CYLINDRICAL WAVE PROPAGATION IN A SELF-GRAVITATING COLLISIONLESS PLASMA WITH INITIAL FLUID VELOCITY IN THE PRESENCE OF MAGNETIC FIELD

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The system of equations derived by Chew *et al.* (1956) is considered for the wave propagation in self gravitating collisionless plasma in the presence of a magnetic field. The equations are linearized and then converted into the cylindrical coordinates. The dispersion relation for the collisionless self-gravitating magnetized plasma with initial fluid velocity is obtained and the effects of gravitational attraction and initial fluid velocity is discussed. It is found that there are three modes of wave propagation subject to certain conditions.

1. INTRODUCTION

A set of equations has been derived by Chew *et al.* (1956) for collisionless plasma in the presence of a magnetic field. Volkov (1966) has discussed the propagation of small amplitude plane waves and Gliddon (1966) has obtained gravitational instability for the case of small plane perturbation in this medium. Tandon and Talwar (1963) have made an extensive investigation of hydromagnetic stability of an infinitely extended uniformly rotating collisionless plasma in a strong magnetic field using C.G.L. equations with the modification to include the effect of self gravitation of the medium. Giulio Mattei (1968) has considered the propagation of cylindrical waves in magnetised collisionless plasma and also considered the effect of gravitational attraction. He discussed the propagation in stationary state. The object of this paper is to consider the propagation of cylindrical waves taking into account the initial fluid velocity in a particular direction and to discuss the effects of gravitational attraction and initial fluid velocity on the different modes of wave propagation.

2. STATEMENT OF THE PROBLEM

We consider the propagation of cylindrical waves in collisionless self-gravitating magnetized plasma. The medium is assumed to have certain initial fluid velocity in z-direction.

3. FUNDAMENTAL EQUATIONS

Chew *et al.* (1956) have shown that the collisionless plasma in a magnetic field is governed by a system of equations which are as follows (if the effect of heat conduction along the magnetic lines of force is neglected):

$$\rho \frac{d\vec{v}}{dt} = -\nabla . \overleftrightarrow{P} + \frac{1}{4\pi\mu}(\nabla \times \vec{B}) \times \vec{B} + \rho \nabla U \quad .. \quad .. \quad (1)$$

$$\frac{\partial \vec{B}}{\partial t} = \nabla \times (\vec{v} \times \vec{B}) \quad .. \quad .. \quad .. \quad (2)$$

$$\frac{d}{dt}\left(\frac{p_{\parallel} B^2}{\rho^3}\right) = 0 \quad .. \quad .. \quad .. \quad (3)$$

$$\frac{d}{dt}\left(\frac{p_{\perp}}{\rho B}\right) = 0 \quad .. \quad .. \quad .. \quad (4)$$

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \vec{v}) = 0, \quad .. \quad .. \quad .. \quad (5)$$

where the anisotropic pressure tensor $\overleftrightarrow{P}$ is given by

$$\overleftrightarrow{P} = p_{\perp} \overleftrightarrow{I} + (p_{\parallel} - p_{\perp}) \vec{n}\,\vec{n}. \quad .. \quad .. \quad .. \quad (6)$$

In the above equations $\vec{v}$ and $\vec{B}$ denote the flow velocity vector and magnetic field vector, ρ the density, B the magnitude of $\vec{B}$, $p_{\parallel}$ and $p_{\perp}$ the scalar pressures parallel and perpendicular to the direction of magnetic field, $\vec{n}$ the unit vector along the direction of magnetic field, $\overleftrightarrow{I}$ the unit tensor and

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \vec{v} \cdot \nabla .$$

The gravitatial potential U is given by

$$\nabla^2 U = -4\pi G \rho. \quad .. \quad .. \quad .. \quad (7)$$

4. Linearization of the Equations

The field variables are perturbed from the equilibrium state as follows:

$$\vec{v} = \vec{v}_0 + \delta \vec{v} \quad .. \quad .. \quad .. \quad (8)$$

where $\vec{v}_0$ is the equilibrium value and $\delta \vec{v}$ is perturbed value. Similar perturbations are assumed for $\vec{B}$, $\overleftrightarrow{P}$, ρ and U.

