HYDROMAGNETIC FLOW OF AN ELECTRICALLY CONDUCTING FLUID DUE TO UNSTEADY ROTATION OF A POROUS DISK OVER A FIXED DISK

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The unsteady hydromagnetic flow of a viscous, incompressible and electrically conducting fluid due to rotatory vibrations of a porous disk about a constant non-zero mean over a fixed disk has been analysed. The fixed disk is supposed to be non-porous and the angular velocity of the other disk which is porous is assumed to consist of basic steady distribution together with a time varying distribution.

1. INTRODUCTION

The steady flow of a viscous incompressible fluid due to an infinite rotating disk was first considered by von Kármán (1921). Stewartson (1953) investigated, experimentally as well as theoretically, the steady motion of a viscous fluid between two coaxial rotating disks. It is found experimentally that when the disks rotate in the same sense the main body of the fluid rotates as well, but if they rotate in the opposite sense the main body of fluid is almost at rest. He has also given an adequate theory to explain this. Further Nihoul (1964) studied the motion of a viscous conducting fluid between two rotating walls under the influence of a magnetic field set up by a line current along the axis of rotation. The flow between a rotating and a fixed disk has recently been studied by Mellore *et al.* (1968).

In the present paper, the flow of a viscous, electrically conducting and incompressible fluid due to rotating vibrations of a porous disk about a constant non-zero mean over a fixed disk has been studied. It is assumed that the angular velocity of the disk with unsteady rotation consists of a basic, steady distribution together with a weak time varying distribution. The amplitude of the superimposed vibration is supposed to be so small that its squares and higher powers can be neglected. The results obtained are valid for small rotational Taylor number of the rotating disk.

2. FUNDAMENTAL EQUATIONS AND BOUNDARY CONDITIONS

Let us take the axis of rotation of the two disks as $\bar{z}$-axis and let the two disks be situated at $\bar{z} = 0$ and $\bar{z} = h$. Let the porous disk at $\bar{z} = 0$ have unsteady rotation with an angular velocity $\Omega(1+\epsilon \cos \omega\bar{t})$ while the disk at $\bar{z} = h$ is at rest.

The equation of motion in cylindrical polar coordinates $(\bar{r}, \bar{\theta}, \bar{z})$ are

$$\frac{\partial \bar{u}}{\partial t}+\bar{u}\frac{\partial \bar{u}}{\partial \bar{r}}+\bar{w}\frac{\partial \bar{u}}{\partial z}-\frac{\bar{v}^2}{\bar{r}} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial \bar{r}}$$

$$+\nu\left[\frac{\partial}{\partial \bar{r}}\left(\frac{\partial \bar{u}}{\partial \bar{r}}+\frac{\bar{u}}{\bar{r}}\right)+\frac{\partial^2 \bar{u}}{\partial \bar{z}^2}\right]-\frac{\sigma B_0{}^2}{\rho}\bar{u} \quad \ldots \quad \ldots \quad \ldots \quad (2.1)$$

$$\frac{\partial \bar{v}}{\partial t}+\bar{u}\frac{\partial \bar{v}}{\partial \bar{t}}+\bar{w}\frac{\partial \bar{v}}{\partial \bar{z}}+\frac{\bar{u}\,.\,\bar{v}}{\bar{r}}$$

$$= \nu\left[\frac{\partial}{\partial \bar{r}}\left(\frac{\partial \bar{v}}{\partial \bar{r}}+\frac{\bar{v}}{\bar{r}}\right)+\frac{\partial^2 \bar{v}}{\partial \bar{z}^2}\right]-\frac{\sigma B_0{}^2}{\rho}\bar{v} \quad \ldots \quad \ldots \quad \ldots \quad (2.2)$$

$$\frac{\partial \bar{w}}{\partial \bar{t}}+\bar{u}\frac{\partial \bar{w}}{\partial \bar{r}}+\bar{w}\frac{\partial \bar{w}}{\partial \bar{z}} = -\frac{1}{\rho}\frac{\partial \bar{p}}{\partial \bar{z}}$$

$$+\nu\left[\frac{1}{r}\frac{\partial}{\partial \bar{r}}\left(\bar{r}\frac{\partial \bar{w}}{\partial \bar{r}}\right)+\frac{\partial^2 \bar{w}}{\partial \bar{z}^2}\right] \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (2.3)$$

and the equation of continuity is

$$\frac{\partial}{\partial \bar{r}}(\bar{r}\,.\,\bar{u})+\frac{\partial}{\partial \bar{z}}(\bar{r}\,.\,\bar{w}) = 0 \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (2.4)$$

where $\bar{u}$, $\bar{v}$, $\bar{w}$ denote, respectively, the radial, transverse and axial components of velocity, $\bar{p}$ the fluid pressure, $\bar{t}$ the time, B_0 the strength of the uniform axial magnetic field, σ the electrical conductivity, ν the kinematic viscosity of the fluid and ρ its density. From symmetry of flow all quantities are independent of $\bar{\theta}$. The induced magnetic field is neglected by assuming magnetic Prandtl number to be small.

