17), we get

$$\frac{\partial^2 \bar{\phi}}{\partial \eta^2} - (1 + \alpha' s)\, \bar{Z} + \beta_1 \, \bar{h}_3^0 - \sigma_0/\gamma T_0 \, s = 0 \text{ at } \eta = 0 \qquad \ldots(28)$$

$$\beta_2 \, s^2 \, \frac{\partial \bar{\phi}}{\partial \eta} - \frac{\partial \bar{h}_3^0}{\partial \eta'} = 0 \text{ at } \eta = \eta' = 0 \qquad \ldots(29)$$

$$\bar{Z}(0, s) = 0 \text{ at } \eta = 0. \qquad \ldots(30)$$

Eliminating $\bar{Z}$ from eqns. (22) and (23), we get

$$\frac{\partial^4 \bar{\phi}}{\partial \eta^4} - [1 + \epsilon + s(1 + \alpha^{*\prime} + \epsilon \alpha')]\, s \, \frac{\partial^2 \bar{\phi}}{\partial \eta^2} + s^3 (1 + s\alpha^{*\prime})\, \bar{\phi} = 0 \text{ for } \eta > 0. \qquad \ldots(31)$$

The general solution of (31) which vanishes at $\eta \to \infty$ is given by

$$\bar{\phi}(\eta, s) = C_1 e^{-\lambda_1 \eta} + C_2 e^{-\lambda_2 \eta} \text{ for } \eta > 0 \qquad \ldots(32)$$

where λ_1, λ_2 are the roots of eqn. (33)

$$\lambda^4 - s[1 + \epsilon + s(1 + \alpha^{*\prime} + \epsilon\, \alpha')]\, \lambda^2 + s^3 (1 + \alpha^{*\prime} s) = 0. \qquad \ldots(33)$$

From equations (22) and (32), we get

$$\bar{Z}(\eta, s) = \frac{1}{(1 + \alpha' s)} [C_1 (\lambda_1^2 - s^2)\, e^{-\lambda_1 \eta} + C_2 (\lambda_2^2 - s^2)\, e^{-\lambda_2 \eta}] \text{ for } \eta > 0 \qquad \ldots(34)$$

Using (32) into (28), (29) and (30), we get

$$s^2 (C_1 + C_2) + \beta_1 C_3 = \sigma_0/\gamma T_0 s \text{ at } \eta = \eta' = 0 \quad ...(35)$$

$$\beta_2 s (C_1 \lambda_1 + C_2 \lambda_2) - \beta C_3 = 0 \text{ at } \eta = \eta' = 0 \quad ...(36)$$

$$C_1 (\lambda_1^2 - s^2) + C_2 (\lambda_2^2 - s^2) = 0 \text{ at } \eta = \eta' = 0. \quad ...(37)$$

From equations (35), (36) and (37), we obtain

$$C_1 = - \sigma_0 \beta (\lambda_2^2 - s^2)/\gamma T_0 s^2 A, C_2 = \sigma_0 \beta (\lambda_1^2 - s^2)/\gamma T_0 s^2 A,$$

$$C_3 = \sigma_0 \beta_2 (\lambda_1 - \lambda_2)(\lambda_1 \lambda_2 + s^2)/\gamma T_0 sA \quad ...(38)$$

where

$$A = (\lambda_1 - \lambda_2) [\beta_1 \beta_2 s^2 + \beta s (\lambda_1 + \lambda_2) + \beta_1 \beta_2 \lambda_1 \lambda_2]. \quad ...(39)$$

