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THE EFFECTS OF VARIABLE SUCTION/INJECTION ON THE FREE CONVECTION FLOW PAST AN ACCELERATED VERTICAL INSULATED PLATE

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The effects of variable suction/injection on the flow field, of an incompressible viscous fluid, past an uniformly accelerated vertical porous plate is studied, when there is no heat transfer between the fluid and the plate. Approximate solution of the problem is obtained when the Prandtl number is taken equal to one. The variations of the velocity, the temperature and the skin friction are shown graphically followed by a quantitative discussion.

1. INTRODUCTION

Boundary layer behaviour on a moving continuous solid surface was first investigated by Sakiadis (1961). Erickson *et al.* (1965) extended this problem to include forced convection heat and mass transfer on a moving plate subjected to suction or injection. Kafousias *et al.* (1979) studied the free convection flow past an impulsively started vertical plate when the fluid is subjected to a constant suction velocity through the porous plate. Recently Gupta *et al.* (1979) investigated the free convection effects on the flow past an accelerated vertical impermeable plate in an incompressible dissipative fluid.

The physical situation discussed above is one of the possible cases. Another physical phenomenon will be that if the plate is perfectly insulated against heat flow, i.e., there is no heat transfer between the fluid and the plate. The heat generated by the fluid through friction serves to heat the plate until the condition $(\partial T/\partial y) = 0$ is reached. Such a condition is satisfied in practice when a so-called plate thermometer is employed to determine high-speed stream temperature (cf. Houghton and Boswell, 1969, p. 261; Schlichting, 1968, p. 267).

Ostrach (1953) presented the first numerical results for free convection flow past a semi-infinite isothermal impermeable vertical flat plate. Recently Takhar and Whitelaw (1976) studied the problem of free convection flow past an insulated vertical impermeable flat plate using matched asymptotic methods.

The significance of suction for the boundary layer control in the field of aerodynamics and space science is well recognized. It is often necessary to prevent separation of the boundary layer to reduce drag and attain high lift values. On the other hand, one of the important problems facing the engineers engaged in high-speed flow is the cooling of the surface to avoid structural failures as a result of frictional heating and other factors. We know the possibility of using injection at the surface to cool the body in a high-temperature fluid.

Hence, the purpose of the present work is to study the free convection effects on the flow field of an incompressible, viscous, dissipative fluid, past an infinite vertical porous plate which is accelerated in its own plane. The fluid is subjected to a normal velocity of suction/injection proportional to $t'^{-1/2}$ and the plate is perfectly insulated, i.e., there is no heat transfer between the fluid and the plate. At time $t' > 0$ the plate starts moving with a velocity $u' = c't'$ (where c' is a positive constant) and the fluid is assumed to have constant properties except that the influence of the density variations with temperature is considered in the body force term.

2. Mathematical Analysis

Two-dimensional free convection flow of an incompressible, viscous fluid past an infinite vertical porous plate is considered. The x'-axis is taken along the plate in the upward direction and the ψ'-axis normal to it. In this situation, all the physical quantities are functions of the space coordinate ψ' and the time t' only. Under these conditions, the problem is governed by the following set of equations:

$$\frac{\partial v'}{\partial \psi'} = 0 \qquad \ldots(1)$$

$$\frac{\partial u'}{\partial t'} + v' \frac{\partial u'}{\partial \psi'} = g_x\beta(T' - T'_\infty) + \nu \frac{\partial^2 u'}{\partial \psi'^2} \qquad \ldots(2)$$

$$\frac{\partial T'}{\partial t'} + v' \frac{\partial T'}{\partial \psi'} = \frac{k}{\rho' c_p} \frac{\partial^2 T'}{\partial \psi'^2} + \frac{\nu}{c_p} \left(\frac{\partial u'}{\partial \psi'}\right)^2 \qquad \ldots(3)$$

where u', v' are the corresponding velocity components along and perpendicular to the plate, g_x is the acceleration due to gravity, β the coefficient of volume expansion, T' the temperature of the fluid, ν the kinematic viscosity, k the thermal conductivity, ρ' the density and c_p the specific heat at constant pressure.

The boundary conditions are

$$t' > 0: \quad \left.\begin{aligned} & u'(0, t') = ct', \ \frac{\partial T'(0, t')}{\partial \psi'} = 0 \\ & u'(\infty, t') = 0, \ T'(\infty, t') = T'_\infty \end{aligned}\right\} \qquad \ldots(4)$$

From eqn. (1), we take

$$v' = -v_0(t') = -\alpha \left(\frac{\nu}{t'}\right)^{1/2} \qquad \ldots(5)$$

where α represents the velocity of suction or injection at the plate according as $\alpha >$ or < 0, respectively.