The boundary conditions are

$$\left.\begin{array}{l} \bar{z} = 0;\ \ \bar{u} = 0,\ \ \bar{v} = \bar{r}\,\Omega(1+\epsilon\cos\omega\bar{t}),\ \ \bar{w} = v_0, \\ \bar{z} = h;\ \ \bar{u} = 0,\ \ \bar{v} = 0,\ \ \bar{w} = 0 \end{array}\right\} \quad \ldots \quad \ldots \quad (2.5)$$

where v_0 is the constant suction velocity at the rotating disk.

Appropriate dimensionless variables are defined by

$$r = \frac{\bar{r}}{h},\ \ z = \frac{\bar{z}}{h},\ \ u = \frac{\bar{u}\,.\,h}{\nu},\ \ v = \frac{\bar{v}\,.\,h}{\nu},$$

$$w = \frac{\bar{w}h}{\nu},\ \ p = \frac{\bar{p}\,.\,h^2}{\rho\nu^2},\ \ t = \frac{\bar{t}\,.\,\nu}{h^2}.$$

In terms of these dimensionless variables the equations of motion and the boundary conditions become

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial r}+w\frac{\partial u}{\partial z}-\frac{v^2}{r}=-\frac{\partial p}{\partial r}+\frac{\partial}{\partial r}\left(\frac{\partial u}{\partial r}+\frac{u}{r}\right)+\frac{\partial^2 u}{\partial_z^2}-M^2u \qquad (2.6)$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial r}+w\frac{\partial v}{\partial z}+\frac{u\,.\,v}{r}=\frac{\partial}{\partial r}\left(\frac{\partial v}{\partial r}+\frac{v}{r}\right)+\frac{\partial^2 v}{\partial z^2}-M^2v \qquad (2.7)$$

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial r}+w\frac{\partial w}{\partial z}=-\frac{\partial p}{\partial z}+\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}+\frac{\partial^2 w}{\partial z^2} \qquad (2.8)$$

$$\frac{\partial}{\partial r}(ru)+\frac{\partial}{\partial z}(r\,.\,w)=0 \qquad (2.9)$$

and

$$\left.\begin{aligned}&z=0;\ u=0,\ v=r\,\alpha(1+\epsilon\cos\beta t),\ w=2K,\\&z=1;\ u=0,\ v=0,\ w=0\end{aligned}\right\} \qquad (2.10)$$

where

$$M^2=\frac{\sigma B^2{}_0h^2}{\rho\nu},\quad \alpha=\frac{\Omega h^2}{\nu},\quad \beta=\frac{\omega h^2}{\nu}\text{ and }K=\frac{v_0h}{2\nu}.$$

3. Solution of the Problem

In solving the above differential equations it is convenient to adopt the complex notation for harmonic functions, the real part of which will have physical significance. The angular velocity of the disk, which can be written as the real part of $\alpha(1+\epsilon\, e^{i\beta t})$, consists of a basic steady distribution α with a superimposed weak time varying distribution $\epsilon\alpha\, e^{i\beta t}$

We now write u, v, w and p as the sum of steady and small oscillatory components. Thus

$$\left.\begin{aligned}u&=u_s+\epsilon u_1e^{i\beta t}\\v&=v_s+\epsilon v_1e^{i\beta t}\\w&=w_s+\epsilon w_1e^{i\beta t}\\p&=p_s+\epsilon p_1e^{i\beta t}\end{aligned}\right\} \qquad (3.1)$$

where u_s, v_s, w_s, p_s is the steady mean flow and satisfies

$$u_s\frac{\partial u_s}{\partial r}+w_s\frac{\partial u_s}{\partial z}-\frac{v_s^2}{r}=-\frac{\partial p_s}{\partial r}+\frac{\partial}{\partial r}\left(\frac{\partial u_s}{\partial r}+\frac{u_s}{r}\right)+\frac{\partial^2 u_s}{\partial z^2}-M^2u_s \qquad (3.2)$$

$$u_s\frac{\partial v_s}{\partial r}+w_s\frac{\partial v_s}{\partial z}+\frac{u_sv_s}{r}=\frac{\partial}{\partial r}\left(\frac{\partial v_s}{\partial r}+\frac{v_s}{r}\right)+\frac{\partial^2 v_s}{\partial z}-M^2v_s \qquad (3.3)$$

$$u_s\frac{\partial w_s}{\partial r}+w_s\frac{\partial w_s}{\partial z}=-\frac{\partial p_s}{\partial z}+\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w_s}{\partial r}\right)+\frac{\partial^2 w_s}{\partial z^2} \qquad (3.4)$$

$$\frac{\partial}{\partial r}(r\,.\,u_s)+\frac{\partial}{\partial z}(r\,.\,w_s)=0 \qquad (3.5)$$

with boundary conditions

$$\left.\begin{aligned} z = 0\,;\; u_s = 0,\, v_s = r\alpha,\, w_s = 2K, \\ z = 1\,;\; u_s = 0,\, v_s = 0,\, w_s = 0 \end{aligned}\right\} \qquad (3.6)$$