All the zero order quantities are independent of space and time. On putting (8) in the fundamental equations (1) to (7) and omitting for simplicity the suffix zero, we get the following linearized equations:

$$\rho \frac{\partial}{\partial t} \frac{(\delta \vec{v})}{} + \rho \vec{v} \cdot \nabla \delta \vec{v} = - \nabla \cdot \delta \overleftrightarrow{P} + \frac{1}{4\pi\mu} (\nabla \times \delta \vec{B}) \times \vec{B} + \rho \nabla \delta U \quad (9)$$

$$\frac{\partial}{\partial t} (\delta \vec{B}) = \nabla \times (\vec{v} \times \delta \vec{B}) + \nabla \times (\delta \vec{v} \times \vec{B}) \quad (10)$$

$$\frac{\delta p_{\parallel}}{p_{\parallel}} + \frac{2\,\delta B}{B} = \frac{3\,\delta p}{\rho} \quad (11)$$

$$\frac{\delta p_{\perp}}{p_{\perp}} = \frac{\delta \rho}{\rho} + \frac{\delta B}{B} \quad (12)$$

$$\frac{\partial}{\partial t} (\delta \rho) + \rho \nabla \cdot \delta \vec{v} + \vec{v} \cdot \nabla \delta \rho = 0 \quad (13)$$

$$\nabla^2 \delta U = -4\pi G \delta \rho \quad (14)$$

$$\delta \overleftrightarrow{P} = \delta p_{\perp} \overleftrightarrow{I} + (\delta p_{\parallel} - \delta p_{\perp}) \vec{n}\,\vec{n} + (p_{\parallel} - p_{\perp}) (\vec{n}\,\delta \vec{n} + \delta \vec{n}\,\vec{n}) \quad (15)$$

where $\vec{\delta n}$ is given by

$$\delta \vec{B} = \delta B\, \vec{n} + B \delta \vec{n}, \quad (16)$$

and δB is the magnitude of $\delta \vec{B}$.

5. Linearized Equations In Cylindrical Coordinates

The linearized equations are now converted in cylindrical polar co-ordinates (r, ϕ, z). Variation is considered only along (r and z) directions. The unperturbed magnetic field $\vec{B}$ is taken along the z-axis of the system. The fluid velocity $\vec{v} = (0, 0, v)$ is along z-direction.

The components of $\delta \vec{B}$ are b_r, b_ϕ, b_z $(= \delta B)$. Then using (15) and (16) and taking the z-axis as the axis of symmetry, the resolutes of $\nabla \cdot \delta \overleftrightarrow{P} = \omega$ are obtained as

$$\begin{aligned} \omega_r &= \frac{\partial (\delta p_{\perp})}{\partial r} + \frac{p_{\parallel} - p_{\perp}}{B} \frac{\partial b_r}{\partial z} \\ \omega_\phi &= \frac{p_{\parallel} - p_{\perp}}{B} \frac{\partial b\phi}{\partial z} \\ \omega_z &= \frac{\partial\, \delta p_{\parallel}}{\partial z} + \frac{p_{\parallel} - p_{\perp}}{B} \frac{1}{r} \frac{\partial (r b_r)}{\partial r}. \end{aligned} \quad (17)$$

Now eqn. (9) is written in (r, ϕ, z) components using eqn. (17) as

$$\rho \frac{\partial}{\partial t}(\delta v_r) + \rho v \frac{\partial}{\partial z}(\delta v_r) + \frac{\partial}{\partial r}(\delta p_\perp) + \left[\frac{p_\parallel - p_\perp}{B} - \frac{B}{4\pi\mu}\right]\frac{\partial b_r}{\partial z} + \frac{B}{4\pi\mu}\frac{\partial b_z}{\partial r} - \rho \frac{\partial}{\partial r}(\delta U) = 0 \quad \ldots \quad (18)$$

$$\rho \frac{\partial}{\partial t}(\delta v_\phi) + \rho v \frac{\partial}{\partial z}(\delta v_\phi) + \left(\frac{p_\parallel - p_\perp}{B} - \frac{B}{4\pi\mu}\right)\frac{\partial b_\phi}{\partial z} = 0 \quad \ldots \quad (19)$$

$$\rho \frac{\partial}{\partial t}(\delta v_z) + \rho v \frac{\partial}{\partial z}(\delta v_z) + \frac{\partial}{\partial z}\delta p_\parallel + \frac{p_\parallel - p_\perp}{B}\frac{1}{r}\frac{\partial}{\partial r}(r b_r) - \rho \frac{\partial}{\partial z}(\delta U) = 0. \quad (20)$$

