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HEAT TRANSFER IN A ROTATING MHD CHANNEL WITH INFINITELY CONDUCTING PLATES

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Exact solution to the temperature field in a MHD rotating channel is derived. The channel plates are assumed to be infinitely electrically conducting. It is observed that the rate of heat transfer decreases due to rotation or an increase in the strength of the magnetic field.

1. INTRODUCTION

MHD channel flows are studied extensively by many researchers because of their practical applications in Nuclear Engineering etc. Hartmann (1937) first studied the flow of a viscous incompressible fluid under transverse magnetic field in a MHD channel flow. The heat transfer aspect of such a flow has been discussed by many researchers, like Soundalgekar (1973). If the channel is rotating, about a vertical axis, there exists a secondary flow. The study of such secondary flows was presented in a rotating channel by Vidyanidhi and Nigam (1967). The effects of a transversely applied magnetic field on such a secondary flow of an electrically conducting fluid were studied by Vidyanidhi (1969). Induced magnetic field was taken into account. The heat transfer aspect of Vidyanidhi (1969) was recently studied by Bhat (1981) for nonconducting plate. How the heat transfer is affected by infinitely electrically conducting plates of a rotating channel, it has not been studied in literature. Hence it is now proposed to study the heat transfer in a rotating plane channel. In section 2, the mathematical analysis is presented and in section 3 conclusions are presented.

2. MATHEMATICAL ANALYSIS

Consider a parallel plate channel rotating about the z'-axis which is taken normal to $x'y'$-plane where x'-axis is taken along the centreline of the channel whose plates are separated by a width of $2L$ and y'-axis is taken normal to x'-axis. If u, v are the velocity components along x' and y' directions, B_x and B_y are the induced magnetic field in dimensionless form, the flow can be shown to be governed by the following equations (Squire 1956) :

$$\frac{d^2q}{dz^2} + M^2 \frac{dH}{dz} - 2\alpha^2 iq = -R \qquad \ldots(1)$$

$$\frac{d^2H}{dz^2} + \frac{dq}{dz} = 0 \qquad \ldots(2)$$

$$\frac{d^2\theta}{dz^2} + PE\left\{\frac{dq}{dz}\frac{\bar{dq}}{dz} + M^2\frac{dH}{dz}\frac{\bar{dH}}{dz}\right\} = 0. \qquad ...(3)$$

Here (-) denotes a complex conjugate. The non-dimensional quantities are defined as follows:

$$z = z'/L,\ u = u'L/\nu,\ v = v'L/\nu,\ B_x = B_{x'}'/\sigma\mu_e\nu B_0$$

$$B_y = B_{y'}'/\sigma\mu_e\nu B_0,\ R = L^3\left(-\frac{\partial P'}{\partial x'}\right)\Big/\rho\nu^2,\ \alpha^2 = \Omega' L^2/\nu$$

$$M^2 = L^2B_0^2\sigma/\rho\nu,\ P = \mu c_p/K,\ E = 2(\nu/L)^2/c_p(T_1 - T_2)$$

$$\theta = \left(T - \frac{T_1 + T_2}{2}\right)\Big/\left(\frac{T_1 - T_2}{2}\right) \qquad ...(4)$$

$$q = u + iv,\ H = B_x + iB_y,\ i = \sqrt{-1}$$

T_1 is the wall temperature at $z' = L$ and T_2 is the wall temperature at $z' = -L$. The boundary conditions are

$$\left.\begin{aligned} &q(\pm 1) = 0, \frac{dH}{dz}(\pm 1) = 0 \\ &\theta(-1) = -1,\ \theta(+1) = 1 \end{aligned}\right\}. \qquad ...(5)$$

The solutions of eqns. (1)—(3), satisfying the boundary conditions (5) are as follows:

$$q/R = \phi(\cosh A - \cosh Az) \qquad ...(6)$$

$$H/R = \phi(\sinh Az - Az\cosh A)/A \qquad ...(7)$$

$$\theta = z + PER^2 G(z) \qquad ...(8)$$

where

$$A = \sqrt{M^2 + 2i\alpha^2},\ \phi = 1/A^2\cosh A$$

$$G(z) = C_1(1 - z^2) + C_2(\cosh 2\alpha' - \cosh 2\alpha' z) + C_3(\cos 2\beta' - \cos 2\beta' z) - \mathrm{Re}\,[C_4(\cosh A - \cosh Az)]$$

$$\alpha' = \mathrm{Re}\,[A],\ \beta' = Im[A]$$

$$C_1 = M^2/2(A\bar{A})^2,\ C_2 = \phi\bar{\phi}(A\bar{A} + M^2)8\alpha'^2$$

$$C_3 = \phi\bar{\phi}(A\bar{A} - M^2)/8\beta'^2,\ C_4 = 4\phi C_1.$$

Particular Cases

(1) When $M = 0$, we get

$$q/R = (\cosh A - \cosh Az)/2i\alpha^2\cosh A,\ A = \alpha(1 + i) \qquad ...(9)$$

$$\theta = z + \frac{PER^2}{8\alpha^4}\left\{1 - \frac{\cosh 2\alpha z + \cos 2\alpha z}{\cosh 2\alpha + \cos 2\alpha}\right\}. \qquad ...(10)$$