Neglecting squares of ϵ and dividing by $e^{i\beta t}$, we find that u_1, v_1, w_1, p_1 satisfy the following differential set

$$i\beta u_1 + u_s \frac{\partial u_1}{\partial r} + u_1 \frac{\partial u_s}{\partial r} + w_s \frac{\partial u_1}{\partial z} + w_1 \frac{\partial u_s}{\partial z} - \frac{2v_s \,.\, v_1}{r}$$

$$= -\frac{\partial p_1}{\partial r} + \frac{\partial^2 u_1}{\partial r^2} + \frac{1}{r}\frac{\partial u_1}{\partial r} + \frac{\partial^2 u_1}{\partial z^2} - \frac{u_1}{r^2} - M^2 u_1 \qquad (3.7)$$

$$i\beta v_1 + u_s \frac{\partial v_1}{\partial r} + u_1 \frac{\partial v_s}{\partial r} + w_s \frac{\partial v_1}{\partial z} + w_1 \frac{\partial v_s}{\partial z} + \frac{u_1 v_s + u_s v_1}{r}$$

$$= \frac{\partial^2 v_1}{\partial r} + \frac{1}{r}\frac{\partial v_1}{\partial r} + \frac{\partial^2 v_1}{\partial z^2} - \frac{v_1}{r^2} - M^2 v_1 \qquad (3.8)$$

$$i\beta u_1 + u_s \frac{\partial w_1}{\partial r} + u_1 \frac{\partial w_s}{\partial r} + w_s \frac{\partial w_1}{\partial z} + w_1 \frac{\partial w_s}{\partial z}$$

$$= -\frac{\partial p_1}{\partial z} + \frac{\partial^2 w_1}{\partial r^2} + \frac{1}{r}\frac{\partial w_1}{\partial r} + \frac{\partial^2 w_1}{\partial z^2} \qquad (3.9)$$

$$\frac{\partial}{\partial r}(r \,.\, u) + \frac{\partial}{\partial z}(r \,.\, w) = 0, \qquad (3.10)$$

with the boundary conditions

$$\left.\begin{aligned} z = 0\,;\; u_1 = 0,\, v_1 = r\alpha,\, w_1 = 0, \\ z = 1\,;\; u_1 = 0,\, v_1 = 0,\, w_1 = 0 \end{aligned}\right\} \qquad (3.11)$$

Equations (3.5) and (3.10) are satisfied by taking

$$u_s = -rf'_s(z),\; v_s = rg_s(z),\; w_s = 2f_s(z),$$

$$u_1 = -rf'_1(z),\; v_1 = rg_1(z),\; w_1 = 2f_1(z).$$

Substituting these values of u_s, v_s, w_s in eqns. (3.2) and (3.3) and the values of u_1, v_1, w_1 in eqns. (3.7) and (3.8) we get the following two systems of equations:

System 1

$$f_1''' + f_s'^2 = M^2 f'_s + 2f_s f''_s + g_s^2 + P_s$$

$$g''_s + 2f'_s g_s = M^2 g_s + 2f_s g'_s$$

where $-\frac{1}{r}\frac{\partial p_s}{\partial r} = P_s$ is assumed to be constant.

System 2

$$f_1''' + 2f'_s f'_1 = (i\beta + M^2) f'_1 + 2f_s f''_1 + 2f''_s f_1 + 2g_s g_1 + P_1$$

$$g''_1 + 2f'_s g_1 + 2f'_1 g_s = (i\beta + M^2) g_1 + 2f_s g'_1 + 2f_1 g'_s$$

where $-\frac{1}{r}\frac{\partial p_1}{\partial r} = P_1$ is assumed to be constant.

4. Solution of System 1

For small values of α, the angular velocity g_s is of order α and the first equation of system 1 shows that f_s will be of order α^2. Hence we may assume

$$f_s = \alpha^2\bar{F}_1+\alpha^3\bar{F}_2+\ldots$$

$$g_s = \alpha\bar{G}_1+\alpha^2\bar{G}_2+\alpha^3\bar{G}_3+\ldots$$

$$P_s = \alpha^2\bar{P}_1+\alpha^3\bar{P}_2+\ldots$$

$$K = \alpha^2K_1+\alpha^3K_2+\ldots$$

Substituting these values of f_s, g_s, P_s and K in system 1 and equating like powers of α on both sides we get an infinite set of linear differential equations with constant coefficients. The first three sets are

$$\bar{G}''_1 = M^2\bar{G}_1 \qquad (4.1)$$

$$\left.\begin{aligned}\bar{F}_1'''-\bar{G}_1^2 &= M^2\bar{F}'_1+\bar{P}_1\\ \bar{G}_2'' &= M^2\bar{G}_2\end{aligned}\right\} \qquad (4.2)$$

$$\left.\begin{aligned}\bar{F}_2'''-2\bar{G}_1\bar{G}_2 &= M^2\bar{F}'_2+\bar{P}_2\\ \bar{G}''_3+2\bar{F}'_1\bar{G}_1-2\bar{F}_1\bar{G}'_1 &= M^2\bar{G}_3\end{aligned}\right\}. \qquad (4.3)$$

