Indian J. pure appl. Math., **14**(12) : 1474–1483, December 1983

SLOW UNIFORM FLOW OF A MICROPOLAR FLUID PAST A PROLATE SPHEROID

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(*Received* 15 *February* 1983)

The slow uniform motion of an incompressible viscous micropolar fluid past a prolate spheroid is studied by the singularity method.

1. INTRODUCTION

Ramkissoon and Majumdar (1976) studied the Stokes' flow of a micropolar fluid past a sphere. Using the Boundary value method, they obtained the expressions for stream function, pressure field, micro-rotation and the drag on the sphere. Chwang and Wu (1975) investigated the Stokes flow of a viscous fluid in a uniform free stream past a prolate spheroid.

In this paper, the slow steady flow of an incompressible viscous micropolar fluid past a prolate spheroid is studied. The exact solutions are obtained by the singularity method for Stokes' flow as employed by Chwang and Wu (1975). The flow is studied for small values of the coupling constant. The approximate expressions for velocity field, pressure field, the microrotation and the drag force on the spheroid are obtained.

2. EQUATIONS OF MOTION

The equations governing the Stokes' flow of an incompressible micropolar fluid are (Ramkissoon and Majumdar 1976),

$$\left.\begin{aligned} &\nabla . \mathbf{u} = 0, \\ &-(\mu+k)\,\nabla\wedge\nabla\wedge\mathbf{u} + k\,\nabla\wedge\underline{\omega} - \nabla p + \rho\,\mathbf{F} = 0, \\ &(\alpha_1+\alpha_2+\alpha_3)\,\nabla\,(\nabla . \underline{\omega}) - \alpha_3\,\nabla\wedge\nabla\wedge\omega + k\nabla\wedge\mathbf{u} - 2k\underline{\omega} = 0 \end{aligned}\right\} \quad (2.1)$$

where $\mathbf{u}$ is the velocity vector, $\underline{\omega}$ the micro-rotation vector, μ the viscosity coefficient, p the pressure, k the coupling constant, ρ the density of the fluid, $\mathbf{F}$ the external force per unit mass, and α_1, α_2, α_3 are constants, characteristics of the particular fluid under consideration.

By an application of three dimensional Fourier Transform, the fundamental singular solutions of (2.1) corresponding to a point force $4\pi(2\mu + k)\underline{\alpha}\,\delta(\mathbf{X})$, $\delta(\mathbf{X})$ being the Dirac delta function, at the origin are found to be

$$\left.\begin{aligned} \mathbf{u} &= \mathbf{U}_s(\mathbf{X};\underline{\alpha}) + \beta\,\mathbf{V}_s(\mathbf{X};\underline{\alpha}) \\ \underline{\omega} &= \underline{\omega}^1(\mathbf{X};\underline{\alpha}) + \beta\,\underline{\omega}^2(\mathbf{X};\underline{\alpha}) \\ p &= P_s(X;\underline{\alpha}). \end{aligned}\right\} \quad \ldots(2.2)$$

where $\mathbf{X} = x\hat{e}_x + y\hat{e}_y + z\hat{e}_z$,

$$\left.\begin{aligned} \mathbf{U}_s(\mathbf{X};\underline{\alpha}) &= \frac{\underline{\alpha}}{R} + \frac{(\underline{\alpha}.\mathbf{X})\mathbf{X}}{R^3} \\ \mathbf{V}_s(\mathbf{X};\underline{\alpha}) &= \frac{\alpha_3}{2\mu+k}\left[(\underline{\alpha}.\mathbf{X})\mathbf{X}\left(\frac{3}{R^5}+\frac{3\lambda}{R^4}+\frac{\lambda^2}{R^3}\right)e^{-\lambda R}\right. \\ &\qquad \left. -\underline{\alpha}\left(\frac{1}{R^3}+\frac{\lambda}{R^2}+\frac{\lambda^2}{R}\right)e^{-\lambda R}\right] \\ \underline{\omega}^1(\mathbf{X};\underline{\alpha}) &= (\underline{\alpha}\wedge\mathbf{X})\frac{1}{R^3} \\ \underline{\omega}^2(\mathbf{X};\underline{\alpha}) &= -(\underline{\alpha}\wedge\mathbf{X})\left(\frac{1}{R^3}+\frac{\lambda}{R^2}\right)e^{-\lambda R} \\ P_s(\mathbf{X};\underline{\alpha}) &= (2\mu+k)\left(\frac{\underline{\alpha}.\mathbf{X}}{R^3}\right) \\ R^2 &= x^2+y^2+z^2 \\ \lambda^2 &= \frac{k(2\mu+k)}{\alpha_3(\mu+k)} \end{aligned}\right\} \quad \ldots(2.3)$$

