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PSEUDOPLANE SYMMETRIC INTERIOR AND EXTERIOR SOLUTIONS FOR A THICK DISK OF FLUID

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The static pseudoplane symmetric interior and exterior solutions of Einstein's equations for a thick disk of fiuid are obtained and discussed.

1. INTRODUCTION

Discussion of self-gravitating systems in general relativity is very important, particularly with the cases where exact solutions of Einstein's equations are possible to obtain. Most of the attempts to solve physically plausible self-gravitating matter distributions concentrate around spherically symmetric or cylindrically symmetric systems, which in fact, is the most reasonable thing to do. However, it would be instructive to look for some configurations which may be plane symmetric, perhaps corresponding to a self-gravitating flat disk. Attempts on these lines have been made by Horsky (1968) and Horsky and Novotny (1969), wherein they have discussed a possible exact static exterior and interior solution of Einstein's equations for a plane thick disk of perfect fiuid, obeying the generalised O'Brien-Synge junction conditions for the energy-momentum tensor. Following the approach of these authors, we now obtain a class of static pseudoplane symmetric solution for the interior and exterior field of a thick disk of perfect fluid.

We start with an anisotropic layer of a fluid, that divides the space-time into two regions V^+ and V^-, the two regions being represented by the space-time metrics :

$$V^+ : ds^2 = dx^2 + (ax + b)^{p_1}\, dy^2 + (ax + b)^{p_2}\, dz^2 - (ax + b)^{p_3}\, dt^2 \qquad \ldots(1a)$$

$$V^- : ds^2 = dx^2 + (-ax + b)^{p_1}\, dy^2 + (-ax + b)^{p_2}\, dz^2 - (-ax + b)^{p_3}\, dt^2 \qquad \ldots(1b)$$

where in the constants p_1, p_2, p_3 satisfy the conditions

$$\left.\begin{aligned} p_1 + p_2 + p_3 &= 2 \\ p_1 p_2 + p_2 p_3 + p_3 p_1 &= 0\,. \end{aligned}\right\} \qquad \ldots(2)$$

Defining the surface energy-momentum tensor

$$t_a^b = \begin{bmatrix} p & 0 & 0 \\ 0 & p' & 0 \\ 0 & 0 & -\sigma \end{bmatrix} \qquad \ldots(3)$$

as taken by Horsky and Novotny, a justification of which will be shown later, we shall in the following, obtain the required solutions by properly choosing the coordinate system and the extrinsic curvature tensor of the layer $K_{ik}^{\pm}$. The notation and units chosen are the same as in the paper of Horsky and Novotny (1969).

2. Finite Number of Plane Layers

The constant b appearing in the solution (1) can be made zero by shifting the origin along x-axis on the surface of singularity and hence we take

$$g_{\alpha\beta}^{\pm} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & (\pm ax)^{p_1} & 0 & 0 \\ 0 & 0 & (\pm ax)^{p_2} & 0 \\ 0 & 0 & 0 & -(\pm ax)^{p_3} \end{bmatrix}. \qquad \ldots(4)$$

Recall the notations x^+ and x^- represent the limiting values of the x co-ordinate as we approach the layer from the right and the left respectively. Similar are the notations for the constant a as it may differ on each side of the plane layer which is obvious in the case of two or more plane layers in space.

In our co-ordinate system

$$4g_{11} = 1, 4g_{i1} = 0$$

and K_{ik}, the extrinsic curvature tensor of the layer, is given by

$$K_{ik}^{\pm} = \frac{1}{2} \frac{\partial^4 g_{ik}}{\partial x} \Big|^{\pm} . \qquad \ldots(5)$$

When passing through the layer, K_{ik} jumps and it is required that

$$\gamma_{ik} = K_{ik}^{+} - K_{ik}^{-} \equiv \left[K_{ik} \right] = -8\pi \left(t_{ik} - \frac{1}{2} g_{ik} t_a^a \right). \qquad \ldots(6)$$

