HEAT TRANSFER FOR LAMINAR FLOW IN A ROTATING STRAIGHT ANNULAR PIPE

by R. S. SRIVASTAVA*, *Defence Science Laboratory, Delhi* 6

and

V. B. SARIN, *Delhi University, Delhi* 7

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Heat transfer for hydrodynamically and thermally fully developed laminar flow in a rotating straight annular pipe is solved by a method of series expansion. For obtaining the solution, the velocity field is determined first. The secondary flow streamlines have been sketched in the plane of cross-section of the pipe. To a first order of approximation the temperature profiles are shown in the plane of the cross-section of the pipe. It is also seen that to a first order of approximation, Nusselt number remains unaffected.

INTRODUCTION

Srivastava (1973) has theoretically determined the temperature distribution for an incompressible viscous fluid in a straight pipe rotating about an axis perpendicular to its own by making use of Barua's (1954) result. Srivastava (1973), in his paper, has introduced some of the assumptions of Ozisik and Topakoglu (1968) who have considered heat transfer effects in a curved pipe.

In the present paper, following Srivastava (1973), we have considered the heat transfer effects in an annular pipe rotating about an axis perpendicular to its own for an incompressible viscous fluid. Following Barua (1954), we assume the angular velocity of the pipe to be small. The velocity field is determined to a first order of approximation. The secondary flow streamlines have been plotted in the plane of the cross-section of pipe.

The temperature profiles are evaluated to a first order of approximation and isotherms are plotted in the plane of cross-section of annular pipe for

*Deputy Director, Directorate of Research Laboratory, R & D Organisation, B Block, Defence Headquarters, P.O., New Delhi-11.

small values of R_1 and R_2, where $R_1 = w_m b/\nu$ and $R_2 = 2\Omega b/w_m$, w_m being the mean velocity of the fluid when the annular pipe is stationary, Ω the angular velocity, b the outer radius of the annular pipe and ν the kinematic coefficient of viscosity.

We introduce the following assumptions for the analysis of the temperature distribution in the pipe :

1. Velocity and temperature fields are fully developed.
2. Flow is laminar.
3. Viscous dissipation is negligible.
4. Wall temperature around the periphery of both the walls of any cross-section and its rate of change in the axial direction are uniform.

Formulation of the Problem

The annular pipe is composed of two coaxial pipes of circular cross-sections. The inner radius of the annular pipe is a and the outer radius is b, and the pipe is rotating about an axis perpendicular to its own plane. A cylindrical polar system of coordinates (r, θ, z) is used, where z is measured from the axis of rotation along the axis of the pipe. The notations used in this paper are the same as those of Barua (1954). Assuming that the motion is the same in each cross-section of the annular pipe, eqns. (5), (6), (7) of Barua hold good and are given as :

$$-2\Omega W \sin\theta + U\frac{\partial U}{\partial r} + \frac{V}{r}\frac{\partial U}{\partial \theta} - \frac{V^2}{r} = -\frac{\partial \psi}{\partial r} + \nu\left[\nabla^2 U - \frac{U}{r^2} - \frac{2}{r^2}\frac{\partial V}{\partial \theta}\right] \quad (1)$$

$$-2\Omega W \cos\theta + U\frac{\partial V}{\partial r} + \frac{V}{r}\frac{\partial V}{\partial \theta} + \frac{UV}{r} = -\frac{1}{r}\frac{\partial \psi}{\partial \theta} + \nu\left[\nabla^2 V + \frac{2}{r^2}\frac{\partial U}{\partial \theta} - \frac{V}{r^2}\right] \quad \ldots (2)$$

$$2\Omega[U \sin\theta + V\cos\theta] + U\frac{\partial W}{\partial r} + V\frac{\partial W}{\partial \theta} = -\frac{\partial \psi}{\partial z} + \nu\nabla^2 W \quad \ldots \quad \ldots (3)$$

where $$\psi = \frac{p}{\rho} - \tfrac{1}{2}\Omega^2[r^2 \sin\theta + z^2] \quad \ldots \quad \ldots \quad \ldots (4)$$

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2}$$

and U, V, W are velocity components along the directions r, θ, z respectively

Following the assumption made, we can write

$$\chi = c'z + \psi(r, \theta) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (5)$$

where c' is a constant and may be termed as pressure gradient along the axis of the pipe.