Similarly eqn. (10) gives

$$\frac{\partial (b_r)}{\partial t} + v \frac{\partial (b_r)}{\partial z} - B \frac{\partial}{\partial z}(\delta v_r) = 0 \quad \ldots \quad (21)$$

$$\frac{\partial (b_\phi)}{\partial t} + v \frac{\partial (b_\phi)}{\partial z} - B \frac{\partial}{\partial z}(\delta v_\phi) = 0. \quad \ldots \quad (22)$$

$$\frac{\partial (b_z)}{\partial t} - \frac{v}{r}\frac{\partial (r b_r)}{\partial r} + \frac{B}{r}\frac{\partial}{\partial r}(r \delta v_r) = 0. \quad \ldots \quad (23)$$

From eqns. (11) and (12), we obtain

$$\delta p_\parallel = \frac{3p_\parallel}{\rho}\delta\rho - \frac{2p_\parallel}{B}\delta B \quad \ldots \quad (24)$$

$$\delta p_\perp = \frac{p_\perp}{\rho}\delta\rho + \frac{p_\perp}{B}\delta B. \quad \ldots \quad (25)$$

Equations (13) and (14) are written in cylindrical coordinates as

$$\frac{\partial (\delta\rho)}{\partial t} + \rho\left[\frac{1}{r}\frac{\partial}{\partial r}\frac{(r\delta v_r)}{\delta v_r} + \frac{\partial (\delta v_z)}{\partial z}\right] + v \frac{\partial (\delta\rho)}{\partial z} = 0 \quad \ldots \quad (26)$$

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial \delta U}{\partial r}\right) + \frac{\partial^2 (\delta U)}{\partial z^2} + 4\pi G \delta\rho = 0. \quad \ldots \quad (27)$$

6. Solution of Equations

Equations (18) to (27) with ten unknown scalars ∂v_r, ∂v_ϕ, ∂v_z, b_r, b_ϕ, b_z, $\partial p_\parallel$, $\delta p_\perp$, $\delta\rho$ and δU and are to be solved.

Equations (19) and (22) are independent from the other equations and having two unknowns δv_ϕ and b_ϕ only. On solving eqns. (19) and (22), it is found that δv_ϕ and b_ϕ satisfy the following equations:

$$\frac{\partial^2}{\partial t^2}(\delta v_\phi) + 2v \frac{\partial^2}{\partial z \partial t}(\delta v_\phi) + \left(v^2 + \frac{p_\parallel - p_\perp - 2p_m}{\rho}\right)\frac{\partial^2}{\partial z^2}(\delta v_\phi) = 0 \quad \ldots \quad (28)$$

$$\frac{\partial^2}{\partial t^2}(b_\phi) + 2v\frac{\partial^2}{\partial z \partial t}(b_\phi) + \left(v^2 + \frac{p_{\parallel} - p_{\perp} - 2p_m}{\rho}\right)\frac{\partial^2}{\partial z^2}(b_\phi) = 0 \quad \ldots \quad (29)$$

$$\text{where } \frac{B^2}{8\pi\mu} = p_m = \text{magnetic pressure.} \quad \ldots \quad (30)$$

We assume the solution of eqns. (28) and (29) of the form

$$\delta v_\phi = f(r) e^{i(\omega t - kz)} \quad \ldots \quad (31)$$

$$b_\phi = g(r) e^{i(\omega t - kz)} \quad \ldots \quad (32)$$

where ω is frequency, k the wave number, $f(r)$ and $g(r)$ the arbitrary functions.