Solving these and applying boundary conditions we get

$$\bar{G}_1(z) = A_1 \cosh Mz+B_1 \sinh Mz \qquad (4.4)$$

where

$$A_1 = 1,\ B_1 = -\coth M.$$

$$\bar{G}_2(z) = \bar{F}_2(z) = 0. \qquad (4.5)$$

$$\begin{aligned}\bar{F}_1(z) = {} & A_2 \cosh Mz+B_2 \sinh Mz\\ & +\frac{A_1^2+B_1^2}{24M^3}\sinh 2Mz+\frac{A_1B_1}{12M^3}\cosh 2Mz\\ & -\frac{1}{M^2}\left(P_1+\frac{A_1^2-B_1^2}{2}\right)z+C_2\end{aligned} \qquad (4.6)$$

where

$$A_2 = K_1-\left(\frac{A_1B_1}{12M^3}+C_2\right),\ B_2 = \frac{1}{M^3}\left(P_1+\frac{A_1^2-B_1^2}{2}-\frac{A_1^2+B_1^2}{12}\right)$$

$$P_1 = \frac{R_2M^2(\cosh M-1)-R_1M^3 \sinh M}{2 \cosh M-M \sinh M-2}$$

$$C_2 = \frac{R_2(\sinh M-M)-R_1M(\cosh M-1)}{M(2 \cosh M-M \sinh M-2)}$$

and

$$R_1 = K_1 \cosh M + \frac{A_1^2+B_1^2}{24M^3}(\sinh 2M - 2\sinh M)$$

$$+\frac{A_1^2-B_1^2}{2M^2}(\sinh M - M) + \frac{A_1B_1}{12M^3}(\cosh 2M - \cosh M)$$

$$R_2 = K_1 M \sinh M + \frac{A_1^2+B_1^2}{12M^2}(\cosh 2M - \cosh M)$$

$$+\frac{A_1^2-B_1^2}{2M^2}(\cosh M - 1) + \frac{A_1B_1}{12M^2}(2\sinh 2M - \sinh M).$$

$$\bar{F}_2(z) = A_3 \cosh Mz + B_3 \sinh Mz - \frac{P_2}{M^2}z + C_3 \quad \ldots \quad \ldots \quad (4.7)$$

where

$$A_3 = K_2 - C_3, \quad B_3 = \frac{P_2}{M^3}$$

$$P_2 = -\frac{K_2 M^3 \sinh M}{2\cosh M - M\sinh M - 2}$$

$$C_3 = \frac{K_2 M(\cosh M - 1) + K_2 M^2 \sinh M}{2\cosh M - M \sinh M - 2}.$$

$$\bar{G}_3(z) = A_4 \cosh Mz + B_4 \sinh Mz - \frac{2(A_2B_1 - A_1B_2)}{M}$$

$$-\frac{(A_1^2+3B_1^2)A_1}{384M^4}\cosh 3Mz - \frac{(3A_1^2+B_1^2)B_1}{384M^4}\sinh 3Mz$$

$$-\left[\frac{A_1^2+3B_1^2}{48M^3} - \left\{\frac{(A_1^2-B_1^2+2P_1)A_1}{2M^3} + B_1C_2\right\}\right] z \sinh Mz$$

$$-\left[\frac{A_1^2-B_1^2}{16M^3} - \left\{\frac{(A_1^2-B_1^2+2P_1)B_1}{2M^3} + A_1C_2\right\}\right] z \cosh Mz$$

$$-\frac{B_1(A_1^2-B_1^2+2P_1)}{4M^2} z^2 \sinh Mz \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (4.8)$$

where

$$A_4 = \frac{2(A_2B_1 - A_1B_2)}{M} + \frac{(A_1^2+3B_1^2)A_1}{384M^4}$$

$$B_4 = \frac{2(A_2B_1 - A_1B_2)}{M \sinh M} + \frac{(A_1^2+3B_1^2)A_1}{384M^4} \cdot \frac{\cosh 3M}{\sinh M} + \frac{(3A_1^2+B_1^2)B_1}{384M^4} + \frac{\sinh 3Mz}{\sinh M}$$

$$+\left[\frac{A_1^2-B_1^2}{16M^3} - \left\{\frac{(A_1^2+3B_1^2)A_1}{384M^4} + \frac{(A_1^2-B_1^2+2P_1)B_1}{2M^3} + \frac{2(A_2B_1-A_1B_2)}{M} + A_1C_2\right\}\right]$$

$$\times \coth M + \left[\frac{A_1^2+3B_1^2}{48M^3} - \left\{\frac{(A_1^2-B_1^2+2P_1)(2A_1+B_1M)}{4M^3} + B_1C_2\right]\right..$$