The equation of continuity is given as

$$\frac{1}{r}\frac{\partial}{\partial r}(ru) + \frac{1}{r}\frac{\partial v}{\partial \theta} = 0. \quad \ldots \quad \ldots \quad (6)$$

If f is the stream function, from (6), we have

$$ru = -\frac{\partial f}{\partial \theta}, \quad v = \frac{\partial f}{\partial r} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (7)$$

Eliminating χ from (1) and (2) and using (7) we have

$$-2\Omega\left[r\cos\theta\frac{\partial W}{\partial r} - \sin\theta\frac{\partial W}{\partial \theta}\right] + \left[\frac{\partial f}{\partial r}\cdot\frac{\partial}{\partial \theta} - \frac{\partial f}{\partial \theta}\cdot\frac{\partial}{\partial r}\right]\nabla^2 f = \nu r\nabla^4 f. \quad \ldots \quad (8)$$

From (3), (5) and (7) we have

$$\frac{2\Omega}{r}\left[r\cos\theta\frac{\partial f}{\partial r} - \sin\theta\frac{\partial f}{\partial \theta}\right] + \frac{1}{r}\left[\frac{\partial f}{\partial r}\cdot\frac{\partial}{\partial \theta} - \frac{\partial f}{\partial \theta}\cdot\frac{\partial}{\partial r}\right] w = -c' + \nu\nabla^2 w. \quad (9)$$

Introducing the non-dimensional variables

$$f = \nu\phi, \quad W = \frac{\nu}{b}w, \quad r = bx \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (10)$$

we obtain from (8) and (9),

$$-R\left[x\cos\theta\frac{\partial w}{\partial x} - \sin\theta\frac{\partial w}{\partial \theta}\right] + \left[\frac{\partial \phi}{\partial x}\cdot\frac{\partial}{\partial \theta} - \frac{\partial \phi}{\partial \theta}\cdot\frac{\partial}{\partial x}\right]\nabla_1^2\phi = x\nabla_1^4\phi \quad \ldots \quad (11)$$

$$\frac{R}{x}\left[x\cos\theta\frac{\partial \phi}{\partial x} - \sin\theta\frac{\partial \phi}{\partial \theta}\right] + \frac{1}{x}\left[\frac{\partial \phi}{\partial x}\cdot\frac{\partial}{\partial \theta} - \frac{\partial \phi}{\partial \theta}\cdot\frac{\partial}{\partial x}\right] w = -C + \nabla_1^2 w \quad \ldots \quad (12)$$

where

$$R = \frac{2}{\nu}\Omega b^2, \qquad C = \frac{c'b^3}{\nu^2} \quad \ldots \quad \ldots \quad \ldots \quad (13)$$

and

$$\nabla_1^2 = \frac{\partial^2}{\partial x^2} + \frac{1}{x}\frac{\partial}{\partial x} + \frac{1}{x^2}\frac{\partial^2}{\partial \theta^2} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (14)$$

Solution for ϕ

Following Barua (1954), we assume

$$\phi = R\phi_1 + R^2\phi_2 + \ldots \quad \ldots \quad \ldots \quad \ldots \quad (15)$$

$$w = w_0 + Rw_1 + R^2w_2 + \ldots \quad \ldots \quad \ldots \quad \ldots \quad (16)$$

Substituting (15) and (16) in (11) and (12) and equating coefficients of similar powers in R, we have series of relations from where $w_0, w_1, \ldots, \phi_1, \ldots$ can be successively found.