The equations (38), (32), (34) and (24) provide us

$$\bar{\phi} (\eta, s) = \frac{\sigma_0 \beta [(\lambda_1^2 - s^2) e^{-\lambda_2 \eta} - (\lambda_2^2 - s^2) e^{-\lambda_1 \eta}]}{\gamma T_0 (\lambda_1 - \lambda_2) [\beta_1 \beta_2 s^2 + \beta s (\lambda_1 + (\lambda_2) + \beta_1 \beta_2 \lambda_1 \lambda_2]} \quad \eta > 0 \quad ...(40)$$

$$\bar{Z} (\eta, s) = \frac{\sigma_0 \beta (\lambda_1^2 - s^2)(\lambda_2^2 - s^2)(e^{-\lambda_1 \eta} - e^{-\lambda_2 \eta})}{\gamma T_0 s^2 (1 + \alpha' s)(\lambda_1 - \lambda_2) [\beta_1 \beta_2 s^2 + \beta s (\lambda_1 + \lambda_2) + \beta_1 \beta_2 \lambda_1 \lambda_2]} \quad \eta > 0 \quad ...(41)$$

$$\bar{h}_3^0 (\eta', s) = \frac{\sigma_0 \beta_2 (\lambda_1 \lambda_2 + s^2) e^{-\beta \eta' s}}{\gamma T_0 s [\beta_1 \beta_2 s^2 + \beta s (\lambda_1 + \lambda_2) + \beta_1 \beta_2 \lambda_1 \lambda_2]} \quad \eta' > 0. \quad ...(42)$$

The transformed displacement is given by

$$\bar{U} (\eta, s) = \frac{\sigma_0 \beta [\lambda_1 (\lambda_2^2 - s^2) e^{-\lambda_1 \eta} - \lambda_2 (\lambda_1^2 - s^2) e^{-\lambda_2 \eta}]}{\gamma T_0 s^2 (\lambda_1 - \lambda_2) [\beta_1 \beta_2 s^2 + \beta s (\lambda_1 + \lambda_2) + \beta_1 \beta_2 \lambda_1 \lambda_2]} \quad \eta > 0. \quad ...(43)$$

3. Small Time Approximations

The dependence of λ_1, λ_2 on s is very complicated and hence the inversion of the Laplace transform is difficult. These difficulties, however, reduce if we use some approximate methods. As the thermal relaxation effects are short-lived so we confine our discussions to small time approximations, i. e., we take s large. A similar approach was used by Sharma[6] to study the thermal shock problem in generalised theory[7] of thermoelasticity. Now the roots λ_1 and λ_2 of eqn. (33) are given by

$$\lambda_{1,2} = s V_{1,2}^{-1} + B_{1,2} + D_{1,2} (1/s) + O (1/s^2) \quad ...(44)$$

where

$$V_{1,2}^{-1} = (K_2 \pm \Gamma^{1/2})^{1/2}/\sqrt{2} \quad ...(45a)$$

$$B_{1,2} = [K_1 \pm (K_1 K_2 - 2)/\sqrt{2}]/2\sqrt{2}\,(K_2 \pm \Gamma^{1/2})^{1/2} \qquad \text{...(45b)}$$

$$D_{1,2} = \{\pm K_1^2/\Gamma^{1/2} \mp (K_1 K_2 - 2)^2/\Gamma^{3/2} - (K_1 \pm (K_1 K_2 - 2)/\Gamma^{1/2})^2/$$
$$2\,(K_2 \pm \Gamma^{1/2})\}/4\sqrt{2}\,(K_2 \pm \Gamma^{1/2})^{1/2} \qquad \text{...(45c)}$$

and

$$\Gamma = K_2^2 - 4\alpha^{*\prime} = (1 + \epsilon\alpha' - \alpha^{*\prime})^2 + 4\,\epsilon\,\alpha'\,\alpha^{*\prime} \qquad \text{...(45d)}$$

$$K_1 = 1 + \epsilon,\ K_2 = 1 + \epsilon\,\alpha' + \alpha^{*\prime}. \qquad \text{...(45e)}$$

Again $(1 + \epsilon\alpha' + \alpha^{*\prime})^2 > \Gamma$ so that $1/V_1^2 > 1/V_2^2$ or $V_1 < V_2$.