For the non-magnetic case, i.e. when $M = 0$, the above functions reduce to

$$\bar{F}_1(z) = K_1 - \left(3K_1 + \frac{1}{20}\right)z^2 + \frac{1}{6}\left(12K_1 + \frac{7}{10}\right)z^3 - \frac{1}{12}z^4 + \frac{1}{60}z^5 \quad .. \quad (4.9)$$

$$\bar{F}_2(z) = K_2 - 3K_2z^2 + 2K_2z^3 \quad .. \quad .. \quad .. \quad .. \quad .. \quad (4.10)$$

$$\bar{G}_1(z) = 1 - z, \quad \bar{G}_2(z) = 0 \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad (4.11)$$

$$\bar{G}_3(z) = \left(\frac{K_1}{10} - \frac{3}{700}\right)z - K_1z^2 + \left(2K_1 + \frac{1}{30}\right)z^3 - \frac{1}{6}\left(9K_1 + \frac{2}{5}\right)z^4$$
$$+ \frac{1}{30}\left(12K_1 + \frac{17}{10}\right)z^5 - \frac{1}{45}z^6 + \frac{1}{315}z^7, \quad .. \quad .. \quad (4.12)$$

and when $K = 0$, i.e. the rotating plate is non-porous, the functions become

$$\bar{F}_1(z) = -\frac{z^2}{20} + \frac{7z^3}{60} - \frac{z^4}{12} + \frac{z^5}{60} \quad .. \quad .. \quad .. \quad .. \quad (4.13)$$

$$\bar{F}_2(z) = 0 \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad .. \quad (4.14)$$

$$\bar{G}_1(z) = 1 - z, \quad \bar{G}_2(z) = 0 \quad .. \quad .. \quad .. \quad .. \quad .. \quad (4.15)$$

$$\bar{G}_3(z) = -\frac{3z}{700} + \frac{z^3}{30} - \frac{z^4}{15} + \frac{17z^5}{300} - \frac{z^6}{45} + \frac{z^7}{315}. \quad .. \quad .. \quad (4.16)$$

These values reduce to those obtained by Mellore *et al.* (1968) if we transfer our origin to the fixed disk.

5. Solution of System 2

We may take

$$f_1 = \alpha^2 F_1 + \alpha^3 F_2 + \ldots$$
$$g_1 = \alpha G_1 + \alpha^2 G_2 + \alpha^3 G_3 + \ldots$$
$$P_1 = \alpha^2 R_1 + \alpha^3 R_2 + \ldots$$

Substituting these values in system 2 and solving the systems of linear differential equations, we get

$$G_1(z) = A_5 \cosh mz + B_5 \sinh mz \quad .. \quad .. \quad .. \quad .. \quad (5.1)$$

where

$$A_5 = 1, \quad B_5 = -\coth m, \quad m = (M^2 + i\beta)^{\frac{1}{2}}.$$

$$F'_1(z) = A_6 \cosh mz + B_6 \sinh mz + \frac{A_5}{2m}(2-z)z \sinh mz$$
$$+ \frac{B_5}{2m}(2-z)z \cosh mz - \frac{R_3}{m^2} \quad .. \quad .. \quad .. \quad .. \quad (5.2)$$

where

$$A_6 = \frac{R_3}{m^2}, \quad B_6 = \frac{B_5}{m^2} + \frac{A_5}{m^3} - C_4 m$$

$$R_3 = \frac{X_2 \sinh m - X_1(\cosh m - 1)}{m \sinh m - 2\cosh m + 2}$$

$$C_4 = \frac{X_2(\cosh m-1)-X_1 m^2(\sinh m-m)}{m^3(m \sinh m-2\cosh m+2)}$$

$$X_1 = \frac{A_5}{2m}(m^2+2)\sinh m+\frac{B_5}{2}(2\sinh m+m\cosh m)$$

$$X_2 = \frac{A_5}{2m}(m^2+2m-2)\cosh m$$
$$+\frac{B_5}{2m}(2m^2\cosh m+m^2\sinh m-2\sinh m).$$