The differential equation determining w_0 is given as :

$$\Delta_1^2 w_0 - C = 0. \quad \ldots \quad \ldots \quad \ldots \quad (17)$$

The solution of differential eqn. (17) satisfying the boundary conditions that w_0 is zero at $x = 1$ and $x = a/b = k$ is given as

$$w_0 = \frac{C}{4}\left[1 - x^2 - \lambda \ln x\right] \quad \ldots \quad \ldots \quad \ldots \quad (18)$$

where

$$\lambda = \frac{1-k^2}{\ln k} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (19)$$

When $k = 0$, from (19), we have $\lambda = 0$ and from (18) we get the value of w_0 as obtained by Barua (1954) for the case of a single straight rotating pipe.

Substituting (15) and (16) in (11) and comparing coefficients of R, we have

$$\nabla_1^2 \phi_1 = -\cos\theta \frac{\partial w_0}{\partial x} \quad \ldots \quad \ldots \quad \ldots \quad (20)$$

where w_0 is given by (18).

From (18) and (20) we have

$$\phi_1 = \left[\frac{C}{4}\left\{-\frac{x^5}{96} - \frac{x^3\lambda \ln x}{16} + \frac{5\lambda x^3}{64}\right\} + A_1x^3 + \frac{B_1}{x} + C_1x + D_1x \ln x\right] \cos\theta \quad \ldots \quad (21)$$

where A_1, B_1, C_1, D_1 are constants of integration and are determined from the fact that

$$\phi_1 = 0, \frac{\partial \phi_1}{\partial x} = 0 \text{ at } x = 1 \quad \text{and} \quad \phi_1 = 0, \frac{\partial \phi_1}{\partial x} = 0 \text{ at } x = k.$$

These are

$$
\left.\begin{aligned}
A_1 &= \Big[A_{11}\Big(-1+\frac{1}{k^2}-\frac{2}{k^2}\ln k\Big)-A_{21}\Big(1-\frac{1}{k^2}-\frac{2}{k^2}\ln k\Big) \\
&\quad +A_{31}\Big(1+2\ln k-\frac{1}{k^2}\Big)-A_{41}\Big(-1+2\ln k+\frac{1}{k^2}\Big)\Big]/\Delta. \\
B_1 &= [-A_{11}(3+2k^2\ln k-3k^2)+A_{21}(1-k^2+2k^2\ln k) \\
&\quad -A_{31}(-3-2\ln k+3k^2)+A_{41}(-1-2\ln k+k^2)]/\Delta. \\
C_1 &= \Big[A_{11}\Big(\frac{3}{k^2}+\frac{6}{k^2}\ln k+k^2-2k^2\ln k-4\Big)-A_{21}\Big(\frac{1}{k^2}+\frac{2}{k^2}\ln k-k^2+2k^2\ln k\Big) \\
&\quad +A_{31}\Big(-4-4\ln k+\frac{1}{k^2}+3k^2\Big)-A_{41}\Big(-4\ln k-\frac{1}{k^2}+k^2\Big)\Big]/\Delta. \\
D_1 &= \Big[-A_{11}\Big(-4-2k^2+\frac{6}{k^2}\Big)+(A_{21}+A_{41})\Big(-4+\frac{2}{k^2}+2k^2\Big) \\
&\quad -A_{31}\Big(-4+6k^2-\frac{2}{k^2}\Big)\Big]/\Delta.
\end{aligned}\right\} \quad (22)
$$

$$\Delta = 8+\frac{4}{k^2}+\frac{4}{k^2}\ln k+4k^2-4k^2\ln k;$$

$$A_{11} = \frac{c}{4}\Big[\frac{1}{96}-\frac{5\lambda}{64}\Big];$$

$$A_{21} = \frac{c}{4}\Big[\frac{5}{96}-\frac{11\lambda}{64}\Big];$$

$$A_{31} = \frac{c}{4}\Big[\frac{k^4}{96}-\frac{5k^2\lambda}{64}+\frac{k^2\lambda\ln k}{16}\Big];$$

$$A_{41} = \frac{c}{4}\Big[\frac{5k^4}{96}-\frac{11k^2\lambda}{64}+\frac{3k^2\lambda\ln k}{16}\Big].$$