Thus V_1 corresponds to the speed of the slowest wave and V_2 to that of fastest wave. Therefore, the points of the solid for which $\eta > V_2\,\tau$ do not experience any disturbance. Also from equations (45) we see that as $\alpha' = \alpha^{*\prime} = 0$, $V_1 \to 1$ and $V_2 \to \infty$. But this corresponds to the case of conventional coupled theory of thermoelasticity, which predicts an infinite speed of heat propagation. Thus we conclude that the wave propagating with speed V_1 must be elastic and that propagating with speed V_2 is the thermal wave. The third wave travelling with velocity c_0 as the Alfven acoustic wave.
Equations (40), (41), (42), and (43) with the help of (44) provide us

$$\bar{\phi}(\eta, s) = \frac{\sigma_0\beta}{\gamma T_0}\Bigg[\Bigg\{\frac{(1 - V_1^2)}{V_1^2}\,\frac{P'}{s^3} + \left(\frac{2B_1 P'}{V_1} + \frac{Q'(1 - V_1^2)}{V_1^2}\right)\frac{1}{s^4}$$
$$+ O\left(\frac{1}{s^5}\right)\Bigg\}e^{-\lambda_2\eta} - \Bigg\{\frac{(1 - V_2^2)\,P'}{V_2^2\,s^3} + \Bigg(\frac{2B_2\,P'}{V_2}$$
$$+ \frac{Q'(1 - V_2^2)}{V_2^2}\Bigg)\frac{1}{s^4} + O\left(\frac{1}{s^5}\right)\Bigg\}e^{-\lambda_1\eta}\Bigg] \qquad \text{...(46)}$$

$$\bar{Z}(\eta, s) = \frac{\sigma_0\,\beta}{\gamma T_0\alpha'}\Bigg[\frac{M_1 P'}{s^2} + \left(M_1 Q' + M_2 P' - \frac{M_1 P'}{\alpha'}\right)\frac{1}{s^3}$$
$$+ O\left(\frac{1}{s^4}\right)\Bigg]\left[e^{-\lambda_2\eta} - e^{-\lambda_1\eta}\right] \qquad \text{...(47)}$$

$$\bar{h}_3^0(\eta, s) = \frac{\sigma_0\,\beta_2}{\gamma\,T_0}\Bigg[\left(\frac{1 + V_1 V_2}{V_1 V_2}\right)\frac{P}{s} + \Bigg\{\left(\frac{1 + V_1 V_2}{V_1 V_2}\right)Q$$
$$+ \frac{(B_1 V_1 + B_2 V_2)P}{V_1 V_2}\Bigg\}\frac{1}{s^2} + O\left(\frac{1}{s^3}\right)\Bigg]e^{-B_3\eta'} \qquad \text{...(48)}$$

$$\bar{U}(\eta, s) = \frac{\sigma_0\,\beta}{\gamma T_0}\Bigg[\Bigg\{\left(\frac{1 - V_2^2}{V_1 V_2^2}\right)\frac{P'}{s^2} + \Bigg\{\left(\frac{B_1(1 - V_2^2)}{V_2^2} + \frac{2B_2}{V_1 V_2}\right)P'$$
$$+ \left(\frac{1 - V_2^2}{V_1 V_2^2}\right)Q'\Bigg\}\frac{1}{s^3} + O\left(\frac{1}{s^4}\right)\Bigg\}e^{-\lambda_1\eta}$$
$$- \Bigg\{\left(\frac{1 - V_1^2}{V_1^2 V_2}\right)\frac{P'}{s^2} + \Bigg\{\left(\frac{B_2(1 - V_1^2)}{V_1^2} + \frac{2B_1}{V_1 V_2}\right)P'$$
$$+ \left(\frac{1 - V_1^2}{V_1^2 V_2}\right)Q'\Bigg\}\frac{1}{s^3} + O\left(\frac{1}{s^4}\right)\Bigg\}e^{-\lambda_2\eta}\Bigg] \qquad \text{...(49)}$$