From (6) and (3), we have

$$\begin{aligned} \gamma_2^2 &= -4\pi (p - p' + \sigma) \\ \gamma_3^3 &= -4\pi (p' - p + \sigma) \\ \gamma_4^4 &= +4\pi (p + p' + \sigma) \end{aligned} \qquad \ldots(7)$$

while using (5), the definition of K_{ik}, we obtain

$$\begin{aligned} \gamma_2^2 &= \frac{p_1}{2} \left(\frac{1}{x^+} - \frac{1}{x^-} \right) \\ \gamma_3^3 &= \frac{p_2}{2} \left(\frac{1}{x^+} - \frac{1}{x^-} \right) \\ \gamma_4^4 &= \frac{p_3}{2} \left(\frac{1}{x^+} - \frac{1}{x^-} \right). \end{aligned} \qquad \ldots(8)$$

Comparing (7) and (8), we get

$$p_1\left(\frac{1}{x^+} - \frac{1}{x^-}\right) = -8\pi(p - p' + \sigma)$$

$$p_2\left(\frac{1}{x^+} - \frac{1}{x^-}\right) = -8\pi(p' - p + \sigma) \qquad \ldots(9)$$

$$p_3\left(\frac{1}{x^+} - \frac{1}{x^-}\right) = 8\pi(p + p' + \sigma)$$

and hence

$$\frac{1}{x^+} - \frac{1}{x^-} = 4\pi(p + p' - \sigma). \qquad \ldots(10)$$

Equations (9), then reduce to the following equations

$$p(p_1 + 2) + p'(p_1 - 2) + \sigma(2 - p_1) = 0$$

$$p(p_2 - 2) + p'(p_2 + 2) + \sigma(2 - p_2) = 0$$

$$p(p_1 - 2) + p'(p_3 - 2) - \sigma(p_3 + 2) = 0 \qquad \ldots(11)$$

which on solving yield

$$p = -\sigma(2 - p_1)/(2 - p_3)$$

$$p' = -\sigma(2 - p_2)/(2 - p_3). \qquad \ldots(12)$$

To have $\left|\frac{p}{\sigma}\right| \leqslant 1$ and $\left|\frac{p'}{\sigma}\right| \leqslant 1$ we must have

$$\left|\frac{2 - p_1}{2 - p_3}\right| \leqslant 1 \text{ and } \left|\frac{2 - p_2}{2 - p_3}\right| \leqslant 1 \qquad \ldots(13)$$

respectively. Prasanna (1969), while discussing the properties of the Narlikar-Karmarkar metric (also Kasner metric) has shown that the possible values of the constants p_1, p_2, p_3 satisfying the conditions (2) above can be expressed as

$$p_1, p_2 = \left(1 - \frac{1}{2}\, p_3\right) \pm \left(1 + p_3 - \frac{3}{4}\, p_3^2\right)^{1/2},\ p_3$$

and that each of them must lie in the range $\left[-\frac{2}{3}, 2\right]$ with only one of them being negative. In the light of these results (13) implies that

$$-\frac{2}{3} \leqslant p_3 < 0 \qquad \ldots(14)$$

and hence either

$$\frac{4}{3} \leqslant p_1 < 2 \text{ and } \frac{4}{3} \geqslant p_2 > 0 \qquad \ldots(15)$$

or

$$\frac{4}{3} \leqslant p_2 < 2 \text{ and } \frac{4}{3} \geqslant p_1 > 0\,. \qquad \ldots(16)$$

Assuming (15) for definitness, it follows that

$$0 < \frac{2 - p_1}{2 - p_3} \leqslant \frac{1}{4} \text{ and } \frac{1}{4} \leqslant \frac{2 - p_2}{2 - p_3} < 1. \qquad \ldots(17)$$

This shows that the tension p' is always strong and p is strong or weak depending on the values of p_1 and p_3.

Let us now denote the thin plane layer by 0 and consider a static system, symmetric with respect to the layer 0, of n thin plane layers. Let the distance between any two layers be δ which is a constant. In addition to conditions (6) the gravitational field must satisfy the conditions

$$[g_{ab}] = 0. \qquad \ldots(18)$$

Hence in this case

$$p_m = -\frac{\sigma_m (2 - p_1)}{2 - p_3}\ ;\ p'_m = -\frac{\sigma_m (2 - p_2)}{2 - p_3}$$

$$\frac{1}{x_m^+} - \frac{1}{x_m^-} = 4\pi (p_m + p'_m - \sigma_m) = -\frac{16\pi\sigma_m}{2 - p_3} \qquad \ldots(19)$$