Streamlines in the Plane of Cross-Section of the Pipe

We shall now carry out the numerical work for $k = \frac{1}{2}$. For this value of k, from (21) and (22), we have

$$\phi_1 = \frac{C}{768}\Big[\frac{0{\cdot}2382}{x}+6{\cdot}385x\ln x+12{\cdot}9852x^3\ln x+7{\cdot}2097x-5{\cdot}4479x^3-2x^5\Big]\cos\theta. \quad \ldots \quad (23)$$

Following Barua (1954), we find that the streamlines in the plane of the cross-section are given by ϕ = constant, i.e. from (23) $\frac{\phi_1 \times 768}{C}$ = const = S. For different values of S, we get different closed curves. These are plotted in Fig. 1. The plotting has been done only in the upper half plane. The interesting feature of the streamlines is that they are divided between positive

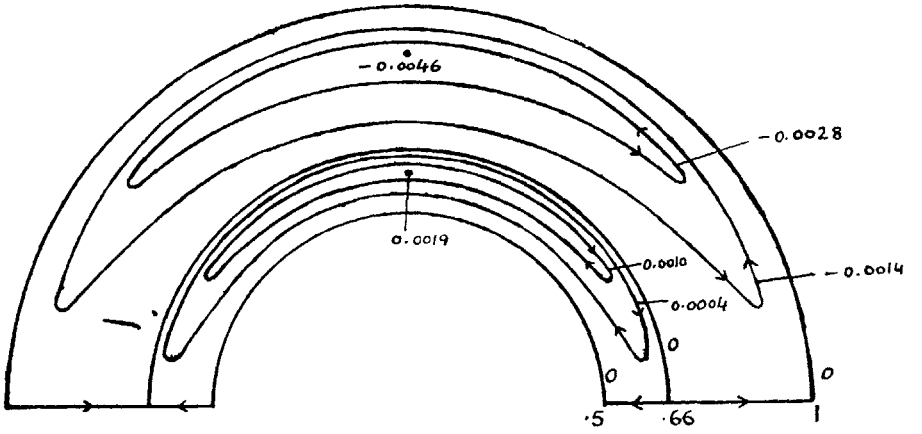

Fig. 1. Project of streamlines on a Cross-section of the annular pipe.

values of S and negative values of S by a curve having a zero value of S (at $x = 0{\cdot}658$, $\phi_1 = 0$). For $x = 0{\cdot}60$, $\theta = 0$, the value of S is maximum ($S = +0{\cdot}0019$) and for $x = 0{\cdot}90$, $\theta = 0$ the value of S is minimum ($-0{\cdot}0046$).

When $k = 0$, from (22) we have B_1, D_1 to be zero and (21) reduces to give the value of ϕ_1 as obtained by Barua (1954) for the case of a single straight rotating pipe.

Solution for w_1

Substituting (15) and (16) in (12) and comparing coefficients of R, we have

$$\nabla_1^2 w_1 = \frac{1}{x}\left[\frac{\partial \phi_1}{\partial x} \cdot \frac{\partial}{\partial \theta} - \frac{\partial \phi_1}{\partial \theta} \cdot \frac{\partial}{\partial x}\right] w_0. \qquad \ldots \quad (24)$$

From (18), (23) and (24), we have

$$w_1 = \left[-\frac{C^2 x^7}{2304 \times 16} + M_1 x^5 + M_2 x^3 + M_3 x + \frac{L_1}{x} - \frac{B_1 \lambda C}{8} \cdot \frac{\ln x}{x} + M_4 x \ln x \right.$$

$$\left. + M_5 x^3 \ln x - \frac{C^2 \lambda}{16 \times 192} x^5 \ln x + \frac{D_1 \lambda C}{16} \cdot x (\ln x)^2 \right] \sin \theta \qquad \ldots \quad (25)$$

where

$$M_1 = \left(32A_1 + \frac{19\lambda C}{24}\right)\frac{C}{4\times 384};$$

$$M_2 = \left(4C_1 + 2A_1\lambda - 3D_1 + \frac{\lambda^2 C}{16}\right)\frac{C}{64};$$

$$M_3 = \left(-2B_1 - C_1\lambda + \frac{D_1\lambda}{2}\right)\frac{C}{16} + \frac{K_1}{2};$$