where

$$P = \frac{1}{\beta_1 \beta_2}\Bigg[1 + \frac{\beta^2 (V_1 + V_2)^2}{\beta_1^2 \beta_2^2 V_1^2 V_2^2} + \frac{2\beta (V_1 + V_2)}{\beta_1 \beta_2 V_1^2 V_2^2} - \frac{\beta^3 (V_1 + V_2)^3}{\beta_1^3 \beta_2^3 V_1^3 V_2^3}$$
$$- \frac{1}{V_1 V_2} + \frac{1}{V_1^2 V_2^2} + \frac{\beta^4 (V_1 + V_2)^4}{\beta_1^4 \beta_2^4 V_1^4 V_2^4}$$
$$- \frac{3\beta^2 (V_1 + V_2)^2}{\beta_1^2 \beta_2^2 V_1^3 V_2^3} - \frac{\beta (V_1 + V_2)}{\beta_1 \beta_2 V_1 V_2}\Bigg] \qquad \ldots(50)$$

$$Q = \frac{1}{\beta_1 \beta_2}\Bigg[\frac{2\beta (B_1 + B_2)(V_1 + V_2)}{\beta_1^2 \beta_2^2 V_1 V_2} - \frac{(B_1 V_1 + B_2 V_2)}{V_1 V_2}$$
$$+ \frac{2\beta}{\beta_1 \beta_2}\left\{ \frac{(V_1 + V_2)(B_1 V_1 + B_2 V_2)}{V_1^2 V_2^2} + \frac{(B_1 + B_2)}{V_1 V_2} \right\}$$
$$- \frac{3\beta^3 (V_1 + V_2)^2 (B_1 + B_2)}{\beta_1^3 \beta_2^3 V_1^2 V_2^2} + \frac{2 (B_1 V_1 + B_2 V_2)}{V_1 V_2} - \frac{3\beta}{\beta_1^2 \beta_1^2}$$
$$\times \left\{ \frac{(V_1 + V_2)^2 (B_1 V_1 + B_2 V_2)}{V_1^3 V_2^3} + \frac{2 (B_1 + B_2)(V_1 + V_2)}{V_1^2 V_2^2} \right\}$$
$$- \frac{4\beta^4 (V_1 + V_2)^3 (B_1 + B_2)}{\beta_1^4 \beta_2^4 V_1^3 V_2^3} - \frac{\beta (B_1 + B_2)}{\beta_1 \beta_2} \Bigg] \qquad \ldots(51)$$

$$\left.\begin{aligned} M_1 &= \frac{1 + V_1 V_2}{V_1 V_2} - \frac{(V_1^2 + V_2^2)}{V_1^2 V_2^2} \\ M_2 &= \frac{2B_2}{V_1^2 V_2^2} - \frac{2 (B_1 V_2 + B_2 V_1)}{V_1 V_2} \end{aligned}\right\} \qquad \ldots(52)$$

and

$$P' = V_1 V_2 P/(V_2 - V_1),\ Q' = [V_1 V_2 Q/V_2 - V_1)] - [V_1^2 V_2^2 (B_1 - B_2) P/(V_2 - V_1)^2]. \qquad \ldots(53)$$

Inverting the Laplace transforms for small times, i.e., for large s, eqns. (46), (47), (48) and (49) provide us

$$\phi(\eta, \tau) = \frac{\sigma_0 \beta}{\gamma T_0}\Bigg[\Bigg\{\left(\frac{1 - V_1^2}{V_1^2}\right) P' (\tau - \eta/V_2)^2 H(\tau - \eta/V_2)$$
$$+ \left(\frac{2B_1 P'}{V_1} + \frac{1 - V_1^2}{V_1^2} Q'\right)(\tau - \eta/V_2)^3 H(\tau - \eta/V_2)\Bigg\} e^{-\beta_2 \eta}$$
$$- \Bigg\{\left(\frac{1 - V_2^2}{V_2^2}\right) P' (\tau - \eta/V_1)^2 H(\tau - \eta/V_1)$$
$$+ \left(\frac{2B_2 P'}{V_2} + \frac{1 - V_2^2}{V_2^2} Q'\right)(\tau - \eta/V_1)^3 H(\tau - \eta/V_1)\Bigg\}$$
$$e^{-\beta_1 \eta}\Bigg] \qquad \ldots(54)$$

