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NATURAL CONVECTION EFFECTS ON MHD FLOW PAST AN IMPULSIVELY STARTED PERMEABLE VERTICAL PLATE

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An exact analysis of the effect of natural convection on an impulsively started permeable vertical plate allowing suction or injection at the wall in presence of uniform magnetic field is obtained for a fluid with unit Prandtl number. The effects of suction/injection, magnetic field and external heating/cooling of the plate by free convection on, velocity field and temperature field, and on skin friction and heat transfer rate, are discussed with the aid of graphs.

NOMENCLATURE LIST

B = intensity of magnetic field
G = Grashof number $(\beta g \nu (T_w - T_0)/U^3)$
g = acceleration due to gravity
h = heat transfer coefficient
k = thermal conductivity of the fluid
L = characteristic length
M = magnetic field parameter $(\sigma\beta^2\nu/\rho U^2)$
N_u = Nusselt number
P = Prandtl number (ν/α)
r = suction/injection parameter (v'/U)
T = temperature of the fluid
T_w = temperature of the plate
T_0 = temperature of the fluid at infinity
t' = time
t = nondimensional time $(t'U^2/\nu)$
u' = velocity of fluid parallel to the plate
u = nondimensional velocity (u'/U)
U = velocity of the free stream
v' = suction velocity
x', y' = coordinate axes parallel and normal to plate respectively
y = nondimensional distance normal to plate $(y' U/\nu)$

ν = kinematic viscosity
σ = electrical conductivity of the fluid
ρ = density of the fluid
θ = nondimensional temperature of the fluid $((T-T_0)/(T_w-T_0))$
β = coefficient of volume expansion
α = thermal diffusivity
τ' = skin friction at the plate
τ = nondimensional skin friction
erf = error function
erfc = complementary error function.

1. Introduction

The flow of an incompressible viscous fluid past an impulsively started infinite plate was first studied by Stokes (1851). This is also known as Rayleigh's problem in the literature. As this problem being important in technology, it has been studied by number of researchers; and they have considered this problem for bodies of different shapes such as cylinder, sphere etc. (c. f. Illingworth 1950, Stewartson 1951, Sakiadis 1961, Hall 1969, Elliott 1969).

The natural convection effect on flow problems is very important in heat transfer studies and hence has attracted the attention of numerous investigators. Ede (1967) has reviewed the modelling and solution of laminar natural convection boundary flows. The flow past a vertical plate moving impulsively in its own plane was studied by Soundalgekar (1977), where the effects of natural convection currents due to the heating or cooling of the plate were discussed. The magnetohydrodynamic case of this problem was considered by Soundalgekar *et al.* (1979). Kafousias *et al.* (1979) studied the same problem for the case of porous plate with constant suction, but in the absence of magnetic field. Recently Soundalgekar *et al.* (1981) considered the problem of impulsively started permeable plate allowing suction or injection at the wall in viscous incompressible fluid and they found that suction and injection have different effects on flow parameters. Because of its importance in industry as well as aerodynamics we extend this problem in case of magnetohydrodynamics. We present the results for a fluid with unit Prandtl number. The results are also compared with hydrodynamic flow case.

2. Analysis and Results

We take coordinate origin O at an arbitrary point on an infinite plate with cartesian coordinate axes Ox' and Oy' along and normal to the plate respectively. We consider an electrically conducting, viscous incompressible fluid filling the semiinfinite space $y' = 0$. The plate is initialy stationary i.e. at $t' \leqslant 0$ and it starts moving impulsively with constant velocity at $t' = 0^+$ parallel to itself along x'-axis. On the physical ground of the problem all the quantities are assumed to be functions of the space coordinate y' and time t'. With usual Buoussinesq approximation for the bouyancy term the system of governing equations for the present problem are given by,

Continuity equation :

$$\frac{\partial v'}{\partial y'} = 0 \quad \text{...(1)}$$

Momentum equation :

$$\frac{\partial u'}{\partial t'} + v' \frac{\partial u'}{\partial y'} = g\beta (T - T_0) + \nu \frac{\partial^2 u'}{\partial y'^2} - \sigma \frac{B^2 u'}{\rho} \quad \text{...(2)}$$