and $\underline{\alpha}$ is a constant vector and β is a constant. The potential doublet in the classical case (Chwang and Wu 1975) is given by

$$\left.\begin{aligned} \mathbf{U}_D(\mathbf{X};\underline{\alpha}) &= -\frac{1}{2}\nabla^2\mathbf{U}_s(\mathbf{X};\underline{\alpha}) \\ &= -\frac{\underline{\alpha}}{R^3} + \frac{3(\underline{\alpha}.\mathbf{X})\mathbf{X}}{R^5} \end{aligned}\right\} \quad \ldots(2.4)$$

The corresponding functions V, $\underline{\omega}^i$ and P are

$$\left.\begin{aligned} \mathbf{V}_D(\mathbf{X};\underline{\alpha}) &= -\frac{1}{2}\nabla^2\mathbf{V}_s(\mathbf{X};\underline{\alpha}) = -\frac{\lambda^2}{2}\mathbf{V}_s(\mathbf{X};\alpha) \\ \underline{\omega}^1_D(\mathbf{X};\underline{\alpha}) &= -\frac{1}{2}\nabla^2\underline{\omega}^1(\mathbf{X};\underline{\alpha}) = 0 \\ \underline{\omega}^2_D(\mathbf{X};\underline{\alpha}) &= -\frac{1}{2}\nabla^2\underline{\omega}^2(\mathbf{X};\underline{\alpha}) = -\frac{\lambda^2}{2}\omega^2(\mathbf{X};\underline{\alpha}) \\ P_D(\mathbf{X};\underline{\alpha}) &= -\frac{1}{2}\nabla^2 P_s(\mathbf{X};\underline{\alpha}) = 0. \end{aligned}\right\} \quad \ldots(2.5)$$

3. Uniform Flow Past a Prolate Spheroid

We now consider the Stokes' flow for a uniform free stream past a prolate spheroid

$$\frac{x^2}{a^2} + \frac{r^2}{b^2} = 1, \; (r^2 = y^2 + z^2, \; a \geqslant b) \qquad \ldots(3.1a)$$

where a is the semi-major axis and b is the semi-minor axis, the focal length $2C$ and the eccentricity e are, as usual, related by

$$C = ae. \; (0 \leqslant e \leqslant 1) \qquad \ldots(3.1b)$$

With no loss of generality, the free stream velocity may be taken as

$$\mathbf{U} = U_1 \hat{e}_x + U_2 \hat{e}_y \qquad \ldots(3.1c)$$

where $\hat{e}_x, \hat{e}_y, \hat{e}_z$ are the unit vectors along x, y, z directions.

The boundary conditions to be satisfied on the surface of the prolate spheroid are the no slip and the no spin conditions i.e.

$$\mathbf{u} = 0, \; \underline{\omega} = 0 \qquad \ldots(3.2)$$

on the surface of the prolate spheroid.