We introduce the following non-dimensional variables and parameters

$$\psi = \psi'\left(\frac{c}{\nu^2}\right)^{1/3}, \quad u = \frac{u'}{(\nu c)^{1/3}}, \quad t = t'\left(\frac{c^2}{\nu}\right)^{1/3},$$

$$T = \frac{(T' - T'_\infty)}{(T'_w - T'_\infty)}, \quad P = \frac{\rho' \nu c_p}{k} \text{ (Prandtl number)}$$

$$E = \frac{(\nu c)^{2/3}}{c_p(T'_w - T'_\infty)} \text{ (Eckert number)}$$

$$G = \frac{1}{c} g_x \beta (T'_w - T'_\infty) \text{ (Grashof number).} \qquad \text{...(6)}$$

On using these non-dimensional quantities and (5), we obtain the following equations governing the motion, in the non-dimensional form

$$\frac{\partial u}{\partial t} - \frac{\alpha}{t^{1/2}}\frac{\partial u}{\partial \psi} = GT + \frac{\partial^2 u}{\partial \psi^2} \qquad \text{...(7)}$$

$$P\frac{\partial T}{\partial t} - \frac{\alpha P}{t^{1/2}}\frac{\partial T}{\partial \psi} = \frac{\partial^2 T}{\partial \psi^2} + PE\left(\frac{\partial u}{\partial \psi}\right)^2. \qquad \text{...(8)}$$

The boundary conditions reduce now to

$$t > 0 : \left.\begin{aligned} & u(0, t) = t, \quad \frac{\partial T(0, t)}{\partial \psi} = 0 \\ & u(\infty, t) = 0, \quad T(\infty, t) = 0. \end{aligned}\right\} \qquad \text{...(9)}$$

In order to solve the non-linear, coupled system of eqns. (7) and (8) we expand u and T in powers of E, under the assumption $E \ll 1$, which is valid for incompressible fluids.

Hence,

$$\left.\begin{aligned} u(\psi, t) &= u_0(\psi, t) + Eu_1(\psi, t) + \ldots \\ T(\psi, t) &= T_0(\psi, t) + ET_1(\psi, t) + \ldots \end{aligned}\right\} \qquad \text{...(10)}$$

By substituting (10) in (7) and (8) and equating the coefficients of the same powers of E, we get the following hierarchy of equations:

$$\frac{\partial u_0}{\partial t} - \frac{\alpha}{t^{1/2}}\frac{\partial u_0}{\partial \psi} = GT_0 + \frac{\partial^2 u_0}{\partial \psi^2} \qquad \text{...(11)}$$

$$P\frac{\partial T_0}{\partial t} - \frac{\alpha P}{t^{1/2}}\frac{\partial T_0}{\partial \psi} = \frac{\partial^2 T_0}{\partial \psi^2} \qquad \text{...(12)}$$

$$\frac{\partial u_1}{\partial t} - \frac{\alpha}{t^{1/2}}\frac{\partial u_1}{\partial \psi} = GT_1 + \frac{\partial^2 u_1}{\partial \psi^2} \qquad ...(13)$$

$$P\frac{\partial T_1}{\partial t} - \frac{\alpha P}{t^{1/2}}\frac{\partial T_1}{\partial \psi} = \frac{\partial^2 T_1}{\partial \psi^2} + P\left(\frac{\partial u_0}{\partial \psi}\right)^2. \qquad ...(14)$$

The boundary conditions (11) become

$$t > 0: \left.\begin{array}{l} u_0(0, t) = t,\ u_1(0, t) = 0,\ \dfrac{\partial T_0}{\partial \psi}(0, t) = 0,\ \dfrac{\partial T_1}{\partial \psi}(0, t) = 0 \\ u_0(\infty, t) = 0,\ u_1(\infty, t) = 0,\ T_0(\infty, t) = 0,\ T_1(\infty, t) = 0. \end{array}\right\} \qquad ...(15)$$

We take the Prandtl number P equal to one. This is a plausible assumption since P is a measure of the relative importance of viscosity and heat conductivity in the fluid. With this assumption the solution of the problem for small t, can be obtained in the form

$$u_0 = tf_0(\eta),\ T_0 = g_0(\eta),\ u_1 = t^3 f_1(\eta),\ T_1 = t^2 g_1(\eta) \qquad ...(16)$$

where $\eta = \psi/2t^{1/2}$.