Energy equation :

$$\frac{\partial T}{\partial t'} + v' \frac{\partial T}{\partial y'} = \alpha \frac{\partial^2 T}{\partial y'^2}. \quad \text{...(3)}$$

The initial and boundary conditions of the problem are,

$$t' \leqslant 0 : u'(y', t') = 0, \quad T(y', t') = T_0 \quad \text{...(4)}$$

$$\left. \begin{array}{l} t' > 0 : u' = U, \quad T = T_w \quad \text{at } y' = 0 \\ t' > 0 : u' = 0, \quad T = T_0 \quad \text{as } y' \to \infty \end{array} \right\}. \quad \text{...(5)}$$

All the quantities have been defined in the nomenclature.
Equation (1) imply that the velocity v' is constant.

With the help of the following non-dimensional variables and parameters,

$$\left. \begin{array}{l} t = \dfrac{t'U^2}{\nu}, \; y = \dfrac{y'U}{\nu}, \; u = \dfrac{u'}{U}, \; \theta = \dfrac{T - T_0}{T_w - T_0} \\ r = \dfrac{v'}{U}, \; p = \dfrac{\nu}{\alpha}, \; M = \dfrac{\sigma B^2 \nu}{\rho U^2}, \; G = \dfrac{\nu g\beta (T_w - T_0)}{U^3} \end{array} \right\} \quad \text{...(6)}$$

eqns. (2) and (3), the initial condition (4) and the boundary conditions (5) reduce to the following equations respectively, as,

$$\frac{\partial u}{\partial t} - r \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + G\theta + Mu \quad \text{...(7)}$$

$$P \frac{\partial \theta}{\partial t} - Pr \frac{\partial \theta}{\partial y} = \frac{\partial^2 \theta}{\partial y^2} \quad \text{...(8)}$$

$$t \leqslant 0 : u(y, t) = 0, \quad \theta(y, t) = 0 \quad \text{...(9)}$$

$$\left. \begin{array}{l} t > 0 : u(0, t) = 1, \quad \theta(0, t) = 1 \\ t > 0 : u(\infty, t) = 0, \quad \theta(\infty, t) = 0. \end{array} \right\} \quad \text{...(10)}$$

Here the parameter r represents suction or injection depending on whether it is positive or negative respectively. The Grashof number, $G > 0$ represents external

cooling of the plate and $G < 0$ represents external heating of the plate. By usual Laplace transform technique the solution for the velocity field and temperature field for fluid with unit Prandtl number are obtained and are given as follows :
Temperature field,

$$\theta = \tfrac{1}{2}\, e^{2r\eta t}\ \mathrm{erfc}\,(\eta - \tfrac{1}{2}rt) + \tfrac{1}{2}\,\mathrm{erfc}\,(\eta + \tfrac{1}{2}rt) \qquad \ldots(11)$$

Velocity field,

$$u = (1 - GM^{-1})\ e^{-2r\eta t^{1/2}} \Big\{ e^{-2\eta t^{1/2}(1/4r^2+M)^{1/2}}\ \mathrm{erfc}\,(\eta - (\tfrac{1}{4}r^2 + M)^{1/2}\sqrt{t})$$
$$+ e^{2\eta t^{1/2}(1/4r^2+M)^{1/2}}\ \mathrm{erfc}\,(\eta + (\tfrac{1}{4}r^2 + M)^{1/2}\sqrt{t})\Big\}$$
$$+ \tfrac{1}{2}GM^{-1}\ e^{-2r\eta t^{1/2}}\ \mathrm{erfc}\,(\eta - \tfrac{1}{2}r\sqrt{t}) + \tfrac{1}{2}\,GM^{-1}\,\mathrm{erfc}\,(\eta + \tfrac{1}{2}r\sqrt{t}). \qquad \ldots(12)$$

In hydrodynamic case the velocity field is,

$$u = \tfrac{1}{2}\,(1 + 2\eta\sqrt{t}\,Gr^{-1})\ e^{-2r\eta\sqrt{t}}\ \mathrm{erfc}\,(\eta - \tfrac{1}{2}r\sqrt{t})$$
$$+ \tfrac{1}{2}\,(-2\eta\sqrt{t}\,Gr^{-1})\ \mathrm{erfc}\,(\eta + \tfrac{1}{2}r\sqrt{t}) \qquad \ldots(13)$$