Now guided by the known solution for a uniform flow past a prolate spheroid based on the singularity method (Chwang and Wu 1975). We construct the solution for a micropolar fluid as

$$\begin{aligned} \mathbf{u} = U_1 \hat{e}_x + U_2 \hat{e}_y &- \int_{-C}^{C} [A_1 \mathbf{U}_s (\mathbf{X}-\underline{\xi}; \hat{e}_x) + A_2 \mathbf{U}_s (\mathbf{X}-\underline{\xi}; \hat{e}_y)] \, d\xi \\ &- \int_{-C}^{C} [B_1 \mathbf{V}_s (\mathbf{X}-\underline{\xi}: \hat{e}_x) + B_2 \mathbf{V}_s (\mathbf{X}-\underline{\xi}: \hat{e}_y)] \, d\xi \\ &+ \int_{-C}^{C} [C_1 \mathbf{U}_D (\mathbf{X}-\underline{\xi}; \hat{e}_x) + C_2 \mathbf{U}_D (\mathbf{X}-\underline{\xi}; \hat{e}_y)] \, d\xi \\ &+ \int_{-C}^{C} [D_1 \mathbf{V}_D](\mathbf{X}-\underline{\xi}; \hat{e}_x) + D_2 \mathbf{V}_D (\mathbf{X}-\underline{\xi}; \hat{e}_y)] \, d\xi \end{aligned} \qquad \ldots(3.3a)$$

$$p = - \int_{-C}^{C} [A_1 P_s (\mathbf{X}-\underline{\xi}; \hat{e}_x) + A_2 P_s (\mathbf{X}-\underline{\xi}; \hat{e}_y)] \, d\xi \qquad \ldots(3.3b)$$

$$\underline{\omega} = -\int_{-C}^{C} [A_1 \underline{\omega}^1 (\mathbf{X} - \underline{\xi}; \hat{e}_x) + A_2 \underline{\omega}^1 (\mathbf{X} - \underline{\xi}; \hat{e}_y)]\, d\xi$$

$$- \int_{-C}^{C} [B_1 \underline{\omega}^2 (\mathbf{X} - \underline{\xi}; \hat{e}_x) + B_2 \underline{\omega}^2 (\mathbf{X} - \underline{\xi}; \hat{e}_y)]\, d\xi$$

$$+ \int_{-C}^{C} [D_1 \underline{\omega}_D^2 (\mathbf{X} - \underline{\xi}; \hat{e}_x) + D_2 \underline{\omega}_D^2 (\mathbf{X} - \underline{\xi}; \hat{e}_y)]\, d\xi \qquad \ldots(3.3c)$$

where A's B's, C's and D's are the densities of the singularities and $\underline{\xi} = \xi \hat{e}_x$.

When λ is small, we have the approximations

$$\left.\begin{aligned}
\mathbf{V}_s (\mathbf{X}; \underline{\alpha}) &= \frac{\alpha_3}{2\mu+k} \left[\mathbf{U}_D (\mathbf{X}; \underline{\alpha}) - \frac{\lambda^2}{2} \mathbf{U}_s (\mathbf{X}; \underline{\alpha}) \right] \\
\underline{\mathbf{V}}_D (\mathbf{X}; \underline{\alpha}) &= -\frac{\lambda^2}{2} \frac{\alpha_3}{2\mu+k} \cdot \mathbf{U}_D (\mathbf{X}; \underline{\alpha}) = -\frac{\lambda^2}{2} \mathbf{V}_s (\mathbf{X}; \underline{\alpha}) \\
\underline{\omega}^2 (\mathbf{X}; \underline{\alpha}) &= -(\underline{\alpha} \wedge \mathbf{X}) \left(\frac{1}{R^3} - \frac{\lambda^2}{2R} \right) \\
\underline{\omega}_D^2 (\mathbf{X}; \underline{\alpha}) &= \frac{\lambda^2}{2} (\underline{\alpha} \wedge \mathbf{X}) \frac{1}{R^3} = \frac{\lambda^2}{2} \underline{\omega}^1 (\mathbf{X}; \underline{\alpha}).
\end{aligned}\right\} \qquad \ldots(3.4)$$

We now consider two cases depending upon the order of smallness of the coupling constant.

