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UNSTEADY FLOW OF AN ELECTRICALLY CONDUCTING DUSTY VISCOUS LIQUID BETWEEN TWO PARALLEL PLATES

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The present problem conisders the two dimensional laminar flow of an electrically conducting viscous incompressible fluid with embedded non-conducting identical spherical particles through a long channel under the influence of a uniform magnetic field and a time varying pressure gradient, taking the fluid and dust particles to be 'initially' at rest. The changes in velocity profiles for liquid and dust particles have been determined and the effect of the strength of the magnetic field on velocity profiles at fixed time has been depicted graphically. Some particular cases have also been given.

1. INTRODUCTION

The influence of dust particles on viscous flows has a great importance in petroleum industry and in the purification of crude oil. Other important applications of dust particles in boundary layer, include soil solvation by natural winds and dust entrainment in a cloud during nuclear explosion. Due to the importance of dusty viscous flows in other technological fields various studies have appeared in the literature.

Saffman[2] studied the stability of the animal flow of a dusty gas with uniform distribution of dust particles. Michael[3] considered the Kelvin-Helmholtz instability of the dusty gas. Liu[4] discussed the flow induced by an oscillating infinite plate in a dusty gas. Miller[5] have discussed the plane parallel flow of a dusty gas.

Singh[6] and Ram[7] have investigated the problems relating to flow of electrically conducting dusty viscous liquid, taking pressure gradient as constant.

FORMULATION OF THE PROBLEM

In the present discussions, the dust particles are assumed to be spherical in shape and are uniformly distributed. To make the analysis simple, some other assumptions have also been taken :

(1) Chemical reaction, mass transfer and radiation between the particles and fluid are not considered. (2) The temperature is uniform within particles. (3) The interaction between particles themseleves has not been considered. (4) The flow is fully developed (i.e., The velocity profile in the channel is independent of the axial

coordinate of the channel). (5) The buoyancy force has been neglected. (6) The number density of the dust particles is constant throughcut the motion. (7) We neglect the displacement current in MHD flows as the flow velocity is very small than the velocity of light ($v \ll c$). (8) The induced magnetic field has been neglected. (9) There is nω external applied electric field. (10) The magnetic field is considered fixed relative to the channel. (11) The Hall-effect is assumed to be negligible. (12) The fluid is electrically neutral, i.e. there is no surplus electric charge distribution present in the fluid. (13) Only the electromagnetic body forces (such as Lorentz force) are present. (14) Fluid properties (density, viscosity) are invariable. (15) Viscous dissipation in negligible.

Governing Equations

The x-axis is taken along the plates and y-axis normal to these. Let the distance between the two plates be $2h$, the axis of the channel is chosen from the origin as shown in Fig. 1. Therefore, based on Saffman's model of dusty viscous incompres-

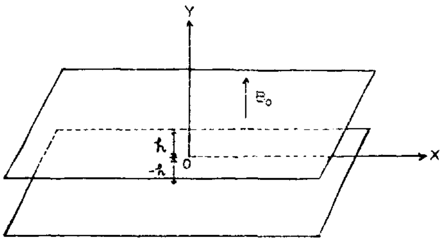

FIG. 1

FIG. 1

sible fluid and Prandtl's boundary layer assumptions, the governing equations of the flow are :

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2} + \frac{KN_0}{\rho}(v-u) - \frac{\sigma B_0^2 u}{\rho} \quad \text{.. (1)}$$

$$0 = \frac{\partial p}{\partial y} \quad \text{.. (2)}$$

$$m\frac{\partial v}{\partial t} = K(u-v) \quad \text{...(3)}$$

where u is the velocity of the fluid; p the pressure, ν the kinematic coefficient of viscosity; K the stakes resistance coefficient (which for spherical particles of radius a is $6\pi\mu a$); N_0 the constant number density of dust particles; σ the conductivity of the fluid; B_0 the magnitude of the magnetic induction $\vec{B}$; μ the coefficient of viscosity; m the mass of dust particles; and t the time.

Equation (6) shows that the pressure is independent of y.