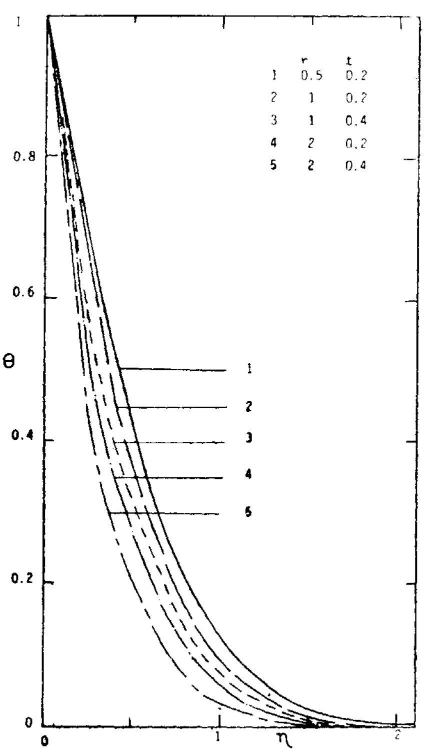

FIG. 1. Temperature profile for suction.

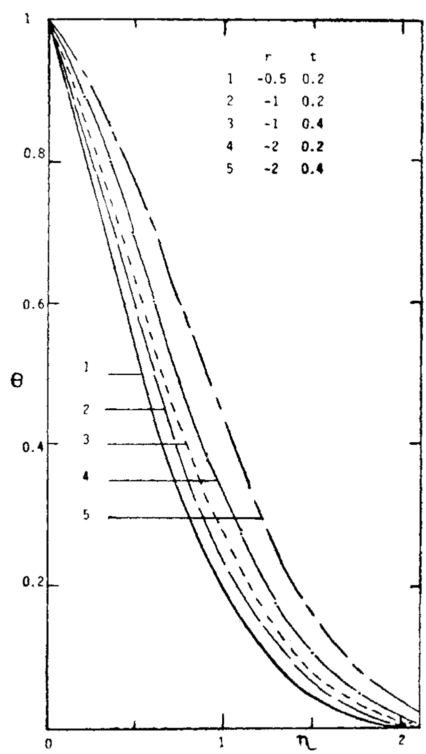

FIG. 2. Temperature profile for injection.

where

$$\eta = \tfrac{1}{2}y/\sqrt{t}.$$

To gain physical insight into the problem numerical values are obtained and are displayed in graphical form. The temperature profiles are shown in Figs. 1 and 2. In the case of suction at the plate (Fig. 1) the temperature of the fluid decreases with the increase in suction velocity, whereas in case of injection at the plate (Fig. 2) the temperature of the fluid increases with increase in injection velocity. Thus in case of injection the thermal boundary layer is extended away from the plate. With the development of time t, the temperature of the fluid decreases in presence of suction at the plate and in case of injection velocity at the plate the fluid temperature increases with increase in time t.

The velocity profiles in presence of magnetic field are displayed in Figs. 3–6. We observe from the Figs. 3 and 4 that for the plate being externally cooled ($G > 0$), an increase in suction velocity decreases the velocity of the fluid, whereas owing to an increase in injection, there is rise in the velocity of the fluid. With the higher

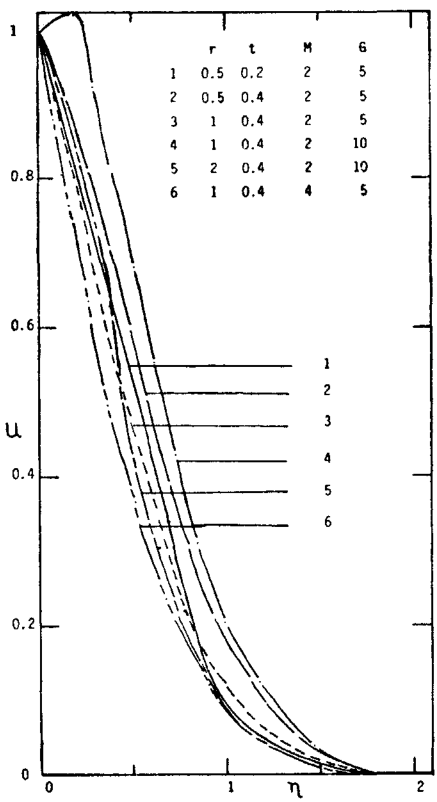

FIG. 3. Velocity profile for suction for cooling of plate.