$$a_m^+ \left| x_m^+ \right| = a_m^- \left| x_m^- \right|$$

as follows from (12), (10) and (18). Because of the symmetry of the system about 0 we have $x_0^- = -x_0^+$ and so from third equation in (19)

$$x_0^+ = -\frac{2 - p_3}{8\pi\sigma}\ . \qquad \ldots(20)$$

Noting $\delta = x_m^- - x_{m-1}^+$ and $a_m^- = a_{m-1}^+$ from (19) one can obtain the reccurence formulae

$$x_m = \frac{x_{m-1} + \delta}{1 - A\sigma_m (x_{m-1} + \delta)} \qquad \ldots(21)$$

$$a_m = [1 - A\sigma_m (x_{m-1} + \delta)]\, a_{m-1} \qquad \ldots(22)$$

where

$$A = \frac{16\pi}{2 - p_3}. \qquad \ldots(23)$$

Here and onwards we have introduced the notation $x_m^+ \equiv x_m$, $a_m^+ \equiv a_m$ for convenience.

3. n-Plane Layers to a Thick Plane Disk

In (21) and (22), we take limit as $\delta \to 0$ such that $\sigma/\delta \to \epsilon(x)$, where $\epsilon(x)$, a density of matter in the plane disk, is a given function and obtain from first principles the derivatives

$$\frac{db}{dx} \equiv \lim_{\substack{\delta \to 0 \\ \sigma/\delta \to \epsilon(x)}} \frac{x_m - x_{m-1}}{\delta} = 1 + A\epsilon\,(x)\, b^2\,(x) \qquad \ldots(24)$$

and

$$\frac{da}{dx} \equiv \lim_{\substack{\delta \to 0 \\ \sigma/\delta \to \epsilon(x)}} \frac{a_m - a_{m-1}}{\delta} = - A\epsilon\,(x)\, a\,(x)\, b\,(x). \qquad \ldots(25)$$

The solution of eqn. (25) is

$$a(x) = c \exp\,[- A \int \epsilon\,(x)\, b\,(x)\, dx] \qquad \ldots(26)$$

where $b\,(x)$ is given by the differential equations (24) and satisfies the condition

$$\lim_{x \to 0^+} b(x) = - \infty \qquad \ldots(27)$$

which follows from (20).

It should be noted that when $\delta \to 0$ the n-plane layers give rise to a thick plane disk. Let $2L$ be the thickness of the disk measured along x-axis. Then the interior solution of the disk can be written as

$$g_{\alpha\beta}(x) = \begin{Bmatrix} 1 & 0 & 0 & 0 \\ 0 & [a(x)b(x)]^{p_1} & 0 & 0 \\ 0 & 0 & [a(x)b(x)]^{p_2} & 0 \\ 0 & 0 & 0 & -[a(x)b(x)]^{p_3} \end{Bmatrix}, \quad |x| \leqslant L \qquad \ldots(28)$$

while for the exterior solution we have

$$g_{\alpha\beta}(x) = \begin{Bmatrix} 1 & 0 & 0 & 0 \\ 0 & [F\,(L, x)]^{p_1} & 0 & 0 \\ 0 & 0 & [F\,(L, x)]^{p_2} & 0 \\ 0 & 0 & 0 & -[F\,(L, x)]^{p_3} \end{Bmatrix}, \quad |x| \geqslant L \qquad \ldots(29)$$

where $F\,(L, x) = a(L)\,[b(L) + x - L]$.

In section 4 different explicit forms of interior solution (28) are given for various assumed values of $\epsilon(x)$.

4. Interior Solutions

Assumption 1 : ϵ = a constant. Then the solution of the eqn. (24) satisfing the condition (27) is

$$b(x) = - (1/\sqrt{A\epsilon}) \cot \sqrt{A\epsilon}\, x \qquad \ldots(30)$$

and therefore from (26)

$$a(x) = C \sin \sqrt{A\epsilon}\, x. \qquad \ldots(31)$$

Then the interior solution is given by

$$g_{\alpha\beta}(x) = \begin{Bmatrix} 1 & 0 & 0 & 0 \\ 0 & \left(-\frac{C}{\sqrt{A\epsilon}}\cos\sqrt{A\epsilon}x\right)^{p_1} & 0 & 0 \\ 0 & 0 & \left(-\frac{C}{\sqrt{A\epsilon}}\cos\sqrt{A\epsilon}x\right)^{p_2} & 0 \\ 0 & 0 & 0 & -\left(-\frac{C}{\sqrt{A\epsilon}}\cos\sqrt{A\epsilon}\,x\right)^{p_3} \end{Bmatrix}, \quad |x| \leqslant L \qquad \ldots(32)$$

