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SOME NON-ISOMORPHIC GRAPHS

D. V. S. SASTRY AND B. SYAM PRASAD RAJU*
Department of Mathematics, Vikram University, Ujjain 456010

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We show that the partial graph of G, $P(G)$ and the total graph $T(G)$ are non-isomorphic for every regular graph of degree $2t$.

§1. For a graph G, with $V(G)$ and $X(G)$ as vertex set and edge set respectively, we define the partial graph $P(G)$ of G with $V(G) \cup X(G)$ as vertex set and two vertices being adjacent if and only if they correspond to two non-adjacent vertices of G, to two adjacent edges of G or to a vertex and an edge incident to it in G.

Let $L(G)$ and $T(G)$ denote the line graph and the total graph of G respectively. The symbol $\cong$ indicates isomorphism between two graphs. We call the vertices of G in $T(G)$ as well as $P(G)$ point vertices and the vertices corresponding to the edges of G line vertices. For undefined terms we refer to Harary (1969).

Let $S(G)$ denote the subdivision graph of G. Then $S(G) \subset P(G)$. Also the vertex set of $P(G)$

$$| V(P(G)) | = | V(G) | + V | L(G) |$$

and the edge set of $P(G)$.

$$| X(P(G)) | = | X(S(G)) | + | X(L(G)) | + | X(C(G)) |$$
$$= | X(T(G)) | - | X(G) | + | X(C(G)) | .$$

For $P(G) \cong T(G)$, we must have G as a self complementary graph.

Hereafter, G denotes a regular graph of degree $2t$ on $4t + 1$ vertices. Then by the definition of $P(G)$ and $T(G)$, both $P(G)$ and $T(G)$ have $4t^2 + 5t + 1$ vertices and are regular of degree $4t$ and so have the same number of edges.

§2. *Lemma* — For $T(G) \cong P(G)$, a line vertex of $T(G)$ does not correspond to a point vertex of $P(G)$.

PROOF : Suppose, if possible, a line vertex say v_{ij} [corresponding to the edge (v_i, v_j) in G] of $T(G)$ corresponds to a point vertex α_k of $P(G)$.

Then in $T(G)$, v_{ij} is on two complete subgraphs say A and B each on $2t + 1$ vertices with v_{ij} as a common vertex lying on both the complete subgraphs A and B and v_i is adjacent to v_j.

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For $T(G) \cong P(G)$ in $P(G)$ also there must be two complete subgraphs, C and D with α_k as the common vertex. Since G is regular of degree $2t$, α_k is adjacent to $2t$ line vertices α_{kn} in $P(G)$, and by the definition of $P(G)$ vertices of type α_{kn} and α_{km} $(n \neq m)$ are adjacent. Therefore the $2t$ line vertices α_{kn} and α_k form a complete subgraph on $2t + 1$ vertices. Call this as C.

By the definition of $P(G)$, α_k cannot be adjacent to any line vertex other than the line vertices in C. So α_k should be adjacent to point vertices only. Two point vertices are adjacent in $P(G)$ iff they are not adjacent in G. Since G is regular of degree $2t$ on $4t + 1$ vertices, in $P(G)$ a point vertex can be adjacent to $2t$ other point vertices. Thus with α_k they form a complete subgraph D on $2t + 1$ vertices.

If there is an isomorphism ϕ of $T(G)$ onto $P(G)$ then either $\phi(A) = C$ or D and if $\phi(A) = C$, then $\phi(B) = D$ and if $\phi(A) = D$ then $\phi(B) = C$. Without loss of generality assume that $\phi(A) = C$ and $\phi(B) = D$. Now we cannot find an edge in $P(G)$ one of whose incident points is among the vertices (except α_k) of the complete subgraph C, and the other among the vertices (except α_k) of the complete subgraph D, because by the definition of $P(G)$, a line vertex is precisely adjacent to two point vertices; and two point vertices are not adjacent if and only if they were adjacent in G. On the other hand in $T(G)$ we have two complete subgraphs A and B, with v_i in A adjacent to v_j in B. Hence the proof.

Theorem — $T(G)$ and $P(G)$ are non-isomorphic.

PROOF: From the above lemma, we observe that a line vertex of $T(G)$ can only correspond to a line vertex of $P(G)$. Since both $T(G)$ and $P(G)$ have the same vertex set, the point vertices and line vertices of $T(G)$ must correspond to point vertices and line vertices of $P(G)$ respectively. But since both $P(G)$ and $T(G)$ have $L(G)$ as induced subgraph, $L(G)$ corresponds to itself under any isomorphism of $P(G)$ onto $T(G)$.

(a) Now let v_i and v_j be the two point vertices adjacent in $T(G)$. Then there is a line vertex v_{ij} [corresponding to the edge (v_i, v_j) of G] in $T(G)$ which is adjacent to v_i and v_j.

Take two point vertices α_k and α_m which are adjacent in $P(G)$. There does not exist a line vertex which is adjacent to both α_k and α_m, since α_k and α_m are not adjacent in G.

Thus from (a) $T(G)$ is not isomorphic to $P(G)$.

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REFERENCE

Harary, F. (1969). Graph Theory. Addison Wesley, Reading, Mass.