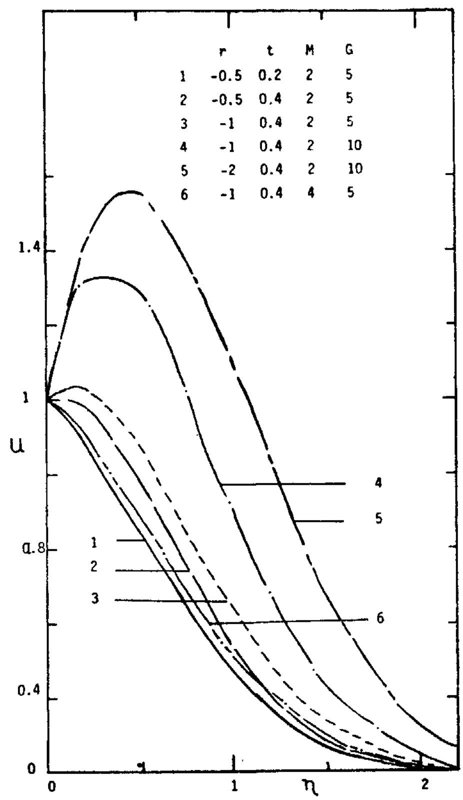

FIG. 4. Velocity profile for injection for heating of the plate.

magnetic field the fluid velocity decreases for both the cases of suction and injection at the plate, when the plate is cooled externally. The effect of time development is to enhance the velocity field for both the cases with suction and injection at the plate. The velocity field is higher in presence of injection at the plate compared with that when there is suction at the plate.

The more cooling of the plate causes higher velocity in the fluid layers near the plate. This is due to the same reasons as explained by Soundalgekar (1977), that the free convection currents are moving away from the plate and hence these currents tend to help the velocity to increase in the case of cooling of plate. This effect is more pronounced with the case of injection, which physically means, by the action of injection we enhance the motion of free convection currents which are moving away from the the plate and hence there is an increase in the velocity field near the plate.

In the case of externally heated plate an increase in the suction velocity decreases the velocity of the fluid up to some layer of the fluid adjascent to the plate and after this thin fluid layer the fluid velocity increases with increasing suction velocity at the plate (Fig. 5). Exactly reverse of these effects are observed in case of injection at the plate (Fig. 6) i.e., the velocity of the fluid is more with higher injection velocity up to some thin layer of the fluid near the plate and after this fluid layer

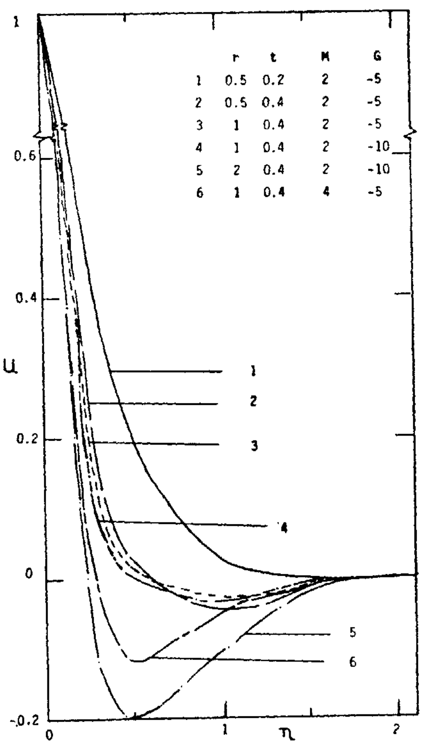

FIG. 5 Velocity profile for suction for heating of the plate.