$$Z(\eta, \tau) = \frac{\sigma_0 \beta}{\gamma T_0 \alpha'}[\{P' M_1 (\tau - \eta/V_2) H(\tau - \eta/V_2) + (M_1 Q' + M_2 P' - \frac{M_1 P'}{\alpha'})(\tau - \eta/V_2)^2 H(\tau - \eta/V_2)\} e^{-B_2 \eta} - \{P' M_1 (\tau - \eta/V_1) H(\tau - \eta/V_1) + (M_1 Q' + M_2 P' - \frac{M_1 P'}{\alpha'})(\tau - \eta/V_1)^2 H(\tau - \eta/V_1)\} e^{-B_1 \eta}] \quad \ldots(55)$$

$$h_3^0(\eta', \tau) = \frac{\sigma_0 \beta_2}{\gamma T_0}\Big[\Big(\frac{1 + V_1 V_2}{V_1 V_2}\Big) P H(\tau - \beta\eta') + \Big\{\Big(\frac{1 + V_1 V_2}{V_1 V_2}\Big) \times Q + \Big(\frac{B_1 V_1 + B_2 V_2)P}{V_1 V_2}\Big\} (\tau - \beta\eta') H(\tau - \beta\eta')\Big] \quad \ldots(56)$$

$$U(\eta, \tau) = \frac{\sigma_0 \beta}{\gamma T_0}\Big[\Big\{\Big(\frac{1 - V_2^2}{V_1 V_2^2}\Big) P' (\tau - \eta/V_1) H(\tau - \eta/V_1) + \Big\{\Big(\frac{B_1(1 - V_2^2)}{V_2^2} + \frac{2B_2}{V_1 V_2}\Big) P' + \frac{1 - V_2^2}{V_2^2} Q'\Big\} \times (\tau - \eta/V_1)^2 H(\tau - \eta/V_1)\Big\} e^{-B_1 \eta} - \Big\{\Big(\frac{1 - V_1^2}{V_1^2 V_2}\Big) P' (\tau - \eta/V_2) H(\tau - \eta/V_2) + \Big\{\frac{B_1(1 - V_1^2)}{V_1^2} + \frac{2B_2}{V_1 V_2}\Big) P' + \frac{1 - V_1^2}{V_1^2} Q'\Big\} (\tau - \eta/V_2) H(\tau - \eta/V_2)\Big\} e^{-B_2 \eta}\Big] \quad \ldots(57)$$

4. Thermal Shock at the Boundary

The thermal shock $\theta(0, t) = \theta_0 H(t)$ also produces disturbances in the elastic medium in the presence of magnetic field. In this case the boundary conditions are given by

$$\left.\begin{aligned} &\sigma_{11} + T_{11} - T_{11}^0 = 0 \text{ at } x = x' = 0 \\ &E_2 = E_2^0 \text{ at } x = x' = 0 \\ &\theta(0, t) = \theta_0 H(t) \text{ at } x = x' = 0 \end{aligned}\right\} \quad \ldots(58)$$

The transformed potential function $\bar{\phi}$, temperature $\bar{Z}$, induced magentic field $\bar{h}_3^0$, and displacement $\bar{U}$, are obtianed as

$$\bar{\phi}(\eta, s) = \frac{\theta_0 (1 + \alpha' s)[(s\beta + \beta_1 \beta_2 \lambda_2) e^{-\lambda_1 \eta} - (s\beta + \beta_1 \beta_2 \lambda_1) e^{-\lambda_2 \eta}]}{s T_0 (\lambda_1 - \lambda_2)[\beta_1 \beta_2 s^2 + \beta s(\lambda_1 + \lambda_2) + \beta_1 \beta_2 \lambda_1 \lambda_2]}$$
for $\eta > 0$ ...(59)

$$\bar{Z}(\eta, s) = \frac{\theta_0 [(\lambda_1^2 - s^2)(\beta s + \beta_1\beta_2\lambda_2) e^{-\lambda_1\eta} - (\lambda_2^2 - s^2)(\beta s + \beta_1\beta_2\lambda_1) e^{-\lambda_2\eta}]}{sT_0 (\lambda_1 - \lambda_2)[\beta_1\beta_2 s^2 + \beta s(\lambda_1 + \lambda_2) + \beta_1\beta_2\lambda_1\lambda_2]}$$
$$\text{for } \eta > 0 \qquad \ldots(60)$$