SOLUTION OF THE PROBLEM

Case A

(i) When $f(t) = Q\,e^{-\lambda t}$, Q is constant, $\lambda > 0$ and both the plates are at rest. In this case, eqns. (1) to (3) are to be solved subject to the initial and boundary conditions as :

at

$$t = 0, u = v = 0$$

for

$$t > 0, u = v = 0 \text{ at } y = \pm h. \qquad \ldots(4)$$

Introducing the following non-dinesional quantities :

$$x' \equiv x/h,\ y' = y/h,\ t' = \nu t/h^2,\ u' = uh/\nu$$
$$v' = vh/\nu,\ p' = ph^2/\rho\nu^2 \qquad \ldots(5)$$

and letting $\lambda' = \lambda h^2/\nu$, eqns. (5) and (7), after removing the primes reduce to,

$$\frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} + \frac{\partial^2 u}{\partial y^2} + \frac{l}{w}(v - u) - M^2 u \qquad \ldots(6)$$

$$w\frac{\partial v}{\partial t} = u - v \qquad \ldots(7)$$

where $l = mN_0/\rho$, $w = m\nu/Kh^2$, and $M = B_0 h\sqrt{\sigma/\mu}$ (Hartmann number). The boundary conditions are :

for

$$t = 0, u = 0, v = 0$$

for

$$t > 0, u = 0, v = 0 \text{ at } y = -1 \qquad \ldots(8)$$
$$u = v = 0 \quad \text{at } y = 1$$

putting $-\dfrac{\partial p}{\partial x} = Qe^{-\lambda t}$ and applying the Laplace's transform to equations (6) and (7):

$$s\,\bar{u} = \frac{Q}{\lambda + s} + \frac{d^2\bar{u}}{dy^2} + \frac{l}{w}(\bar{v} - \bar{u}) - M^2\,\bar{u} \qquad \ldots(9)$$

$$ws\,\bar{v} = \bar{u} - \bar{v} \qquad \ldots(10)$$

where we define the Laplace's transform of u as :

$$\bar{u} = \int_0^\infty u\,e^{-st}\,dt \text{ and } \bar{v} = \int_0^\infty ve^{-st}\,dt.$$

The boundary conditions (8) are now transformed to,

$$\left.\begin{aligned} &\text{at } t = 0, \quad \bar{u} = 0, \quad \bar{v} = 0 \\ &\text{at } t > 0, \quad \bar{u} = 0, \quad \bar{v} = 0 \text{ at } y = -1 \\ &\qquad\qquad \bar{u} = 0, \quad \bar{v} = 0 \text{ at } y = 1. \end{aligned}\right\} \qquad ...(11)$$

Eliminating $\bar{v}$ from eqn. with the help of eqn. (10), we get,

$$\frac{d^2\bar{u}}{dy^2} - p^2\,\bar{u} + \frac{Q}{\lambda + s} = 0 \qquad ...(12)$$

where

$$p^2 = \frac{sl + (s + M^2)(1 + ws)}{1 + ws}. \qquad ...(13)$$

On solving eqn. (12) subject to the boundary conditions (11), we get,

$$\bar{u} = \frac{Q\,(1 + ws)}{(\lambda + s)\,[sl + (s + M^2)(1 + ws)]}\left[1 - \frac{\cosh py}{\cosh p}\right] \qquad ...(14)$$

$$\bar{v} = \frac{Q}{(\lambda + s)\,[sl + (s + M^2)(1 + ws)]}\left[1 - \frac{\cosh py}{\cosh p}\right]. \qquad ...(15)$$

Applying invesion theorem,

$$u = \frac{1}{2\pi i}\int_{\gamma - i\infty}^{\gamma + i\infty} \frac{Q\,(1 + ws)\,e^{st}}{(\lambda + s)\,[sl + (s + M^2)(1 + ws)]}\left[1 - \frac{\cosh py}{\cosh p}\right] ds \qquad ...(16)$$

where γ is so large that all the singularities of $\bar{u}$ lie to the left of the line $(\gamma + i\infty, \gamma - i\infty)$. The path $(\gamma - i\infty, \gamma + i\infty)$ may be replaced by an arc of a circle c of radius R containing all the poles

$$s = -\lambda,\ p = \frac{\pi i\,(2n + 1)}{2},\ n = 0, 1, 3...$$

of the integrand as shown in Fig. 2 (see Carslaw and Jaeger[1], §§ 31) and u is equal to the sum of the residues at these poles.

Following this procedure, u can be computed as :

$$\frac{u}{Q} = \frac{e^{-\lambda t}}{q^2}\left[1 - \frac{\cosh qy}{\cosh q}\right] + \frac{4}{\pi}\sum_{n=0}^{\infty} \frac{(-1)^n\,(1 - w\,\alpha_1)^2\,e^{-\alpha_1 t}\,P}{(2n+1)\,(\lambda - \alpha_1)\,(l + 1 - 2w\alpha_1 + w^2\alpha_1^2)}$$

$$+ \frac{4}{\pi}\sum_{n=0}^{\infty} \frac{(-1)^n\,(1 - w\,\alpha_2)^2\,e^{-\alpha_2 t}\,p}{(2n + 1)\,(\lambda - \alpha_2)\,(l + 1 - 2w\,\alpha_2 + w\,\alpha_2^2)} \qquad ...(17)$$