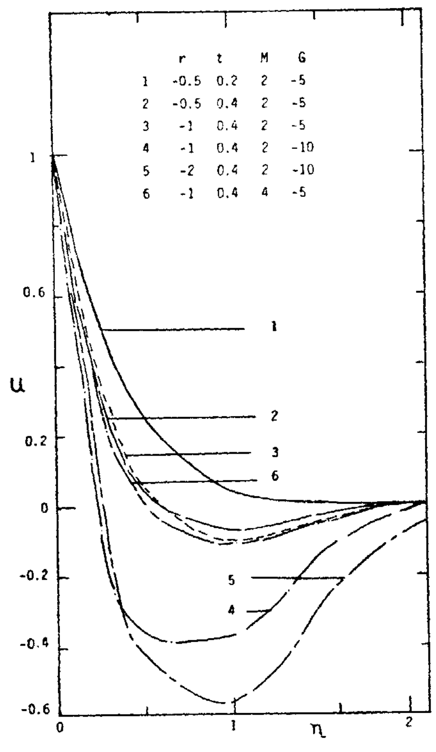

FIG. 6. Velocity profile for injection for heating of the plate.

the velocity field decreases with increasing injection velocity at the plate. With increase in magnetic field a decrease in the fluid velocity up to some fluid layer is observed in both the cases of suction and injection at the plate. There is a decrease in the fluid velocity with development of time t for both the cases of suction or injection at the plate, when the plate is being heated externally.

Increase in heating of the plate causes considerable decrease in the fluid velocity in either of the cases of suction or injection at the plate. This is because of the fact that the free convection currents are moving towards the plate and hence the fluid motion is opposed by these currents, resulting in decrease of fluid velocity for $G < 0$. With higher heating of the plate by free convection currents a flow reversal of the fluid is predicted. The velocity of the reverse flow increases with increase in injection velocity, because the injection velocity is an additional opposition to the convection currents, resulting, totally increase in reverse flow velocity in case of plate being heated.

In hydrodynamic case the effects of time, suction/injection for both heating or cooling of the plate on velocity profiles as predicted by Figs. 7 and 8 are as same as that in magnetohydrodynamic case. After we know the velocity field it is interesting to study the skin friction at the plate. It is given by the following equation :

$$\tau = \frac{\tau'}{\rho} U^2 = - \left(\frac{du}{dy} \right)_{y=0} . \qquad \ldots(14)$$

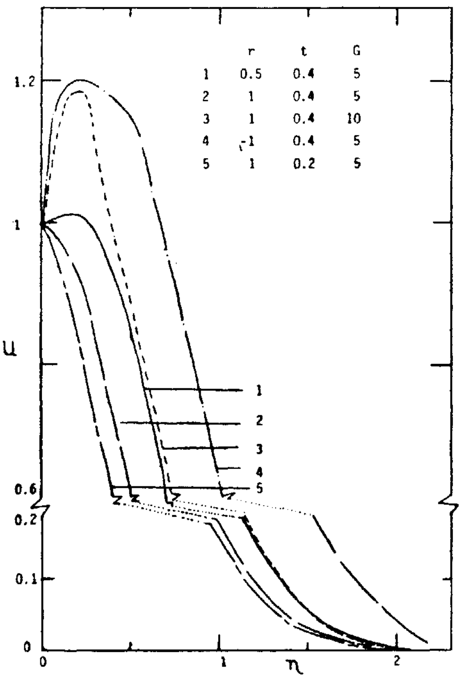

FIG. 7. Velocity profile for cooling of the plate (Hydrodynamic case).

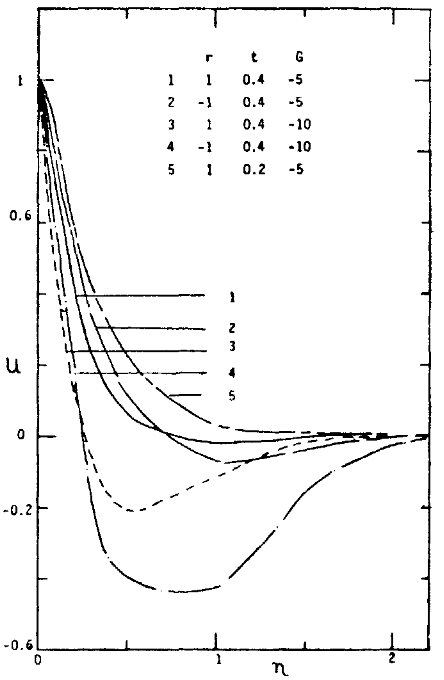

FIG. 8. Velocity profile for heating of the plate (Hydrodynamic case).