Case I: $k = 0(\lambda)$—Let us take $k = q_1 \mu \lambda$, where q_1 is some constant, we then have

$$\frac{\alpha_3}{2\mu+k} = \frac{k}{(\mu+k)\lambda^2} = \frac{(1-q_1\lambda)\, q_1}{\lambda}$$

We now set the densities of the singularities as

$$\left.\begin{aligned}
A_1 &= A_{1,0} + \lambda A_{1,1}, \\
A_2 &= A_{2,0} + \lambda A_{2,1}, \\
B_1 &= B_{1,0} + \lambda \xi^2 B_{1,1} \\
B_2 &= B_{2,0} + \lambda \xi^2 B_{2,1} \\
C_1 &= \frac{C_{1,-1}}{\lambda} + (C_{1,0} + \lambda C_{1,1})(C^2 - \xi^2) + C_{1,2} \\
C_2 &= \frac{C_{2,-1}}{\lambda} + (C_{2,0} + \lambda C_{2,1})(C^2 - \xi^2) + C_{2,2} \\
D_1 &= -\frac{2}{\lambda} D_{1,0} (C^2 - \xi^2) \\
D_2 &= -\frac{2}{\lambda} D_{2,0} (C^2 - \xi^2)
\end{aligned}\right\} \qquad \ldots(3.5)$$

where A's, B's, C's and D's are constants. Using (3.4) and (3.5) in (3.3) we get following approximate forms for the functions $\mathbf{u}$, p and $\underline{\boldsymbol{\omega}}$

$$\mathbf{u} = U_1 \hat{e}_x + U_2 \hat{e}_y - \int_{-C}^{C} [(A_{1,0} + \lambda A_{1,1})\, \mathbf{U}_s (\mathbf{X}-\underline{\xi}; \hat{e}_x)$$

$$+ (A_{2,0} + \lambda A_{2,1})\, \mathbf{U}_s (\mathbf{X}-\underline{\xi}; \hat{e}_x)]\, d\xi - \int_{-C}^{C} [(B_{1,0} + \lambda \xi^2 B_{1,1})$$

$$\times \left\{ \mathbf{U}_D (\mathbf{X}-\underline{\xi}; \hat{e}_x) - \frac{\lambda^2}{2} \mathbf{U}_s (\mathbf{X}-\underline{\xi}; \hat{e}_x) \right\} \frac{q_1}{\lambda} (1-q_1 \lambda)$$

$$+ (B_{2,0} + \lambda \xi^2 B_{2,1}) \left\{ \mathbf{U}_D (\mathbf{X}-\underline{\xi}; \hat{e}_y) - \frac{\lambda^2}{2} \mathbf{U}_s (\mathbf{X}-\underline{\xi}; \hat{e}_y) \right\}$$

$$\frac{q_1}{\lambda}(1-q_1 \lambda) \Big]\, d\xi$$

$$+ \int_{-C}^{C} \left[\left\{ \frac{C_{1,-1}}{\lambda} + (C_{1,0} + \lambda C_{1,1})(C^2-\xi^2) + C_{1,2} \right\} \mathbf{U}_D (\mathbf{X}-\underline{\xi}; \hat{e}_x) \right.$$

$$\left. + \left\{ \frac{C_{2,-1}}{\lambda} + (C_{2,0} + \lambda C_{2,1})(C^2-\xi^2) + C_{2,2} \right\} \mathbf{U}_D (\mathbf{X}-\underline{\xi}; \hat{e}_y) \right] d\xi$$

$$+ \int_{-C}^{C} (C^2-\xi^2)\, q_1 (1-q_1\lambda)\, [D_{1,0}\, \mathbf{U}_D (\mathbf{X}-\underline{\xi}; \hat{e}_x) + D_{2,0} \mathbf{U}_D (\mathbf{X}-\underline{\xi}; \hat{e}_y)]\, d\xi \quad \ldots (3.6a)$$

$$p = -\int_{-C}^{C} [(A_{1,0} + \lambda A_{1,1})\, P_s (\mathbf{X}-\underline{\xi}; \hat{e}_x) + (A_{2,0} + \lambda A_{2,1})\, P_s (\mathbf{X}-\underline{\xi}; \hat{e}_y)]\, d\xi \quad \ldots (3.6b)$$

$$\underline{\omega} = -(A_{1,0} + \lambda A_{11}) \int_{-C}^{C} (y\hat{e}_z - z\hat{e}_y) \frac{1}{R_\xi^3}\, d\xi$$