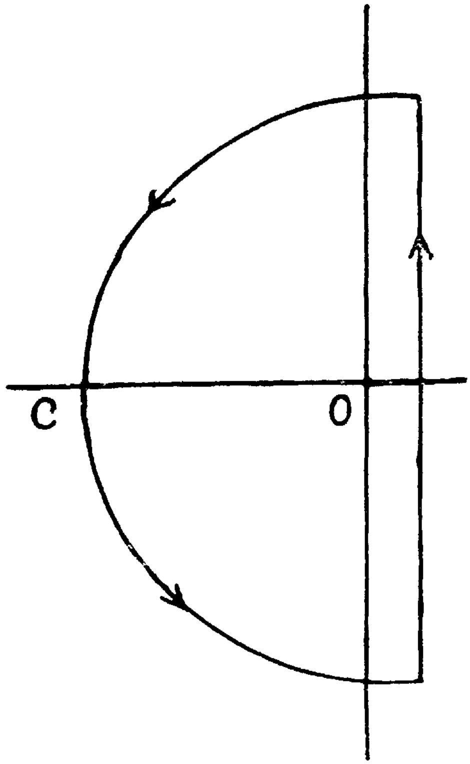

FIG. 2

and velocity profile for dust particles is given by,

$$\frac{v}{Q} = \frac{e^{-\lambda t}}{-\lambda l + (M^2 - \lambda)(1 - \lambda w)} \left[1 - \frac{\cosh qy}{\cosh q}\right]$$

$$+ \frac{4}{\pi} \sum_{n=0}^{\infty} \frac{(-1)^n (1 - w\alpha_1) e^{-\alpha_1 t} p}{(2n+1)(\lambda - \alpha_1)(l + 1 - 2w\alpha_1 + w^2 \alpha_1^2)}$$

$$+ \frac{4}{\pi} \sum_{n=0}^{\infty} \frac{(-1)^n (1 - w\alpha_2) e^{-\alpha_2 t}}{(2n+1)(\lambda - \alpha_2)(l + 1 - 2w\alpha_2 + w^2\alpha_2^2)} p \quad \ldots(18)$$

where

$$q^2 = \frac{(M^2 - \lambda)(1 - \lambda w) - \lambda l}{1 - \lambda w} \quad \ldots(19)$$

$$2w\alpha_1 = l + 1 + wM^2 + \frac{(2n+1)^2 \pi^2}{4} w + X \quad \ldots(20)$$

$$2w\alpha_2 = l + 1 + wM^2 + \frac{(2n+1)^2\pi^2}{4}w - X \qquad \ldots(21)$$

with

$$X = \sqrt{[l + 1 + wM^2 + \frac{(2n+1)^2\pi^2}{4}w]^2 - [4wM^2 + (2n+1)^2\pi^2 w]}$$

$$p = \cos\frac{(2n+1)\pi}{2}y.$$

(ii) When $f(t) = Q\,e^{-\lambda t}$ and the lower plate is stationary and upper plate is moving with uniform velocity U.

(iii) When $f(t) = Q\,e^{-\lambda t}$ and both the plates are moving in the same direction with a uniform velocity U.

The solutions can be obtained following the same procedure.

Case B

(i) When $f(t) = A\cos\Omega t$, A is constant, $\Omega > 0$, and both the plates are at rest.

In this case,

$$\frac{u}{A} = \frac{e^{i\Omega t}}{2r_1^2}\left[1 - \frac{\cosh r_1 y}{\cosh r_1}\right] + \frac{e^{-i\Omega t}}{2r_1^2}\left[1 - \frac{\cosh r_2 y}{\cosh r_2}\right]$$

$$- \frac{4}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^n\alpha_1(1 - w\alpha_1)^2 e^{-\alpha_1 t}}{(2n+1)(\Omega^2 + \alpha_1^2)(l + 1 - 2w\alpha_1 + w^2\alpha_1^2)}p$$

$$- \frac{4}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^n\alpha_2(1 - w\alpha_2)^2 e^{-\alpha_2 t}}{(2n+1)(\Omega^2 + \alpha_2^2)(l + 1 - 2w\alpha_2 + w^2\alpha_2^2)}p \qquad \ldots(22)$$

and the velocity profile for dust particles is given by

$$\frac{v}{A} = \frac{e^{\Omega it}}{2r_1^2(1 + i\Omega w)}\left[1 - \frac{\cosh r_1 y}{\cosh r_1}\right] + \frac{e^{-i\Omega t}}{2r_2^2(1 - i\Omega w)}$$