which can be written in the form

$$g_{\alpha\beta}(x) = \begin{Bmatrix} 1 & 0 & 0 & 0 \\ 0 & (\cos\sqrt{A\epsilon}\,x)^{p_1} & 0 & 0 \\ 0 & 0 & (\cos\sqrt{A\epsilon}\,x)^{p_2} & 0 \\ 0 & 0 & 0 & -(\cos\sqrt{A\epsilon}\,x)^{p_3} \end{Bmatrix}, \quad |x| \leqslant L \qquad \ldots(33)$$

after a simple transformation

$$x' = x,\ y' = (-C/\sqrt{A\epsilon})^{p_{1/2}}\, y,\ z' = (-C/\sqrt{A\epsilon})^{p_{2/2}}\, z$$
$$t' = (-C/\sqrt{A\epsilon})^{p_{3/2}}\, t. \qquad \ldots(34)$$

Assumption 2 : $\epsilon(x) = \dfrac{AB^2}{AB\,e^{ABx} - 1}$ where B is a positive constant, A is given by (23) and $AB > 1$. In this case the solution of the eqn. (24) satisfying the condition (27) is found to be

$$b(x) = \frac{AB\,e^{ABx} - 1}{AB} - \frac{e^{2ABx}}{e^{ABx} + \dfrac{1}{AB}\log\left(\dfrac{ABe^{ABx} - 1}{AB - 1}\right) - 1} \qquad \ldots(35)$$

Equation (26) then gives

$$a(x) = C\,e^{-ABx}\left[1 - e^{ABx} - \frac{1}{AB}\log\left(\frac{AB\,e^{ABx} - 1}{AB - 1}\right)\right] \qquad \ldots(36)$$

and the interior solution is

$$g_{\alpha\beta}(x) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & [G(x)]^{p_1} & 0 & 0 \\ 0 & 0 & [G(x)]^{p_2} & 0 \\ 0 & 0 & 0 & -[G(x)]^{p_3} \end{bmatrix}, \quad |x| \leqslant L \qquad \ldots(37a)$$

where

$$G(x) = \frac{C}{AB}\left\{1 + \frac{AB\,e^{ABx} - 1}{e^{ABx}}\left[1 - \frac{1}{AB}\log\left(\frac{AB\,e^{ABx} - 1}{AB - 1}\right)\right]\right\}. \qquad \ldots(37b)$$

The transformation

$$x' = x,\ y' = (C/AB)^{p_{1/2}}\,y,\ z' = (C/AB)^{p_{2/2}}\,z,\ t' = (C/AB)^{p_{3/2}}\,t \qquad \ldots(38)$$

reduces the solution (37) into

$$g_{\alpha\beta}(x) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & [G'(x)]^{p_1} & 0 & 0 \\ 0 & 0 & [G'(x)]^{p_2} & 0 \\ 0 & 0 & 0 & -[G'(x)]^{p_3} \end{bmatrix}, \quad |x| \leqslant L \qquad \ldots(39a)$$

where

$$G'(x) = 1 + \frac{AB\,e^{ABx} - 1}{e^{ABx}}\left[1 - \frac{1}{AB}\log\left(\frac{AB\,e^{ABx} - 1}{AB - 1}\right)\right]. \qquad \ldots(39b)$$

Assumption 3 : $\epsilon(x) = \dfrac{B}{[(1 + AB)\,x + \alpha]^2}$ where B is a positive constant, α a non-zero constant and A is given by (23). For this value of $\epsilon(x)$ the solution of (24) satisfying the condition (27) is

$$b(x) = \frac{X}{AB}\left[1 - AB\,(\alpha/X)^{(AB-1)/(AB+1)}\right]\left[1 - (\alpha/X)^{(AB-1)/(AB+1)}\right]^{-1} \qquad \ldots(40)$$

where

$$X = (1 + AB)\,x + \alpha \qquad \ldots(41)$$

and hence $a(x)$ is given by

$$a(x) = \frac{CAB}{AB - 1}\,X^{-1/(AB+1)}\left[(\alpha/X)^{(AB-1)/(AB+1)} - 1\right]. \qquad \ldots(42)$$