$$M_4 = \left(2B_1 + C_1\lambda - \frac{D_1\lambda}{8}\right)\frac{C}{8};$$

$$M_5 = \left(2D_1 - \frac{\lambda^2 C}{64}\right)\frac{C}{32}.$$

K_1, L_1 are constants of integration and determined from the fact that $w_1 = 0$ at $x = 1$, $x = k$. These are

$$L_1 = \left[\frac{C^2}{2304\times 16} - M_1 - M_2 - M_3\right]$$

$$K_1 = \frac{2}{k}\left[\frac{C^2 k^7}{2304\times 16} - M_1 k^5 - M_2 k^3 + \left(2B_1 + C_1\lambda - \frac{D_1\lambda}{2}\right)\frac{C}{16}k - \frac{L_1}{k}\right.$$

$$\left. + \frac{B_1\lambda C}{8}\frac{\ln k}{k} - M_4 l \ln k - M_5 k^3 \ln k + \frac{C^2\lambda k^5 \ln k}{16\times 192} - \frac{D_1\lambda C k}{16}(\ln k)^2\right].$$

When $k = 0$, (25) reduces to give the value of w_1 as obtained by Barua (1954) for the case of a single straight rotating pipe.

Analysis for Temperature Distribution

According to the assumptions introduced in the introductory part, the temperature T at any point in the cross-section has the following form :

$$T = MZ + N(r, \theta) \quad \ldots \quad \ldots \quad \ldots \quad (27)$$

$$N(r, \theta) = \text{constant at both the walls} \quad \ldots \quad \ldots \quad (28)$$

where M denotes the temperature gradient in the direction of axis of the pipe. We will assume $N(r, \theta)$ to be zero at either of the walls.

The energy equation can be written as

$$U\frac{\partial T}{\partial r} + \frac{V}{r}\frac{\partial T}{\partial \theta} + W\frac{\partial T}{\partial z} = \chi\left[\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2}\right] \ldots \quad (29)$$

where χ is the thermal diffusivity.

We assume the temperature distribution to be same in each cross-section of the pipe.

From (7) and (27), energy equation (29) takes the following form

$$-\frac{1}{r}\cdot\frac{\partial f}{\partial\theta}\cdot\frac{\partial N}{\partial r}+\frac{1}{r}\frac{\partial f}{\partial r}\cdot\frac{\partial N}{\partial\theta}+WM=\chi\left[\frac{\partial^2 N}{\partial r^2}+\frac{1}{r^2}\frac{\partial^2 N}{\partial\theta^2}+\frac{1}{r}\frac{\partial N}{\partial r}\right]\ldots \quad (30)$$

We introduce additional non-dimensional variables m and n as

$$M=\frac{E}{K}m, \quad \text{and} \quad N=\frac{bE}{K}n \qquad \ldots \quad \ldots \quad (31)$$

where E is the reference flux and K is the conductivity.

Using (10) and (31), eqn. (30) reduces to

$$P_r\left[\frac{\partial\theta}{\partial x}\cdot\frac{\partial n}{\partial\theta}-\frac{\partial\phi}{\partial\theta}\cdot\frac{\partial n}{\partial x}\right]\frac{1}{x}+P_r wm=\frac{\partial^2 n}{\partial r^2}+\frac{1}{x}\cdot\frac{\partial n}{\partial x}+\frac{1}{x^2}\frac{\partial^2 n}{\partial\theta^2} \qquad \ldots \quad (32)$$

where $$P_r=\frac{\nu}{\chi}\text{ (Prandtl number).}$$

Following Srivastava (1973), we assume

$$n=n_0+Rn_1+R^2n_2+\ldots \qquad \ldots \quad \ldots \quad \ldots \quad (33)$$

Substituting (15), (33) and (16) in (32) and comparing coefficients of R^0, we have

$$\nabla_1^2 n_0=P_r\, mw_0. \qquad \ldots \quad \ldots \quad \ldots \quad (34)$$

From (18) and (34) we have

$$n_0=\frac{-P_r mC}{4}\left[\frac{x^2}{4}-\frac{x^4}{16}-\frac{\lambda x^2\ln x}{4}+\frac{\lambda x^2}{4}\right]+A\ln x+B \qquad \ldots \quad (35)$$

where A, B are constants of integration and are determined from the fact that n_0 is zero at $x=1$ and $x=k$.