Substituting solution (31) and (32) in eqn. (28) and (29) we get the dispersion relation

$$(\omega - kv)^2 = \frac{(2p_m + p_\perp - p_\parallel)\, k^2}{\rho}. \quad \ldots \quad (33)$$

Now, using eqns. (24) and (25), remembering ($\delta B = b_z$) and eliminating $\delta p_\parallel$ and $\delta p_\perp$ from (18) and (20), we get

$$\rho\frac{\partial(\delta v_r)}{\partial t} + \left(\frac{p_\perp}{B} + \frac{B}{4\pi\mu}\right)\frac{\partial b_z}{\delta r} + \left(\frac{p_\parallel - p_\perp}{B} - \frac{B}{4\pi\mu}\right)\frac{\partial b_r}{\partial z}$$

$$+\frac{p_\perp}{\rho}\frac{\partial(\delta\rho)}{\partial r} - \rho\frac{\partial(\delta U)}{\partial r} + \rho v\frac{\partial(\delta v_r)}{\partial z} = 0 \quad \ldots \quad (34)$$

$$\rho\frac{\partial(\delta v_z)}{\partial t} + \rho v\frac{\partial(\delta v_z)}{\partial z} + \left(\frac{p_\parallel - p_\perp}{B}\right)\frac{1}{r}\frac{\partial(rb_r)}{\partial r}$$

$$-\frac{2p_\parallel}{B}\frac{\partial b_z}{\partial z} + \frac{3p_\parallel}{\rho}\frac{\partial(\delta\rho)}{\partial z} - \rho\frac{\partial(\delta U)}{\partial z} = 0. \quad \ldots \quad (35)$$

Equations (21), (23), (26), (27), (34) and (35) consist of a system of six linear differential equations for the six unknowns δv_r, δv_z, b_r, b_z, $\delta\rho$ and δU. If these unknowns are determined then $\delta p_\parallel$ and $\delta p_\perp$ can be found.

Assuming cylindrical wave propagation along z-direction, we assume the following solution:

$$(\delta v_r,\ b_r) = (\overline{\delta v^r}.\ \overline{b_r})\, J_1(\gamma r)\, e^{i(\omega t - kz)}$$

$$(\delta v_z, b_z, \delta\rho, \delta U) = (\overline{\delta v_z}, \overline{b_z}, \overline{\delta\rho}, \overline{\delta U})\, J_0(\gamma r)\, e^{i(\omega t - kz)} \quad \ldots \quad (36)$$

where J_0 and J_1 are Bessel's function of first kind of order zero and unity, γ the real constant and $\overline{\delta v_r}$, $\overline{\delta v_z}$, $\overline{b_r}$ $\overline{b_z}$, $\overline{\delta\rho}$ and $\overline{\delta U}$ the small constants in linearized approximation.

Solution (36) is substituted in set of eqns. (21), (23), (26), (27), (34) and (35). Thus,

$$\omega\overline{b_r}-Kv\overline{b_r}+KB\,\overline{\delta v_r}=0 \qquad (37)$$

$$i\omega\overline{b_z}-v\gamma\overline{b_r}+\gamma B\,\overline{\delta v_r}=0 \qquad (38)$$

$$i\omega\,\overline{\delta\rho}+\rho(\gamma\,\overline{\delta v_r}-iK\overline{\delta v_z})-iKv\,\overline{\delta\rho}=0 \qquad (39)$$

$$(\gamma^2+K^2)\,\overline{\delta U}-4\pi G\,\overline{\delta\rho}=0 \qquad (40)$$

$$i\rho\omega\,\overline{\delta v_r}-iK\rho v\,\overline{\delta v_r}-iK\left(\frac{p_{\parallel}-p_{\perp}}{B}-\frac{B}{4\pi\mu}\right)\overline{b_r}$$
$$-\gamma\frac{p_{\perp}}{\rho}\overline{\delta\rho}-\gamma\left(\frac{B}{4\pi\mu}+\frac{p_{\perp}}{B}\right)\overline{b_z}+\rho\gamma\,\overline{\delta U}=0 \qquad (41)$$

$$i\rho\omega\,\overline{\delta v_z}-iK\rho\,\overline{\delta v_z}+\frac{p_{\parallel}-p_{\perp}}{B}\gamma\overline{b_r}+\frac{2ip_{\parallel}}{B}K\overline{b_z}$$
$$-3iK\frac{p_{\parallel}}{\rho}\overline{\delta\rho}+i\rho K\overline{\delta U}=0. \qquad (42)$$