$$G_2(z) = 0, \quad F_2(z) = 0 \qquad \ldots \ldots \ldots \quad (5.3)$$

$$G_3(z) = A_7\cosh mz+B_7\sinh mz+\frac{1}{m^2}\left(C_4-\frac{2R_3}{m^2}\right)$$
$$+\frac{1}{m}\left(mK_1A_5-B_6-\frac{A_6}{m}+\frac{B_5}{m^4}+\frac{A_5}{m^3}\right)z\cosh mz$$
$$+\frac{1}{m}\left(mK_1B_5-A_6-\frac{B_6}{m}+\frac{A_5}{m^4}+\frac{B_5}{m^3}\right)z\sinh mz$$
$$+\left[\frac{B_6}{2m}+A_5\left\{\frac{7K_1}{3m^2}+\frac{131}{36m^2}+\frac{278731}{388800m^4}\right\}\right.$$
$$\left.+B_5\left\{\frac{4K_1}{m}+\frac{1}{15m}-\frac{124}{135m^3}\right\}\right]z^2\cosh mz$$
$$+\left[\frac{A_6}{2m}+A_5\left\{\frac{4K_1}{m}+\frac{1}{15m}-\frac{124}{135m^3}\right\}\right.$$
$$\left.+B_5\left\{\frac{7K_1}{3m^2}+\frac{131}{36m^2}+\frac{278731}{388800m^4}\right\}\right]z^2\sinh mz$$
$$+\left[A_5\left\{-\frac{1}{3}\left(3K_1+\frac{1}{20}\right)+\frac{145}{432m^2}\right\}\right.$$
$$\left.+B_5\left\{-\frac{17}{6m}\left(K_1+\frac{7}{120}\right)+\frac{1}{m^2}+\frac{7381}{38880m^3}\right\}\right]z^3\cosh mz$$
$$+\left[A_5\left\{-\frac{17}{6m}\left(K_1+\frac{7}{120}\right)+\frac{1}{m^2}+\frac{7381}{38880m^3}\right\}\right.$$
$$\left.+B_5\left\{-\frac{1}{3}\left(3K_1+\frac{1}{20}\right)+\frac{145}{432m^2}\right\}\right]z^3\sinh mz$$
$$+\left[A_5\left\{\frac{1}{24}\left(12K_1+\frac{7}{10}\right)-\frac{811}{10800m^2}\right\}+\frac{29B_5}{240m}\right]z^4\cosh mz$$
$$+\left[\frac{29A_5}{240m}+B_5\left\{\frac{1}{24}\left(12K_1+\frac{7}{10}\right)-\frac{811}{10800m^2}\right\}\right]z^4\sinh mz$$
$$-\left[\frac{A_5}{60}+\frac{11B_5}{450m}\right]z^5\cosh mz-\left[\frac{11A_5}{450m}+\frac{B_5}{60}\right]z^5\sinh mz$$
$$+\frac{A_5}{360}z^6\cosh mz+\frac{B_5}{360}z^6\sinh mz. \qquad \ldots \ldots \ldots \quad (5.4)$$

The constants A_7 and B_7 are easily determined with the help of the boundary conditions (3.11).

6. Discussion

Curves have been drawn showing the variation of $\bar{F}'_1(z)$, $\bar{F}'_2(z)$, $\bar{G}_1(z)$, $\bar{G}_3(z)$, $F'_1(z)$ and $G_1(z)$ with z when $M = 0{\cdot}5$, $K_1 = 0{\cdot}1$ and $K_2 = 0{\cdot}5$.

From Fig. 1, it is seen that the variation of $\bar{F}'_1(z)$ with z for magnetic as well as for the non-magnetic case is of an oscillatory nature. The effect of magnetic field is to damp the value of $\bar{F}'_1(z)$. The phase of oscillations also changes with the magnetic parameter M. Also it is obvious from the graph that the effect of porosity is to increase its value.