(2) When $\alpha \to 0$

$$u/R = (\cosh M - \cosh Mz)/M^2\cosh M,\ v = 0 \qquad ...(11)$$

$$B_x/R = (\sinh Mz - Mz\cosh M)/M^3\cosh M,\ B_y = 0 \qquad ...(12)$$

$$\theta = z + PER^2\left\{\frac{1-z^2}{2M^2} + \frac{\cosh 2M - \cosh 2Mz}{4M^4 \cosh^2 M} - \frac{2(\cosh M - \cosh Mz)}{M^4 \cosh M}\right\}. \quad \ldots(13)$$

(3) When $M \to 0$, $\alpha \to 0$, we get

$$u/R = \frac{1}{2}(1 - z^2), \quad v = 0$$

$$\theta = z + \frac{PER^2}{12}(1 - z^4).$$

In order to get a physical insight of the problem, we have carried out the numerical calculations for $G(z)$ and these are shown on Fig. 1. We observe from Fig. 1 that because of the rotation of the channel, there is a fall in the value of $G(z)$. An increase in M also leads to a decrease in the value of $G(z)$.

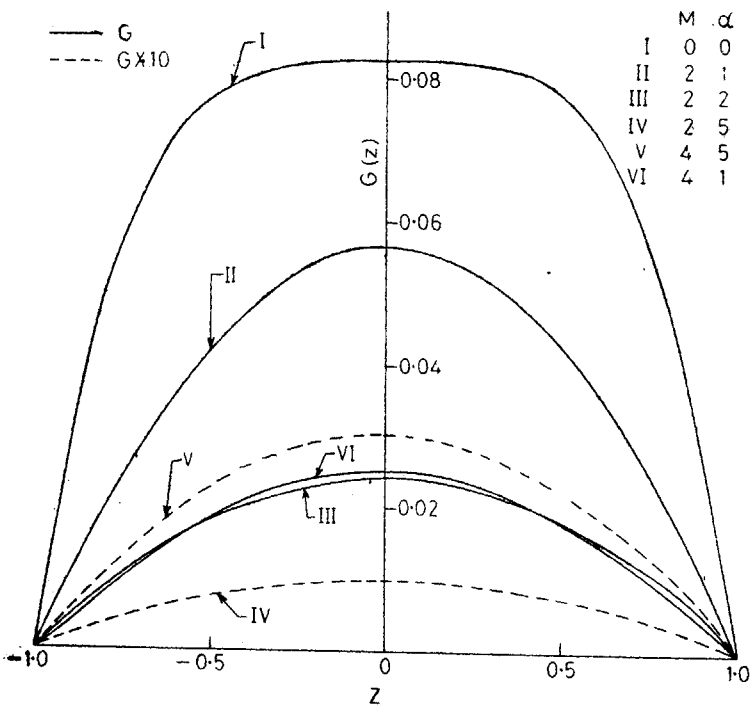

FIG. 1. The function $G(z)$.

From the temperature field, we now study the effects of rotation on the rate of heat transfer. It is given in dimensionless form as

$$Nu = \left.\frac{d\theta}{dz}\right|_{z=-1} = 1 + PER^2\, G'(-1). \quad \ldots(14)$$

We have calculated the numerical values of $G'(-1)$ and these are entered in Table I.

TABLE I
Values of $G'(-1)$

M	α	$G'(-1)$
0	0	0.33333
2	1	0.11854
2	2	0.05340
2	5	0.00333
4	5	0.00663
4	1	0.04637

We conclude from this table that due to the rotation of the channel, there is a decrease in the value of $G'(-1)$. For small values of α an increase in M leads to a decrease in the value of $G'(-1)$.

3. CONCLUSIONS

(i) Because of the rotation of the channel there is a fall in the temperature of the fluid.

(ii) An increase in M also leads to a decrease in the temperature.

(iii) Due to the rotation of the channel, the rate of heat transfer decreases.

(iv) For small values of the Ekman number α, an increase in M leads to a decrease in the value of the rate of heat transfer.

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