The interior solution is then

$$g_{\alpha\beta}(x) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & [H(x)]^{p_1} & 0 & 0 \\ 0 & 0 & [H(x)]^{p_2} & 0 \\ 0 & 0 & 0 & -[H(x)]^{p_3} \end{bmatrix}, \quad |x| \leqslant L \qquad \ldots(43a)$$

where

$$H(x) = \frac{C}{AB-1} X^{AB/(AB+1)} \left[AB\,(\alpha/X)^{(AB-1)/(AB+1)} - 1 \right]. \qquad \ldots(43b)$$

This solution transforms into

$$g_{\alpha\beta}(x) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & [H'(x)]^{p_1} & 0 & 0 \\ 0 & 0 & [H'(x)]^{p_2} & 0 \\ 0 & 0 & 0 & -[H'(x)]^{p_3} \end{bmatrix}, \; |x| \leqslant L \qquad \ldots(44a)$$

where

$$H'(x) = X^{AB/(AB+1)} \left[AB\,(\alpha/X)^{(AB-1)/(AB+1)} - 1 \right] \qquad \ldots(44b)$$

by a simple transformation

$$x' = x,\; y' = [C/(AB-1)]^{p_1/2}\, y,\; z' = [C/(AB-1)]^{p_2/2}\, z,$$
$$t' = [C/(AB-1)]^{p_3/2}\, t. \qquad \ldots(45)$$

5. Discussions

(5.1) Here we justify the choice of the surface energy-momentum tensor t_{ab} (3). The energy-momentum tensor $T_{\alpha\beta}$ of a thick plane disk is calculated directly from the Einstein's equations ($G_{\alpha\beta} = -8\pi T_{\alpha\beta}$) and is found to be given by

$$T_\alpha^\beta = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -\frac{2-p_1}{2-p_3}\,\epsilon(x) & 0 & 0 \\ 0 & 0 & -\frac{2-p_2}{2-p_3}\,\epsilon(x) & 0 \\ 0 & 0 & 0 & -\epsilon(x) \end{bmatrix}. \qquad \ldots(46)$$

The surface energy-momentum tensor t_{ab} and the energy momentum tensor $T_{\alpha\beta}$ must satisfy the relation

$$t_{ab} = \lim_{\delta \to 0} \int_{x_0}^{x_0+\delta} T_{ab}\, dx\,. \qquad \ldots(47)$$

Now we calculate t_{ab} from (47) using (46), in case of the solutions (33). For solutions (39) and (44) the calculations are similar. For the component t_{22} we have

$$t_{22} = \lim_{\delta \to 0} \int_{x_0}^{x_0+\delta} T_{22}\, dx$$

(*equation continued on p. 57*)

$$= \lim_{\delta \to 0} \int_{x_0}^{x_0+\delta} \frac{p_1 - 2}{2 - p_3} \frac{\sigma}{\delta} (\cos \sqrt{A\epsilon}\, x)^{p_1}\, dx$$

$$= \frac{p_1 - 2}{2 - p_3} \lim_{\delta \to 0} \left[\frac{\sigma}{\delta} \cos \sqrt{A\epsilon}\, x_0)^{p_1}\, \delta \right]$$

$$= - \frac{\sigma (2 - p_1)}{2 - p_3} (\cos \sqrt{A\epsilon}\, x_0)^{p_1} \qquad \ldots(48)$$

so that

$$t_2^2 = g^{22} t_{22} = - \frac{\sigma (2 - p_1)}{2 - p_3} = p \qquad \ldots(49)$$

from (12). Similar calculations yeild

$$t_3^3 = p' , \; t_4^4 = - \sigma \qquad \ldots(50)$$

justifying the choice of the tensor t_{ab}.

(5.2). Taking into consideration that $\psi\,|^+ = 0$, $T_1^1 = 0$, $m_\alpha = \delta_\alpha^1$ and $l_{(a)}^\alpha = \delta_a^\alpha$ at the point $x = L$, one can see that the generalised O'Brein-Synge junction conditions (Israel 1966)

$$G_{\alpha\beta}\, m^\alpha\, m^\beta\, |^+ = G_{\alpha\beta} m^\alpha\, m^\beta\, |^-$$

$$G_{\alpha\beta}\; l_{(a)}^\alpha\, m^\beta\, |^+ = G_{\alpha\beta}\; l_{(a)}^\alpha\, m^\beta\, |^- \qquad \ldots(51)$$

are satisfied, whatever be the value of $\epsilon(x)$. It can also be verified that $g_{\alpha\beta}$ and their first derivatives are continuous at the boundary $x = L$.