$$-(A_{2,0} + \lambda A_{2,1}) \int_{-C}^{C} (z\hat{e}_x - x\hat{e}_z) \frac{1}{R_{\xi 1}^3}\, d\xi$$

$$+ \int_{-C}^{C} (B_{1,0} + \lambda \xi^2 B_{1,1})\, (y\hat{e}_z - z\hat{e}_y) \left(\frac{1}{R_\xi^3} - \frac{\lambda^2}{2R_\xi} \right) d\xi$$

$$+ \int_{-C}^{C} (B_{2,0} + \lambda \xi^2 B_{2,1})\, (z\hat{e}_x - x\hat{e}_z) \left(\frac{1}{R_\xi^3} - \frac{\lambda^2}{2R_\xi} \right) d\xi$$

(*equation continued on p.* 1479)

$$-\int_{-C}^{C} \lambda (C^2-\xi^2)\, D_{1,0}\, (y\hat{e}_z - z\hat{e}_y) \left(\frac{1}{R_\xi^3}\right) d\xi$$

$$-\int_{-C}^{C} \lambda (C^2-\xi^2)\, D_{2,0}\, (z\hat{e}_x - x\hat{e}_z) \frac{1}{R_\xi^3}\, d\xi \qquad \ldots(3.6c)$$

where

$$R_\xi^2 = (x-\xi)^2 + y^2 + z^2.$$

Applying the boundary conditions (3.2) on the surface of the spheroid, we get

$$\left.\begin{aligned}
A_{1,0} &= U_1 e^2 [-2e + L(1+e^2)]^{-1},\; A_{1,1} = \frac{q_1}{2} A_{1,0}\\
A_{2,0} &= 2U_2 e^2 [2e + L(3e^2-1)]^{-1},\; A_{2,1} = \frac{q_1}{2} A_{2,0}\\
C_{1,0} &= \left(\frac{1-e^2}{2e^2}\right) A_{1,0}\\
C_{2,0} &= \frac{1-e^2}{2e^2} A_{2,0}\\
B_{1,0} &= A_{1,0}\; B_{2,0} = A_{2,0}\\
B_{1,1} &= \frac{q_1}{2C^2} A_{1,0}, \quad B_{2,1} = \frac{q_1}{2C^2} A_{2,0}\\
C_{1,-1} &= q_1 A_{1,0},\; C_{2,1} = q_1 A_{2,0}\\
C_{1,1} &= q_1^2 D_{1,0},\; C_{2,1} = q_1^2 D_{2,0}\\
C_{1,2} &= -\frac{q_1}{2} A_{1,0},\; C_{2,2} = -\frac{q_1}{2} A_{2,0}\\
D_{1,0} &= -\frac{q_1}{2C^2} A_{1,0},\; D_{2,0} = -\frac{q_1}{2C^2} A_{2,0}
\end{aligned}\right\} \qquad \ldots(3.7)$$

using these constants in (3.6), the expressions for $\mathbf{u}$, p and $\underline{\omega}$ reduce to

$$\left.\begin{aligned}
\mathbf{u} &= U_1 \hat{e}_x + U_2 \hat{e}_y - [2A_{1,0}\hat{e}_x + A_{2,0}\hat{e}_y] I_{1,0}\\
&\quad - (A_{1,0}\, r\hat{e}_r + A_{2,0}\, y\hat{e}_x)\left(\frac{1}{R_2} - \frac{1}{R_1}\right)\\
&\quad + (A_{1,0}\, r\hat{e}_x - A_{2,0}\, y\hat{e}_r)\, rI_{3,0}\\
&\quad + \left(\frac{1-e^2}{2e^2}\right)\nabla\left\{-2A_{1,0} I_{1,1} + A_{2,0}\, y\left[\frac{X-C}{r^2} R_1\right.\right.\\
&\quad \left.\left. - \frac{x+C}{r^2} R^2 + I_{1,0}\right]\right\}\\
p &= 2\mu(1+q_1\lambda)\left[A_{1,0}\left(\frac{1}{R_1} - \frac{1}{R_2}\right) + A_{2,0}\left(\frac{x-C}{R_2} - \frac{x+C}{R_1}\right)\right]\\
\omega &= 0
\end{aligned}\right\} \qquad \ldots(3.8)$$