These are

$$B=\frac{P_r mC}{4}\left[\frac{3}{16}+\frac{\lambda}{4}\right]$$

$$A=\frac{1}{\ln k}\left[\frac{P_r mC}{4}\left\{\frac{k^2}{4}-\frac{k^4}{16}-\frac{\lambda k^2\ln k}{4}+\frac{\lambda k k^2}{4}\right\}-B\right]. \qquad \ldots \quad (36)$$

When $k=0$, A is zero from (36) and (35) reduces to give the value of n_0 as obtained by Srivastava (1973) for the case of a single straight rotating pipe.

Substituting (15), (16) and (33) in (32) and comparing coefficients of R, we have

$$\nabla_1^2 n_1 = P_r m w_1 - \frac{P_r}{x} \cdot \frac{\partial \phi_1}{\partial \theta} \cdot \frac{\partial n_0}{\partial x} \quad \ldots \quad \ldots \quad (37)$$

where ϕ_1, w_1, n_0 are given by (21), (25), (35) respectively.

Equations (21), (25), (35) and (37) suggest the form of n_1 as

$$n_1 = l_1 \sin\theta \quad \ldots \quad \ldots \quad \ldots \quad (38)$$

where l_1 is a function of x only.

The differential equation determining l_1 is given as

$$\Delta_1^2 l_1 = P_r m X_2 - \frac{P_r}{x} \cdot F_2 \frac{dn_0}{dx}. \quad \ldots \quad \ldots \quad \ldots \quad (39)$$

where

$$\phi_1 = F_2 \cos\theta, \quad w_1 = X_2 \sin\theta \quad \ldots \quad \ldots \quad \ldots \quad (40)$$

and F_2, X_2 are functions of x only.

The solution of differential equation (39) satisfying the boundary conditions that l_1 is zero at $x = 1$ and at $x = k$, is given as :

$$l_1 = P_r m f_1(x) + P_r m f_2(x) \quad \ldots \quad \ldots \quad \ldots \quad (41)$$

where

$$f_1(x) = \Big[-\frac{C_1}{2304\times 16} x^9 + P_1 x^7 + P_2 x^5 + P_3 x^3 + P_4 x + P_5 x \ln x + P_6 x^3 \ln x + \frac{M_5}{24} x^5 \ln x - \frac{C^2\lambda}{2304\times 64} x^7 \ln x - \frac{B_1\lambda C}{32} x(\ln x)^2 + \frac{D_1\lambda C}{128} x^3 (\ln x)^2 + \frac{A_2 x}{2} + \frac{B_2}{x} \Big] \quad (42)$$

and

$$P_1 = \frac{N_1}{8} + \frac{C^2\lambda}{2304\times 512}; \quad P_2 = \frac{N_2}{6} - \frac{M_5}{24}; \quad P_3 = \frac{N_3}{4} - \frac{N_4}{16};$$

$$P_4 = -\frac{B_1\lambda C}{64} - \frac{L_1}{4}; \quad P_5 = \frac{L_1}{2} + \frac{B_1\lambda C}{32}; \quad P_6 = \frac{N_4}{4} - \frac{D_1\lambda C}{256};$$

$$N_1 = \frac{M_1}{6} + \frac{C^2\lambda}{144\times 2304}; \quad N_2 = \frac{M_2}{4} - \frac{M_5}{16}; \quad N_3 = \frac{M_3}{2} - \frac{M_4}{4} + \frac{D_1\lambda C}{64};$$

$$N_4 = \frac{M_4}{2} - \frac{D_1\lambda C}{32}; \quad B_2 = \frac{B_3 - A_3 k}{\left(\frac{1}{k} - k\right)}; \quad A_2 = 2\frac{\left(\frac{A_3}{R} - B_3\right)}{\left(\frac{1}{k} - k\right)};$$