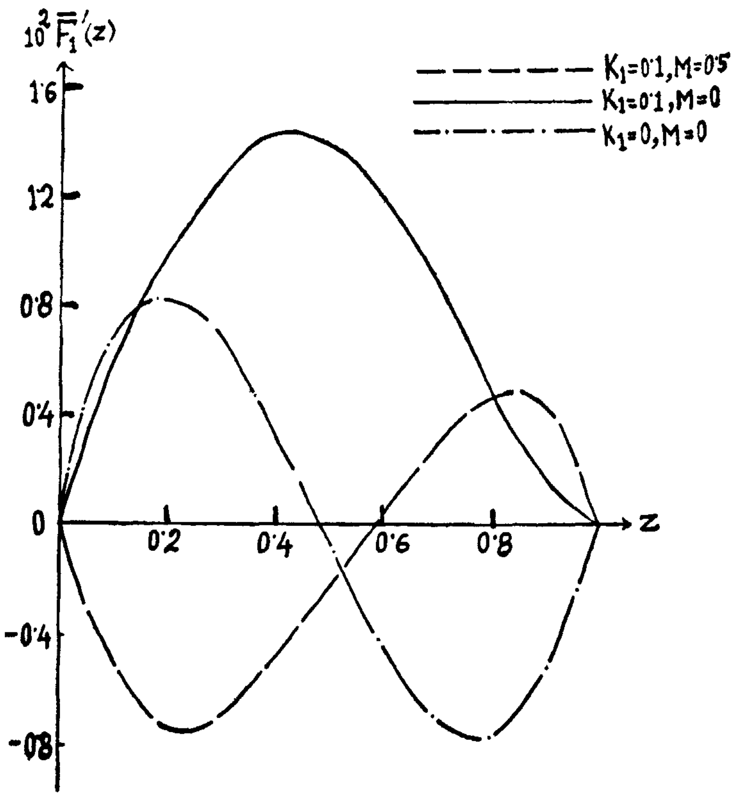

Fig. 1. Variation of $\bar{F}'_1(z)$ with z.

Again from Fig. 2 we see that the variation of $\bar{F}'_2(z)$ with z is of parabolic nature and is symmetrical about $z = 0{\cdot}5$. The effect of porosity is to increase the value of $\bar{F}'_2(z)$ while magnetic field damps it.

From Fig. 3, it is seen that at $z = 0$, the fluid rotates with the disk. As z increases $\bar{G}_1(z)$ increases linearly (for the non-magnetic case) and finally it is

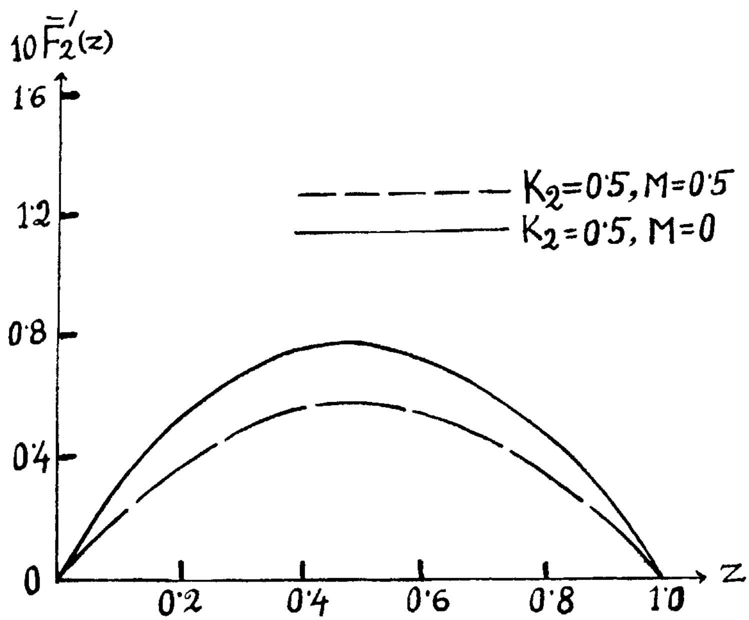

FIG. 2. Variation of $\bar{F}'_2(z)$ with z.

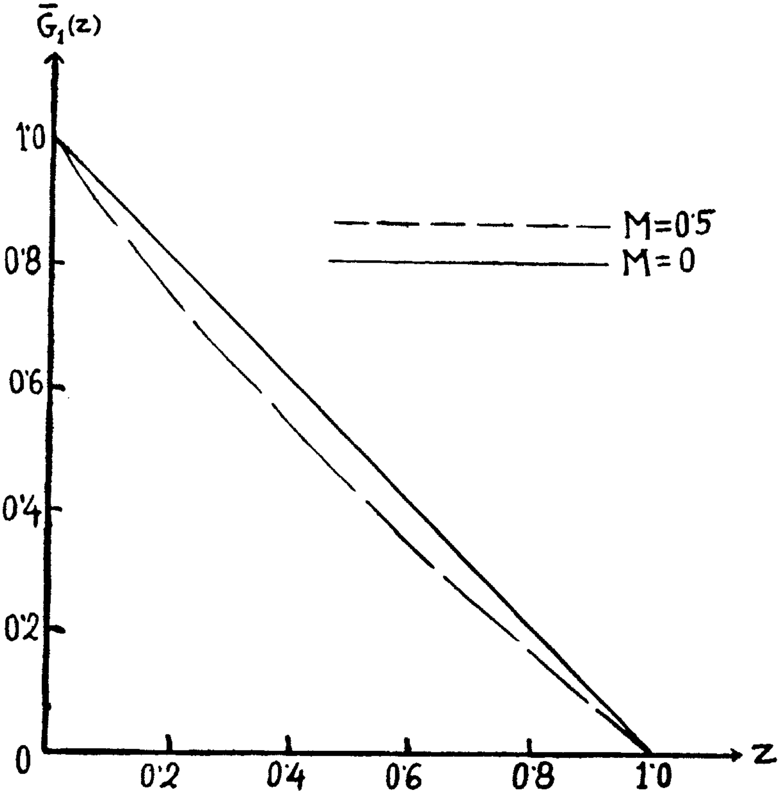

FIG. 3. Variation of $\bar{G}_1(z)$ with z.

zero at $z = 1$. The effect of magnetic field is to damp the value of $\bar{G}_1(z)$ but there is no effect of porosity on it. Further from Fig. 4 it may easily be seen that the effect of magnetic field is to increase the value of $\bar{G}_3(z)$ while porosity decreases it.