where $r\hat{e}_r = y\hat{e}_y + z\hat{e}_z$ and

$$R_1 = [(x+C)^2 + r^2]^{1/2},\ R_2 = [(x-C) + r^2]^{1/2}$$

$$I_{m,n}(\mathbf{X}) = \int_{-C}^{C} \frac{\xi^n}{|\mathbf{X}-\underline{\xi}|^m}\, d\xi \begin{pmatrix} n = -0, 1, 2\ldots \\ m = 1, 1, 3, 5\ldots \end{pmatrix}$$

The drag force on the spheroid is given by the contribution of Stockes lets $\mathbf{U}_s$ occurring in eqn. (3.6a).

$$\mathbf{D} = 4\pi(2\mu+k)\int_{-C}^{C} [(A_{1,0} + \lambda A_{1,1})\hat{e}_x + (A_{2,0} + \lambda A_{2,1})\hat{e}_y]\, d\xi$$

$$= 16\pi\mu a e^3 (1+q_1\lambda)\left[\frac{U_1\hat{e}_x}{L(1+e^2)-2e} + \frac{2U_2\hat{e}_y}{2e+(3e^2-1)L}\right]$$

$$= (1+q_1\lambda)\,\mathbf{D}_c = \left(1+\frac{k}{\mu}\right)\mathbf{D}_c \qquad \ldots(3.9)$$

where $\mathbf{D}_c$ is the drag force for a classical viscous fluid. In the limiting case of a sphere, $e \to 0$ and

$$\mathbf{D} \to 6\pi\mu a\,\mathbf{U}\,(1+q_1\lambda)$$

$$= 6\pi\mu a\left(1+\frac{k}{\mu}\right)\mathbf{U}. \qquad \ldots(3.10)$$

Case II: $k = O(\lambda^2)$—Let us take $k = q_2.\,\mu\lambda^2$, where q_2 is some constant. We then have

$$\frac{\alpha_3}{2\mu+k} = \frac{k}{\lambda^2(\mu+k)} = q_2(1-q_2\lambda^2). \qquad \ldots(3.11)$$

We now set the densities of the singularities as

$$\left.\begin{aligned} A_1 &= A_0^1 + \lambda^2 A_1^1 \\ A_2 &= A_0^2 + \lambda^2 A_1^2 \\ B_1 &= B_0^1 + \lambda^2 (C^2-\xi^2) B_1^1 \\ B_2 &= B_0^2 + \lambda^2 (C^2-\xi^2) B_1^2 \\ C_1 &= (C_0^1 + \lambda^2 C_1^1)(C^2-\xi^2) + C_2^1 + C_3^1\lambda^2 \\ C_2 &= (C_0^2 + \lambda^2 C_1^2)(C^2-\xi^2) + C_2^2 + C_3^2\lambda^2 \\ D_1 &= -2D_0^1,\ D_2 = -2D_0^2 \end{aligned}\right\} \qquad \ldots(3.12)$$

where A's, B's, C's and D's are constants. Using (3.4) and (3.12) in (3.3) following approximations for $\mathbf{u}$, p and ω emerge

$$\mathbf{u} = U_1 \hat{e}_x + U_2 \hat{e}_y - \int_{-C}^{C} (A_0^1 + \lambda^2 A_1^1)\, \mathbf{U}_s (\mathbf{X}-\underline{\xi}; \hat{e}_x)\, d\xi$$

$$- \int_{-C}^{C} (A_0^2 + \lambda^2 A_1^2)\, \mathbf{U}_s (\mathbf{X}-\underline{\xi}; \hat{e}_y)\, d\xi$$

$$- \int_{-C}^{C} (B_0^1 + \lambda^2 (C^2-\xi^2) B_1^1)\, \mathbf{V}_s (\mathbf{X}-\underline{\xi}; \hat{e}_x)\, d\xi$$