Substituting (16) into (11) - (14), we get

$$\left.\begin{array}{l} f_0'' + 2(\eta + \alpha) f_0' - 4f_0 = -4Gg_0 \\ f_1'' + 2(\eta + \alpha) f_1' - 12f_1 = -4Gg_1 \\ g_0'' + 2(\eta + \alpha) g_0' = 0 \\ g_1'' + 2(\eta + \alpha) g_1' - 8g_1 = -f_0'^2 \end{array}\right\} \qquad ...(17)$$

where the prime denotes differentiation with respect to η. The boundary conditions (15) become

$$\left.\begin{array}{l} f_0(0) = 1,\ g_0'(0) = 0,\ f_0(\infty) = 0,\ g_0(\infty) = 0 \\ f_1(0) = 0,\ g_1'(0) = 0,\ f_1(\infty) = 0,\ g_1(\infty) = 0. \end{array}\right\} \qquad ...(18)$$

The solution of the (17), under their boundary conditions (18) is

$$g_0(\eta) = 0 \qquad ...(19)$$

$$f_0(\eta) = \frac{Hh_2(\sqrt{2}\,\xi)}{Hh_2(\sqrt{2}\,\alpha)} \qquad ...(20)$$

$$g_1(\eta) = \frac{Hh_1(\sqrt{2}\,\alpha)}{Hh_2(\sqrt{2}\,\alpha)} \cdot \frac{Hh_4(\sqrt{2}\,\xi)}{Hh_3(\sqrt{2}\,\alpha)} - \frac{Hh_2^2(\sqrt{2}\,\xi)}{2Hh_2^2(\sqrt{2}\,\alpha)} \qquad ...(21)$$

$$f_1(\eta) = G\left(\frac{Hh_1(\sqrt{2}\,\alpha)}{Hh_2(\sqrt{2}\,\alpha)} \cdot \frac{Hh_4(\sqrt{2}\,\xi)}{Hh_3(\sqrt{2}\,\alpha)} + \tfrac{1}{2} \cdot \frac{Hh_3^2(\sqrt{2}\,\xi)}{Hh_2^2(\sqrt{2}\,\alpha)} - \left(\frac{2Hh_2(\sqrt{2}\,\alpha)\,.\,Hh_1(\sqrt{2}\,\alpha)\,.\,Hh_4(\sqrt{2}\,\alpha) + Hh_3^3(\sqrt{2}\,\alpha)}{2Hh_2^2(\sqrt{2}\,\alpha)\,.\,Hh_3(\sqrt{2}\,\alpha)\,.\,Hh_6(\sqrt{2}\,\alpha)}\right) \times Hh_6(\sqrt{2}\,\xi)\right) \quad \text{...(22)}$$

where $\xi = \eta + \alpha$. The function $Hh_n(\sqrt{2}\,\xi)$ is defined in Jeffreys and Jeffreys (1972) and is related with the complementary error function (erfc) as in Appendix. Using (10) and (16) the skin friction τ_w, in dimensionless form, may be found as

$$\tau_w = \tfrac{1}{2}\, t^{1/2}[f_0'(0) + Et^2 f_1'(0)]$$

$$= -\frac{\sqrt{2}}{2}\, t^{1/2}\left[\frac{Hh_1(\sqrt{2}\,\alpha)}{Hh_2(\sqrt{2}\,\alpha)} + G\,.\,E\,.\,t^2\left[\frac{Hh_1(\sqrt{2}\,\alpha)}{Hh_2(\sqrt{2}\,\alpha)} + \frac{Hh_3(\sqrt{2}\,\alpha)}{Hh_2(\sqrt{2}\,\alpha)} - \frac{[2Hh_2(\sqrt{2}\,\alpha)\,.\,Hh_3(\sqrt{2}\,\alpha)\,.\,Hh_4(\sqrt{2}\,\alpha) + Hh_3^3(\sqrt{2}\,\alpha)]}{2Hh_2^2(\sqrt{2}\,\alpha)\,.\,Hh_3(\sqrt{2}\,\alpha)} \times \frac{Hh_5(\sqrt{2}\,\alpha)}{Hh_6(\sqrt{2}\,\alpha)}\right.\right. \cdot \quad \text{...(23)}$$

Also, the velocity and temperature field are given by

$$\left.\begin{aligned} u &= tf_0(\eta) + Et^3 f_1(\eta), \\ \text{and}\qquad T &= Et^2 g_1(\eta), \end{aligned}\right\} \quad \text{...(24)}$$

respectively, where f_0, f_1 and g_1 are given by (20) - (22).

3. Results

The obtained numerical results, for various combinations of the dimensionless parameters G, E, α and time t, are shown in Figs. 1 - 3. Figure 1 shows the variation of the dimensionless velocity when $G = 10$ and $E = 0.02$. It is observed that application of suction ($\alpha = 0.5$) helps in reducing the dimensionless velocity and this phenomenon is more evident for higher values of time t.