$$\bar{h}_3^0(\eta', s) = \frac{\theta_0 (1 + \alpha' s)\beta_2 s\, e^{-B_3\eta'}}{T_0[\beta_1\beta_2 s^2 + \beta s(\lambda_1 + \lambda_2) + \beta_1\beta_2\lambda_1\lambda_2]} \text{ for } \eta' > 0 \qquad \ldots(61)$$

$$\bar{U}(\eta, s) = \frac{\theta_0 (1 + \alpha' s)[\lambda_2(\beta s + \beta_1\beta_2\lambda_1) e^{-\lambda_2\eta} - \lambda_1(\beta s + \beta_1\beta_2\lambda_2) e^{-\lambda_1\eta}]}{T_0 s(\lambda_1 - \lambda_2)[\beta_1\beta_2 s^2 + \beta s(\lambda_1 + \lambda_2) + \beta_1\beta_2\lambda_1\lambda_2]}$$
$$\text{for } \eta > 0. \qquad \ldots(62)$$

Inverting the Laplace transform for small times, eqns. (59), (60), (61), and (62) provide us

$$\phi(\eta, \tau) = \frac{\theta_0}{T_0}\Big[\Big\{P^*(\tau - \eta/V_1) H(\tau - \eta/V_1) + Q^*(\tau - \eta/V_1)^2$$
$$\times H(\tau - \eta/V_1)\Big\} e^{-B_1\eta} - \{P^{*\prime}(\tau - \eta/V_2) H(\tau - \eta/V_2)$$
$$+ Q^{*\prime}(\tau - \eta/V_2)^2 H(\tau - \eta/V_2)\} e^{-B_2\eta}\Big] \qquad \ldots(63)$$

$$Z(\eta, \tau) = \frac{\theta_0}{T_0}[\{P' N_1 H(\tau - \eta/V_1) + (N_1 Q' + N_2 P')(\tau - \eta/V_1)$$
$$H(\tau - \eta/V_1)\} e^{-B_1\eta} - \{P' N_1' H(\tau - \eta/V_2) + (N_1' Q'$$
$$+ N_2' P')(\tau - \eta/V_2) H(\tau - \eta/V_2)\} e^{-B_2\eta}] \qquad \ldots(64)$$

$$h_3^0(\eta', \tau) = \frac{\theta_0 \beta_2}{T_0}[\alpha' P\,\delta(\tau - \beta\eta') + (P + Q\alpha') H(\tau - \beta\eta')] \qquad \ldots(65)$$

$$U(\eta, \tau) = \frac{\theta_0}{T_0}[\{M_1' H(\tau - \eta/V_2) + M_2'(\tau - \eta/V_2) H(\tau - \eta/V_2)\} e^{-B_2\eta}$$
$$- \{M_1'' H(\tau - \eta/V_1) + M_2''(\tau - \eta/V_1) H(\tau - \eta/V_1)\} e^{-B_1\eta}]$$
$$\ldots(66)$$

where

$$P^* = P'(\beta_1\beta_2\alpha' + \alpha'\beta V_2)/V_2,\ Q^* = [P'(\beta V_2 + \beta_1\beta_2 B_2 V_2 + \beta_1\beta_2)/V_2]$$
$$+ Q'(\beta_1\beta_2\alpha' + \alpha'\beta V_2)/V_2,$$

and

$$P^{*\prime} = P'(\beta_1\beta_2\alpha' + \alpha'\beta V_1)/V_1,\ Q^{*\prime} = [P'(\beta V_1 + \beta_1\beta_2 B_1 V_1$$
$$+ \beta_1\beta_2)/V_1] + Q'(\beta_1\beta_2\alpha' + \alpha'\beta V_1)/V_1$$
$$N_1 = [\beta V_2 + \beta_1\beta_2(1 - V_1^2)]/V_1^2 V_2,\ N_2 = [2B_1(\beta V_2 + \beta_1\beta_2)/V_1 V_2]$$
$$+ (1 - V_1^2)\beta_1\beta_2 B_2/V_1^2 V_2$$