$$- \int_{-C}^{C} (B_0^2 + \lambda^2 (C^2-\xi^2) B_1^2\, (\mathbf{V}_s (\mathbf{X}-\xi; \underline{\hat{e}_y})\, d\xi$$

$$+ \int_{-C}^{C} [(C_0^1 + \lambda^2 C_1^1)(C^2-\xi^2) + C_2^1 + C_3^1 \lambda^2]\, \mathbf{U}_D (\mathbf{X}-\underline{\xi}; \hat{e}_x)\, d\xi$$

$$+ \int_{-C}^{C} [(C_0^2 + \lambda^2 C_1^2)(C^2-\xi^2) + C_2^2 + C_3^2 \lambda^2]\, \mathbf{U}_D (\mathbf{X}-\underline{\xi}; \hat{e}_x)]\, d\xi$$

$$+ \int_{-C}^{C} [D_0^1 \lambda^2\, \mathbf{V}_s (\mathbf{X}-\underline{\xi}; \hat{e}_x) + D_0^2 \lambda^2\, \mathbf{V}_s (\mathbf{X}-\underline{\xi}; \hat{e}_y)]\, d\xi. \qquad \ldots(3.13a)$$

$$p = - \int_{-C}^{C} [(A_0^1 + \lambda^2 A_1^1)\, \mathbf{P}_s (\mathbf{X}-\underline{\xi}; \hat{e}_x) + (A_0^2 + \lambda^2 A_1^2)\, \mathbf{P}_s (\mathbf{X}-\underline{\xi}; \hat{e}_y)]\, d\xi \qquad \ldots(3.13b)$$

$$\underline{\omega} = - \int_{-C}^{C} [(A_0^1 + \lambda^2 A_1^1)\, \underline{\omega}^1 (\mathbf{X}-\underline{\xi}; \hat{e}_x) + (A_0^2 + \lambda^2 A_1^2)\, \underline{\omega}^1 (\mathbf{X}-\underline{\xi}; \hat{e}_y)]\, d\xi$$

$$- \int_{-C}^{C} [(B_0^1 + \lambda^2 (C^2-\xi^2) B_1^1)\, \underline{\omega}^2 (\mathbf{X}-\underline{\xi}; \hat{e}_x) + (B_0^2 + \lambda^2 (C^2-\xi^2) B_1^2)$$

$$\times\, \underline{\omega}^2 (\mathbf{X}-\underline{\xi}; \hat{e}_y)]\, d\xi$$

$$- \int_{-C}^{C} [\lambda^2 D_0^1\, \underline{\omega}^1 (\mathbf{X}-\underline{\xi}; \hat{e}_x) + \lambda^2 D_0^2\, \underline{\omega}^1 (\mathbf{X}-\underline{\xi}; \hat{e}_y)]\, d\xi. \qquad \ldots(3.14)$$

Applying the boundary conditions (3.2) on the surface of the spheroid, we obtain after some calculation

$$\left.\begin{aligned}
A_0^1 &= \frac{U_1 e^2}{[-2e + L(1+e^2)]},\ C_0^1 = \frac{(1-e^2)}{2e^2} A_0^1 \\
A_0^2 &= \frac{2U_2 e^2}{[2e+L(-1+3e^2)]},\ C_0^2 = \frac{(1-e^2)}{2e^2} A_0^2 \\
B_0^1 &= A_0^1,\ B_1^1 = -\frac{L}{2T_1} A_0^1,\ D_0^1 = -\frac{q_2}{2} A_0^1 \\
B_0^2 &= A_0^1,\ B_1^2 = -\frac{L}{2T_1} A_0^2,\ D_0^2 = -\frac{q_2}{2} A_0^2 \\
A_1^1 &= \frac{q_2}{2} A_0^1,\ A_1^2 = \frac{q_2}{2} A_1^2 \\
C_1^1 &= \frac{L}{2T_1} q_2 A_0^1,\ C_1^2 = \frac{L}{2T_1} q_2 A_0^2 \\
C_2^1 &= q_2 A_0^1,\ C_2^2 = q_2 A_0^2 \\
C_3^1 &= -\frac{q_2^2}{2} A_0^1,\ C_3^2 = -\frac{q_2^2}{2} A_0^2
\end{aligned}\right\} \qquad \ldots(3.15)$$