$$\times\left[1 - \frac{\cosh r_2 y}{\cosh r_2}\right]$$

$$- \frac{4}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^n\alpha_1(1 - w\alpha_1)e^{-\alpha_2 t}}{(2n+1)(\Omega^2 + \alpha_1^2)(l + 1 - 2w\alpha_1 + w^2\alpha_1^2)}p$$

$$- \frac{4}{\pi}\sum_{n=0}^{\infty}\frac{(-1)^n\alpha_2(1 - w\alpha_2)e^{-\alpha_2 t}}{(2n+1)(\Omega^2 + \alpha_2^2)(l + 1 - 2w\alpha_2 + w^2\alpha_2^2)}p \ldots(23)$$

where

$$r_1^2 = \frac{i\Omega + (M \quad + i\Omega)(1 + iw\Omega)}{+ iw\Omega} \qquad \ldots(24)$$

$$r_2^2 = \frac{-i\Omega + (M^2 - i\Omega)(1 - i\Omega w)}{1 - iw\Omega}. \qquad \ldots(25)$$

(ii) When $f(t) = A \cos \Omega t$ and lower plate is stationary and upper plate is moving with a uniform velocity U.

(iii) When $f(t) = A \cos \Omega t$ and both the plates are moving in the same direction with a uniform velocity U.

Solutions can be obtained as per procedure following earlier.

Particular Case—If in Case A, we take $\lambda = 0$ or in Case B, $\Omega = 0$ then all the results will be reduced for a constant pressure gradient.

Conclusion

Figures 3 and 4 show the velocity profiles for the liquid and particles at a fixed Hartmann number when both the plates are at rest and $f(t)\, Q\, e^{-\lambda t}$. From these graphs it can easily be seen that the maximum velocities for liquid and dust particles occur at $t = 1.5$ and $t = 3.0$ respectively. After that, velocities for liquid and dust particles decrease and reach to zero for very large values of t which is desirables in physical situation. The velocity is symmetrical with the centre of the channel.

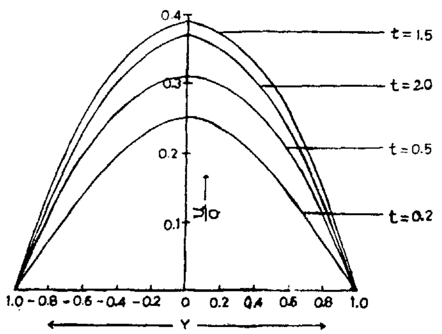

FIG. 3 Velocity profile for liquid at different times and fixed value of Hartmann number (l=0.2, ω=0.8, M=0.5 8λ=0.1)

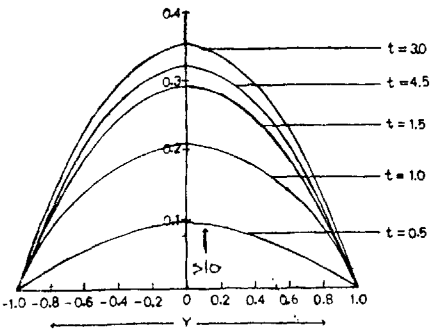

FIG. 4 Velocity profile for dust particles at different times and fixed value of hartmann number (l=0.2, ω=0.8, M=0.5 8λ=0.1).

Figures 5 and 6 show the effect of the strength of the magnetic field on velocity profiles at fixed time. It is seen that the Hartmann number is not favourable to the

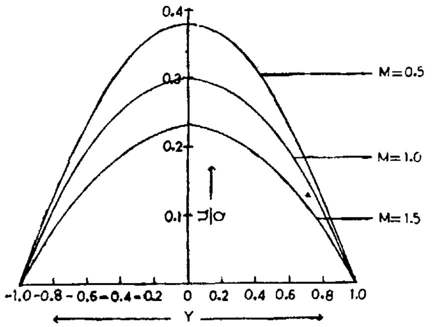

FIG. 5 Velocity profile for liquid at fixed time different values of Hartmann number (l=0.2, ω=0.8, t=1.0 & λ=0.1).

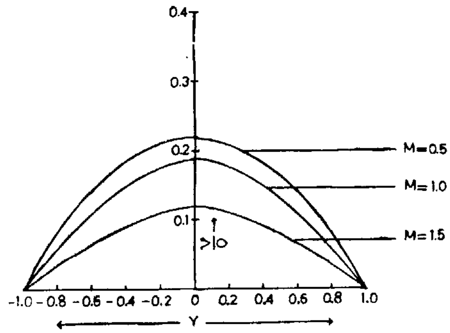

FIG. 6 Velocity profile for dust particles at fixed time & different values of Hartmann number (l=0.2, ω=0.8, t=1.0 & λ=0.1).

velocity field, i.e., for a constant valve of t, the velocitp profiles for liquid and dust particle decrease as the Hartmann number increases.

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