On solving eqn. (37) to (40), we get

$$\overline{b_r}=\frac{-KB\,\overline{\delta v_r}}{(\omega-Kv)} \qquad (43)$$

$$\overline{b_z}=\frac{-B\gamma\,\overline{\delta v_r}}{i(\omega-Kv)} \qquad (44)$$

$$\overline{\delta\rho}=\frac{-\rho\,(\gamma\,\overline{\delta v_r}-iK\,\overline{\delta v_z})}{i(\omega-Kv)} \qquad (45)$$

$$\overline{\delta U}=\frac{-4\pi G\rho(\gamma\,\overline{\delta v_r}-iK\,\overline{\delta v_z})}{i(\gamma^2+K^2)\,(\omega-Kv)}. \qquad (46)$$

Substituting the values of $\overline{b_r}$, $\overline{b_z}$, $\overline{\delta\rho}$ and $\overline{\delta U}$ from eqns. (43) to (46) in eqns. (41) and (42), we get

$$\left[-(\omega-Kv)^2+\frac{2p_m+p_{\perp}-p_{\parallel}}{\rho}K^2+2\frac{p_m+p_{\perp}}{\rho}\gamma^2\right.$$
$$\left.-\frac{4\pi G\rho\gamma^2}{\gamma^2+K^2}\right]\overline{\delta v_r}+iK\gamma\left[\frac{4\pi G\rho}{\gamma^2+K^2}-\frac{p_{\perp}}{\rho}\right]\overline{\delta v_z}=0, \qquad (47)$$

$$\left[-(\omega-Kv)^2-\frac{4\pi G\rho}{\gamma^2+K^2}K^2+3\frac{K^2p_{\parallel}}{\rho}\right]\overline{\delta v_z}-iK\gamma\left[\frac{4\pi G\rho}{\gamma^2+K^2}-\frac{p_{\perp}}{\rho}\right]\overline{\delta v_r}=0. \qquad (48)$$

If the matrix of eqns. (47) and (48) is made equal to zero then we get the following dispersion relation:

$$(\omega-Kv)^4-C(\omega-Kv)^2+D=0 \qquad (49)$$

where

$$C=\frac{1}{\rho}\left[2(p_m+p_{\parallel})+p_{\perp}\right]K^2+\frac{2}{\rho}\quad(p_m+p_{\perp})\gamma^2-4\pi G\rho\,,$$

$$D=K^2\left\{\left(\frac{3p_{\parallel}}{\rho^2}-\frac{4\pi G}{\gamma^2+K^2}\right)\left[2p_m\gamma^2+(p_{\perp}-p_{\parallel}+2p_m)K^2\right]\right.$$
$$\left.+\gamma^2\left[3p_{\parallel}\quad\frac{2p_{\perp}}{\rho^2}-\frac{4\pi G}{\gamma^2+K^2}\quad-\frac{p_{\perp}{}^2}{\rho^2}\right]\right\}.$$

Thus, the linearized equations are satisfied by the solutions (31), (32) and (36) for the propagation of cylindrical waves in collisionless magnetised plasma having initial fluid velocity in z-direction and ω and K satisfy the dispersion relation (33) and (49).

7. Discussions

The dispersion relation (33) and (49) give three distinct roots for ω^2 when K is real. This suggests that there are three modes of wave propagation in the medium.

If we put in the dispersion relation (33) and (49)

$$\omega-Kv=\omega' \qquad (50)$$

we have

$$\omega'^2=K^2\quad A^2+\frac{p_{\perp}-p_{\parallel}}{\rho} \qquad (51)$$

where

$$A^2=\frac{2p_m}{\rho}=\frac{B^2}{4\pi\mu\rho},\quad A \text{ is Alfvén velocity}$$

$$\omega'^4-C\omega'^2+D=0. \qquad (52)$$

If $v=0$, then we have from eqn. (50) $\omega=\omega'$, i.e. when there is no initial motion of the plasma we have frequency ω' and when plasma is in motion the frequency is ω. We can obtain the dispersion relations for plasma at rest by putting $v=0$ in the eqns. (51) and (52). Thus frequency ω of moving plasma is sum of frequency of plasma at rest and a correction term due to Doppler effect. The moving observer measures a frequency which depends upon the wave number (Oster 1960).