From eqns. (12) and (14) the expression for skin friction is given as

$$\tau = (1 - GM^{-1})\tfrac{1}{2}r\,\{1 + \text{erf}\,((\tfrac{1}{4}r^2 + M)^{1/2}\,t^{1/2})\} + \tfrac{1}{2}rGM^{-1}$$

$$\times\,(1 + \text{erf}\,(\tfrac{1}{2}r\,t^{1/2})) + (1 - GM^{-1})\,\frac{e^{-(1/4r^2+M)t}}{2\sqrt{t\pi}} + \frac{G\,e^{-1/4r^2t}}{2M\sqrt{t\pi}}. \quad \ldots(15)$$

From eqns. (13) and (14) the expression for skin friction in case of hydromagnetic flow is

$$\tau = \tfrac{1}{2}r + (\tfrac{1}{2}r - Gr^{-1})\,\text{erf}\,(\tfrac{1}{2}r\,t^{1/2}) + \frac{e^{-1/4r^2t}}{2\sqrt{t\pi}}. \quad \ldots(16)$$

The numerical plots of skin friction with magnetic field are presented in Figs. 9 and 10. For the case of heated plate the skin friction decreases with magnetic field with suction at the plate. Same effect is ture with cooled plate for injection at the plate. In case of heated plate with injection at plate a negative skin friction is predicted and this negative skin friction decreases with magnetic field increasing on the fluid. Skin friction decreases with increase in suction or time t for all M for the cooled plate. For the case with injection at the plate more cooling of the plate causes increase in τ.

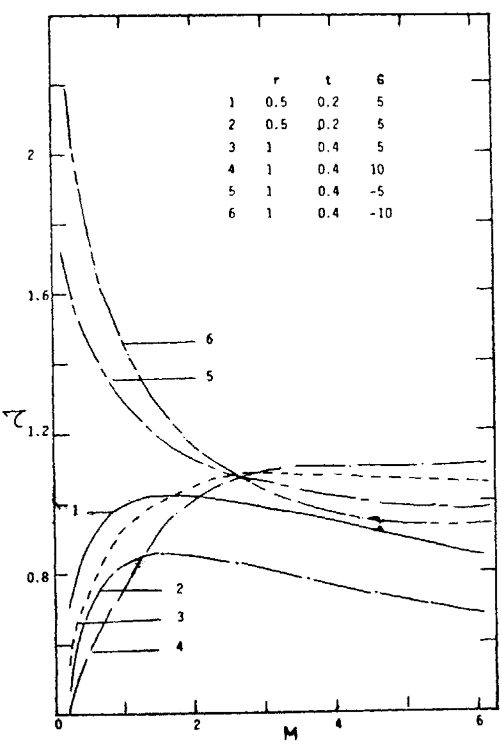

FIG. 9. Skin friction at the plate for suction.

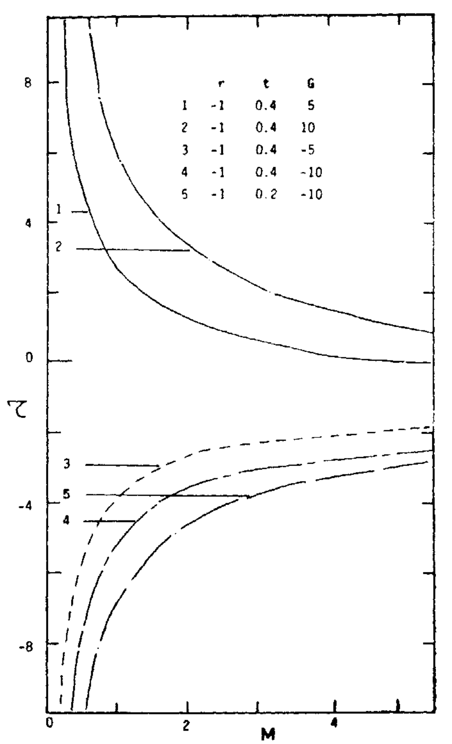

FIG. 10. Skin friction at the plate for injection.