$$N_1' = (\beta V_1 + \beta_1 \beta_2 (1 - V_2^2)]/V_1 V_2^2, \; N_2' = [2B_2 (\beta V_1 + \beta_1\beta_2)/V_1 V_2] + (1 - V_2^2)\beta_1 \beta_2 B_1/V_1V_2^2$$

$$M_1' = \beta \alpha' P'/V_2, \; M_2' = [\beta (P' + \alpha' Q')/V_1] + \alpha' P' (\beta_1 \beta_2 B_1 V_1 + \beta_1 \beta_2 V_1V_2\beta_1\beta_2V_2)/V_1 V_2$$

$$M_1'' = \beta \alpha' P'/V_1, \; M_2'' = [\beta (P' + \alpha' Q')/V_1] + \alpha' P'(\beta_1 \beta_2 B_2 V_2 + \beta_1 \beta_2 V_1 V_2 + \beta_1 \beta_2 B_1 V_1)/V_1 V_2.$$

The stresses in the vacuum and the elastic medium can be easily obtained by using various expressions in eqns. (6) and (7).

5. Discussion of the Results

The short time solutions above show that they consist of three waves, i. e., the elastic wave, thermal wave, and Alfven-acoustic wave travelling with velocities V_1, V_2, and c_0 respectively. The terms containing $H(\tau - \eta/V_1)$ represent the contribution of the elastic wave in vicinity of its wavefront ($\eta = V_1 \tau$), the terms with $H(\tau - \eta/V_2)$ represent the contribution of the thermal wave in the vicinity of its wavefront ($\eta = V_2 \tau$), and those with $H(\tau - \beta\eta')$ represent the contribution of the Alfven acoustic wave in the vicinity of its wavefront ($\eta' = \tau/\beta$). The displacement and temperature are found to be continuous at the wave fronts and the perturbed magnetic is discontinuous in case of normal load. The discontinuty is given by

$$\left[h_3^{0+} - h_3^{0-}\right]_{\eta'=\tau/\beta} = \sigma_0 \beta_2 (1 + V_1 V_2) P/V_1 V_2 \gamma T_0.$$

In case of thermal shock the deformation, temperature, and the perturbed magnetic field are all found to be discontinuous and the jumps at the wave-fronts are given by

$$[U^+ - U^-]_{\eta=V_1\tau} = -\theta_0 M_1'' \exp(-B_1 V_1 \tau)/T_0,$$

$$[U^+ - U^-]_{\eta=V_2\tau} = \theta_0 M_1' \exp(-B_2 V_2 \tau)/T_0,$$

$$[Z^+ - Z^-]_{\eta=V_1\tau} = \theta_0 N_1 P' \exp(-B_1 V_1 \tau)/T_0$$

$$[Z^+ - Z^-]_{\eta=V_2\tau} = -\theta_0 N_1' P' \exp(-B_2 V_2 \tau)/T_0$$

$$\left[h_3^{0+} - h_3^{0-}\right]_{\eta'=\tau/\beta} = \theta_0 \beta_2 (P + Q \alpha')/T_0.$$

Clearly the discontinuities in deformation and temperature decay exponentially with time. In case of conventional coupled theory of thermoelasticity $\alpha = \alpha^* = 0$ and hence

$$K_1 = 1 + \epsilon,\ K_2 = 1,\ V_1 = 1,\ V_2 \to \infty,\ \Gamma = 1,\ B_1 = \epsilon/2$$

$$B_2 \to \infty,\ D_1 = \epsilon\,(4 - \epsilon)/8,\ D_2 \to \infty.$$

It is observed that the perturbed magnetic field experiences finite and infinite jumps in case of normal load and thermal shock, respectively. In case of normal load the displacement and temperature are found to be continuous at both the wave fronts whereas in case of thermal shock these quantities are continuous at the thermal wave front but experience finite jumps at the elastic wave fronts.

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