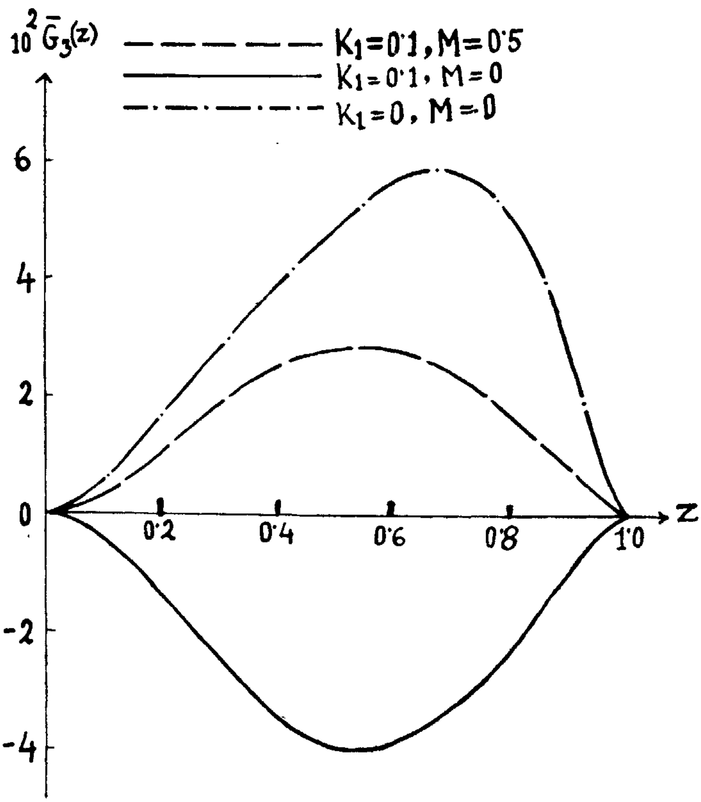

FIG. 4. Variation of $\bar{G}_3(z)$ with z.

From the above discussion we may infer that the magnetic field damps the radial as well as the transverse components of velocity while porosity increases the radial component of velocity and damps the transverse component.

From Fig. 5, it may easily be observed that the amplitude of fluctuations in the main transverse velocity v_s due to superimposed vibration is maximum at the rotating disk and decreases almost linearly as we move from the rotating disk towards the fixed disk where it is zero. Similarly the amplitude of fluctuations in the main radial velocity u_s is zero at the two disks while it varies almost parabolically as z varies from $z = 0$ to $z = 1$. The effect of magnetic field is to damp the amplitudes of these fluctuations.

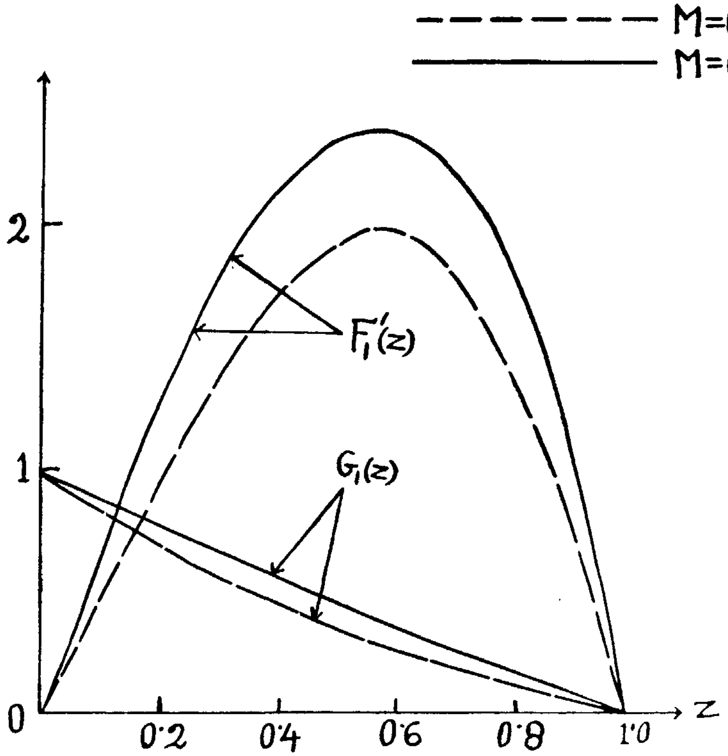

FIG. 5. Variation of $F'_1(z)$ and $G_1(z)$ with z.

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