However for more heating of the plate, with injection at plate the negative skin friction decreases and same effect is true with increase in time t.

In hydrodynamic case (Fig. 11), with increase in suction there is increase in τ in case of cooled plate. The skin friction decreases with higher cooling of the plate, in presence of suction at the plate. In the case of heated plate the skin friction increases with more heating plate.

As we known the temperature field, we now study the heat transfer rate at the plate which is given in terms of Nusselt number N_u, defined as,

$$N_u = \frac{hL}{k} \qquad \ldots(17)$$

where L has dimension of length, the heat transfer coefficient h is given as

$$h = \frac{-k}{(T_w - T_0)} \left(\frac{\partial T}{\partial y'}\right)_{y'=0}.$$

With introduction of non-dimensional quantities the Nusselt number is given as

$$N_u = -\left(\frac{d\theta}{dy}\right)_{y=0}.$$

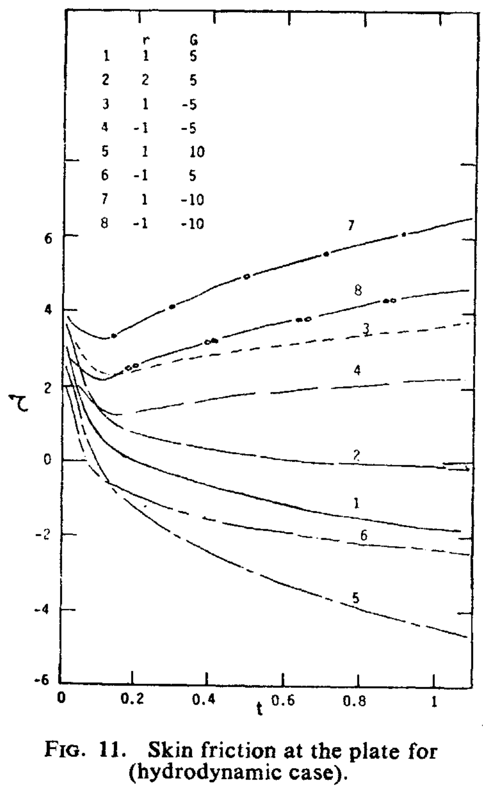

FIG. 11. Skin friction at the plate for (hydrodynamic case).

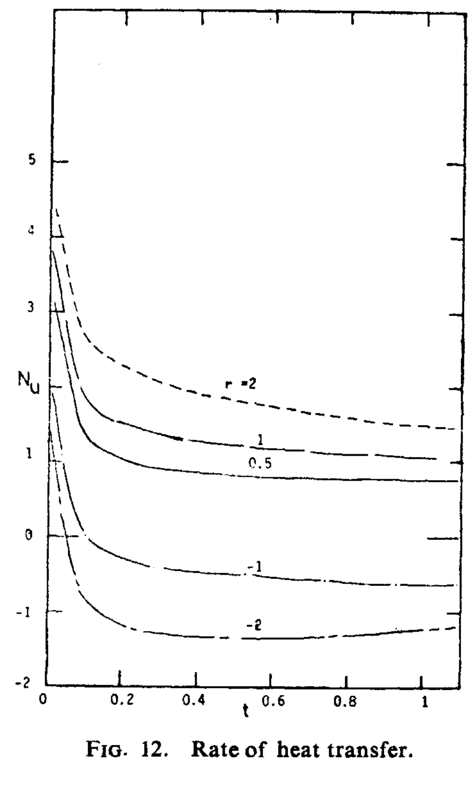

FIG. 12. Rate of heat transfer.

Hence the Nusselt number is given as,

$$N_u = \tfrac{1}{2}r\,(1 + \text{erf}\,(\tfrac{1}{2}r\,t^{1/2})) + \frac{e^{-1/4r^2t}}{2\sqrt{t\pi}}\,. \qquad \ldots(18)$$

Figure 12 shows the variation of heat transfer rate at the plate with time. With increase in suction/injection there is increase/decrease in the rate of heat transfer. A negative N_u is predicted with higher injection velocities. The steady state heat transfer rate is determined by the suction/injection velocity at the plate as can be seen byFig. 12 as $t \to \infty$.

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