where

$$L = \log\frac{1+e}{1-e},\ T_1 = L - \frac{2e}{1-e_2} \qquad \ldots(3.16)$$

Using the constants given by (3.15) in (3.13), we get

$$\begin{aligned}
\mathbf{u} = U_1 \hat{e}_x + U_2 \hat{e}_y &- (2A_0^1 \hat{e}_x + A_0^2 \hat{e}_y) I_{1,0} \\
&- (A_0^1 r\hat{e}_r + A_0^2 y \hat{e}_x)\left(\frac{1}{R_2} - \frac{1}{R_1}\right) + (A_0^1 r\hat{e}_x - A_0^2 y \hat{e}_r) r I_{3,0} \\
&+ \nabla\left\{-2C_0^1 I_{1,1} + C_0^2 y\left[\frac{x-C}{r^2} R_1 - \frac{x+C}{r^2} R_2 + I_{1,0}\right]\right\}.
\end{aligned} \qquad \ldots(3.17a)$$

$$p = 2\mu(1+q_2\lambda^2)\left[\left(\frac{1}{R_1} - \frac{1}{R_2}\right) A_0^1 + A_0^2 \frac{y}{r^2}\left(\frac{x-C}{R_2} - \frac{x+C}{R_1}\right)\right] \qquad \ldots(3.17b)$$

$$\underline{\omega} = \lambda^2 (y\hat{e}_z - z\hat{e}_y) A_0^1 \left\{\frac{L}{2T_1}(C^2 I_{3,0} - I_{3,2}) - \frac{1}{2} I_{1,0}\right\}. \qquad \ldots(3.17c)$$

The velocity field u is seen to be independent of λ and is same as obtained by Chwang and Wu for the non-polar case.

The drag force on the prolate spheroid is given by

$$\mathbf{D} = 4\pi(2\mu + k) \int_{-C}^{C} [(A_0^1 + \lambda^2 A_1^1)\hat{e}_x + (A_0^2 + \lambda^2 A_1^2)\hat{e}_y)] \, d\xi$$

$$= 16\pi\mu a e^3 (1 + q_2\lambda^2) \left[\frac{U_1 \hat{e}_x}{L(1+e^2) - 2e} + \frac{2U_2 \hat{e}_y}{L(3e^2 - 1) + 2e} \right]$$

$$= (1 + q_2\lambda^2)\,\mathbf{D}_c \qquad \ldots(3.18)$$

$$= \left(1 + \frac{k}{\mu}\right)\mathbf{D}_c$$

and in the limiting case of a sphere, $e \to 0$ and the drag force reduces to

$$\mathbf{D} = 6\pi\mu a\,(1 + q_2\lambda^2)(U_1\hat{e}_x + U_2\hat{e}_y) = 6\pi\mu a\,\mathbf{U}\,(1 + q^2\lambda^2)$$

$$= 6\pi\mu a \left(1 + \frac{k}{\mu}\right)\mathbf{U}.$$

4. Conclusion

It is interesting to note that in both the cases although the velocity field remains unaffected by the micropolarity of the medium, we are yet able to determine the first order corrections in the value of drag force. It is also seen from (3.9) and (3.18) that for any free stream direction, we can deduce the value of the drag for the polar case from the non-polar case simply through multiplication by the factor $(1 + k/\mu)$. These coefficients are also independent of e, representing the shape of the spheroid. In particular for $e = 0$. our results correspond to the approximate values of drag force for a sphere as determined by Ramkissoon and Majumdar.

The above discussion implies that the drag force on an arbitrary shaped particle of small size placed in a slow stream of small micropolarity may also be computed from the known non-polar value by multiplication through the factor $(1 + k/\mu)$.

ReferencFs

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