$$A_3 = -\left[-\frac{C^2}{2304\times 1280}+P_1+P_2+P_3+P_4\right]$$

$$B_3 = -\left[-\frac{C^2}{2304\times 1280}\,k^9+P_1k^7+P_2k^5+P_3k^3+P_4k+P_5\,k\ln k+P_6k^3\ln k\right.$$

$$\left.+\frac{M_5}{24}\,k^5\ln k-\frac{C^2\lambda}{2304\times 64}\,k^7\ln k-\frac{B_1\lambda C}{32}\,k(\ln k)^2+\frac{D_1\lambda C}{128}\,k^3(\ln k)^2\right] \quad (43)$$

$$f_2(x) = \frac{C^2}{16}\left[-\frac{x^9}{30720}+Q_{17}x^7+Q_{18}x^5+Q_{19}x^3+Q_{20}x+Q_{21}x\ln x+Q_{22}x^3\ln x\right.$$

$$+Q_{23}x^5\ln x+\frac{Q_1}{64}\,x^7\ln x+\frac{Q_{11}}{4}\,x(\ln x)^2-Q_1D_1x^3(\ln x)^2-\frac{Q_1^2}{3}\,x^5(\ln x)^2$$

$$\left.+\frac{A_5B_1}{2}\,\frac{\ln x}{x}\right]+\frac{A_4x}{2}+\frac{B_4}{x}.$$

$$A_5 = \frac{1}{\ln k}\left[\frac{k^2}{4}-\frac{k^4}{16}-\frac{\lambda k^2\ln k}{4}+\frac{\lambda k^2}{4}-\frac{3}{16}-\frac{\lambda}{4}\right]. \quad \ldots \quad \ldots \quad \ldots \quad (44)$$

where

$$Q_1 = -\frac{\lambda_1}{16}\,;\quad Q_2 = \frac{5\lambda}{64}+A_1\,;\quad Q_3 = \left[-\frac{1}{2}-\frac{\lambda}{4}\right];\quad Q_5 = \frac{Q_2}{4}-\frac{Q_3}{96}\,;$$

$$Q_6 = \frac{C_1}{4}+Q_3Q_2+\frac{A_5}{96}\,;\quad Q_7 = \frac{B_1}{4}+Q_3C_1-A_5Q_2;\quad Q_8 = Q_3D_1-A_5Q_1-8C_1Q_1;$$

$$Q_9 = \frac{D_1}{4}+Q_3Q_1-8Q_1Q_2;\quad Q_{10} = Q_3B_1-A_5C_1;\quad Q_{11} = -8Q_1B_1-A_5D_1;$$

$$Q_{12} = \frac{Q_5}{6}-\frac{Q_1}{108};\quad Q_{13} = \frac{Q_6}{4}-\frac{Q_9}{16}-4Q_1^2;\quad Q_{14} = \frac{Q_7}{2}-\frac{Q_8}{4}-2Q_1D_1\,;$$

$$Q_{15} = \frac{Q_8}{2}+4Q_1D_1;\quad Q_{16} = \frac{Q_9}{4}+Q_1^2;\quad Q_{17} = \frac{Q_{12}}{8}-\frac{Q_1}{512}\,;$$

$$Q_{18} = \frac{Q_{13}}{6}-\frac{Q_1^2}{54}-\frac{Q_{16}}{36}\,;$$

$$Q_{19} = \frac{Q_{14}}{4}-\frac{Q_{15}}{16}-\frac{Q_1D_1}{8};\quad Q_{20} = -\frac{Q_{10}}{4}+\frac{Q_{11}}{8};\quad Q_{21} = \frac{Q_{10}}{2}-\frac{Q_{11}}{4};$$

$$Q_{22} = \frac{Q_{15}}{4}+\frac{Q_1D_1}{2}\,;\quad Q_{23} = \frac{Q_{16}}{6}+\frac{Q_1^2}{9}\,;\quad A_4 = \frac{2\left[\frac{A_{33}}{k}-B_{33}\right]}{\left(\frac{1}{k}-k\right)};$$