On writing the dispersion relation (33) in terms of phase velocity, we have

$$v_f = v + v' \qquad (53)$$

where

$$v' = \pm\sqrt{A^2 + \frac{p_\perp - p_\parallel}{\rho}} \qquad (54)$$

v' is the phase velocity when plasma is at rest and v_f is the phase velocity when plasma is moving with initial velocity v.

Thus, we see that the phase velocity of the moving fluid is the sum of translatory velocity v in the z-direction and the phase velocity v' when plasma is at rest. This new feature in the dispersion relation when plasma is initially moving in the direction of magnetic field as compared to plasma at rest is due to the Doppler effect (Oster 1960). This is also true with dispersion equation (49).

On considering dispersion relation (33) we see that the perturbation δv_ϕ and b_ϕ are propagated along z-axis with phase velocity v_f and with an amplitude [eqns. (31) and (32) which is an arbitrary function of (r), only if

$$2p_m + p_\perp - p_\parallel > 0. \qquad (55)$$

If,

$$2p_m + p_\perp - p_\parallel < 0 \qquad (56)$$

then there will be no wave propagation and we will have an instability named hose instability.

If we put $p_\parallel = p_\perp$ in eqn. (33), then phase velocity is equal to Alfvén velocity and there will be no instability. Thus hose instability is due to anisotropy in the pressure.

From dispersion relation (33) we conclude that it is independent of gravity, so there is no effect of gravitational attraction on this type of wave propagation.

Now, eqn. (52) gives two distinct roots $\omega_1'^2$ and $\omega_2'^2$ when K is real.

$$\omega_1'^2 + \omega_2'^2 = C$$
$$\omega_1'^2 \omega_2'^2 = D.$$

If $D>0$, then there will be wave propagation.

If $D<0$, then one of the two modes must be unstable.

On neglecting action of gravitational attraction we have

$$D = K^2 \left\{ K^2(2p_m + p_\perp - p_\parallel) \quad \frac{3p_\parallel}{\rho^2} + \frac{\gamma^2}{\rho^2}\left[6p_m p_\parallel + (6p_\parallel - p_\perp)p_\perp\right]\right\}. \qquad (57)$$

So, from eqn. (57) we see that in addition to condition (55) we have

$$(6p_\parallel - p_\perp)p_\perp + 6p_\parallel p_m > 0 \qquad (58)$$

for wave propagation. Thus, conditions (55) and (58) are satisfied then there will be always wave propagation in all the three modes in the absence of gravitational attraction.

Now, we consider two cases:

(a) *Propagation along Axial Direction* ($\gamma=0$)

We have from eqn. (52)

$$\frac{\omega_1'^2}{K^2}=\left[\frac{p_\perp-p_{\parallel}}{\rho}+A^2\right] \quad \quad (59)$$

$$\frac{\omega_2'^2}{K^2}=\frac{3p_{\parallel}}{\rho}-\frac{4\pi G\rho}{K^2}. \quad \quad (60)$$

From eqn. (59) we see that the hose instability occurs under the same condition (56). From eqn. (60) we conclude that there will be wave propagation when $K>K_j$ and instability when $K<K_j$

$$\text{where } K_j=\left[\frac{4\pi G\rho}{3p_{\parallel}/\rho}\right]^{\frac{1}{2}}. \quad \quad (61)$$

This type of instability is called Jean's instability and is due to gravitational attraction.

In the absence of gravitational effect we will have no instability of this type. Equation (51) is also true in this case. Thus, we will have all the three modes of wave propagation when $K>K_j$ and condition (55) is satisfied.

(b) *Propagation along Radial Direction* ($K=0$)

From (52) we have

$$\frac{\omega'^2}{\gamma^2}=\frac{2}{\rho}(p_m+p_\perp)-\frac{4\pi G\rho}{\gamma^2}. \quad \quad (62)$$

The wave will propagate when $\gamma>\gamma_j$

$$\gamma_j=\left[\frac{4\pi G\rho}{2(p_m+p_\perp)/\rho}\right]^{\frac{1}{2}}. \quad \quad (63)$$

When $\gamma<\gamma_j$, there will be instability. This type of instability is due to gravitational attraction. In the absence of gravitational attraction, we will not have any instability and there will be always wave propagation. This type of instability criterion also depends upon magnetic field strength. Again in this case we have only one mode of wave propagation, when $\gamma>\gamma_j$.

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