(5.3). The exterior solution (29) has the singularity at the point

$$x = L - b(L). \qquad \ldots(52)$$

The singularity lies outside/on/inside the disk according as

$$b(L) \gtreqless 0. \qquad \ldots(53)$$

The conditions (53) imply

$$\left.\begin{aligned} &\sqrt{A\epsilon}\, L \lesseqgtr \frac{\pi}{2} \text{ , when } \epsilon = \text{a constant and } 0 < \sqrt{A\epsilon}\, L < 2\pi \\ &A\, e^{ABL}\, \epsilon(L) + 1 \lesseqgtr \log\, [\epsilon(0)/\epsilon(L)] \\ &\qquad \text{when } \epsilon(x) = AB^2/(AB\, e^{ABx} - 1) \\ &(L) \lesseqgtr \frac{\alpha}{1 + AB} \left[AB^{(AB+1)/(AB-1)} - 1 \right] \\ &\qquad \text{when } \epsilon(x) = B/[(AB + 1)\, x + \alpha]^2 . \end{aligned}\right\} \qquad \ldots(54)$$

However for physically reasonable solution, the singularity must be inside or on the disk.

(5.4). In each of the assumptions 2 and 3 of $\epsilon(x)$ we note that $\epsilon(x)$ decreases as we move away from the plane of the symmetry.

(5.5). When ϵ = a constant, we compare our results with those in Newtonian theory. Consider a free probe particle immediately above the disk. If this particle starts with zero velocity it will move along the straight line y = constant, z = constant. At this moment its acceleration (measured by a standard clock at rest at this point) is given by

$$\frac{d^2x}{dT^2} \equiv g_{\text{rel}} = -\frac{p_3}{2b(L)} . \qquad \ldots(55)$$

Using (30), we get

$$g_{\text{rel}} = \frac{p_3}{2} \sqrt{A\epsilon} \tan \sqrt{A\epsilon}\, L. \qquad \ldots(56)$$

Let $\sqrt{A\epsilon}\, L \ll 1$, so that $\tan \sqrt{A\epsilon}\, L \simeq \sqrt{A\epsilon}\, L$ and

$$g_{\text{rel}} = \frac{8\pi p_3 \epsilon}{2 - p_3} L. \qquad \ldots(57)$$

In Newtonian theory, by use of the Gauss Theorem one can easily show that the acceleration of the probe praticle is given by

$$g_{\text{clas}} = -4\pi\epsilon L. \qquad \ldots(58)$$

Thus $g_{\text{clas}} \neq g_{\text{rel}}$. This can be explained in the following way: according to Landau and Lifshitz (1971) the total energy of matter and the static gravitational field is

$$-P_4 = M = \int (T_1^1 + T_2^2 + T_3^3 - T_4^4)(-g)^{1/2}\, dv. \qquad \ldots(59)$$

In our case the total energy of a piece of the disk is

$$M \simeq \int (T_2^2 + T_3^3 - T_4^4)\, dv = -\frac{2p_3}{2 - p_3} \epsilon \int dv = \frac{2p_3}{2 - p_3} \epsilon v \qquad \ldots(60)$$

because $(-g)^{1/2} = 1$ and $T_1^1 = 0$. As the Newtonian density of matter is the relation between the total mass and its volume, (60) shows that this density is $-\frac{2p_3}{2 - p_3}\epsilon$ (and not ϵ).

$$g_{\text{clas}} = \frac{8\pi p_3 \epsilon}{2 - p_3} L \qquad \ldots(61)$$

and so

$$g_{\text{clas}} = g_{\text{rel}} \qquad \ldots(62)$$

within our approximation.

(5.6). We get plane thick disk of perfect fluid if

$$p_1 = p_2 = \frac{4}{3}\ ,\ p_3 = -\frac{2}{3} . \qquad \ldots(63)$$

For these values of p_1, p_2 and p_3 our case of ϵ = a constant coincides with that of Horsky and Novotny (1969).

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