$$B_4 = \frac{B_{33}-A_{33}k}{\left(\frac{1}{k}-k\right)} \quad A_{33} = -\frac{C^2}{16}\left[-\frac{1}{30720}+Q_{17}+Q_{18}+Q_{19}+Q_{20}\right];$$

$$B_{33} = -\frac{C^2}{16}\left[-\frac{k^9}{30720}+Q_{17}k^7+Q_{18}k^5+Q_{19}k^3+Q_{20}k+Q_{21}l\ln k+Q_{22}k^3\ln k\right.$$

$$+Q_{23}k^5\ln k+\frac{Q_1}{64}k^7\ln k+\frac{Q_{11}}{4}k(.\ln k)^2-Q_1D_1k^3(\ln k)^2$$

$$\left.-\frac{Q_1^2}{3}k^5(\ln k)^2+\frac{A_5B_1}{2}\frac{\ln k}{k}\right]. \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (45)$$

When $k=0$, B_2, P_4, P_5 are zero from (43), A_5, Q_1, Q_8, Q_9, Q_{10}, Q_{11}, Q_{15}, Q_{16}, Q_{20}, Q_{21}, Q_{22}, Q_{23}, B_4 are all zero from (45). In this case, with the help of (42) and (44), (41) reduces to give the value of n_1 as obtained by Srivastava (1973) for the case of a single straight rotating pipe.

Isotherms in the Plane of Cross-section

From (33), to a first order of approximation, we have

$$n = n_0+Rn_1 = n_0+Rl_1\sin\theta \quad \ldots \quad \ldots \quad (46)$$

where n_0 and l_1 are functions of x only.

Proceeding on the lines of Barua (1954), we have $C = 8R_1$, $R = R_1R_2$.

For $k = \frac{1}{2}$, the isotherms are plotted in Fig. 2. The isotherms are plotted

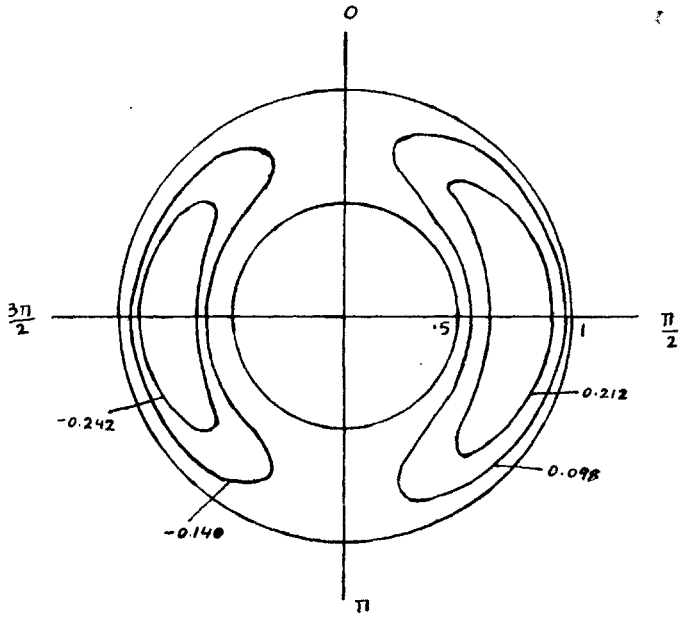

Fig. 2. Isotherms for $P_r = 100$, $C = 24$ and $R = 9$.

for Prandtl number 100, $C = 24$, $R = 9$. For the purpose of illustration, the isotherms are plotted for $10^{-4}n$ = constant. The region towards the outer side of the axis of rotation represents the heated region while the region towards the inner side represents the cooled region. A plane through $\theta = 0$ line and perpendicular to $\theta = \pi/2$ line divides the cross-section in two regions. The isotherms are symmetrical about $\theta = \pi/2$ line. The maximum drop in temperature is 0·3124 and maximum rise in temperature is 0·3129.

Nusselt Number

To a first order of approximation, there is no contribution of w_1, to the mean bulk temperature and as such the Nusselt number for the rotating pipe is the same as that for the stationary pipe.

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