The dimensionless temperature is shown in Fig. 2 when $G = 5$ and for different values of α, E and t. In this case the viscous dissipation term causes the formation of a temperature field in the thermal boundary layer. As mentioned earlier, it is assumed that the plate is at a higher temperature than that of the free stream (i.e., $T'_w > T'_\infty$) and that no heat transfer takes place between the fluid and the plate. Under these conditions, as expected, the fluid temperature decreases with the distance from the plate.

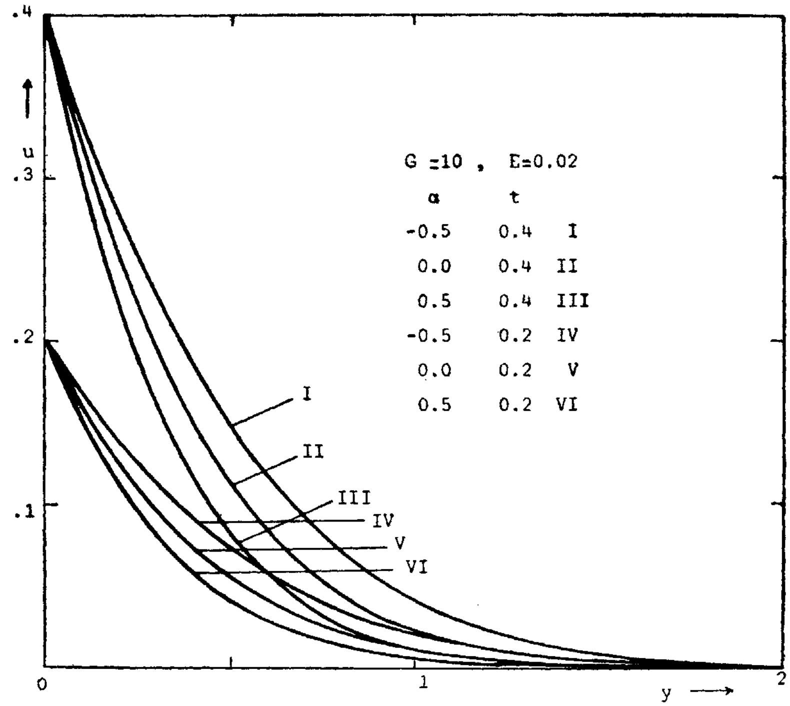

FIG. 1. The velocity profiles.

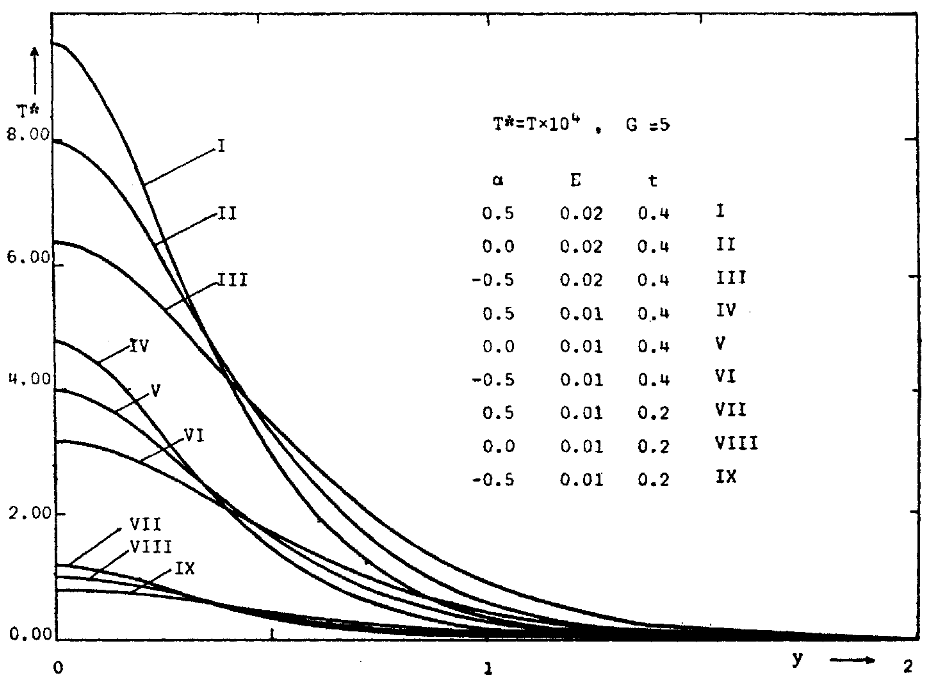

FIG. 2. The temperature profiles.

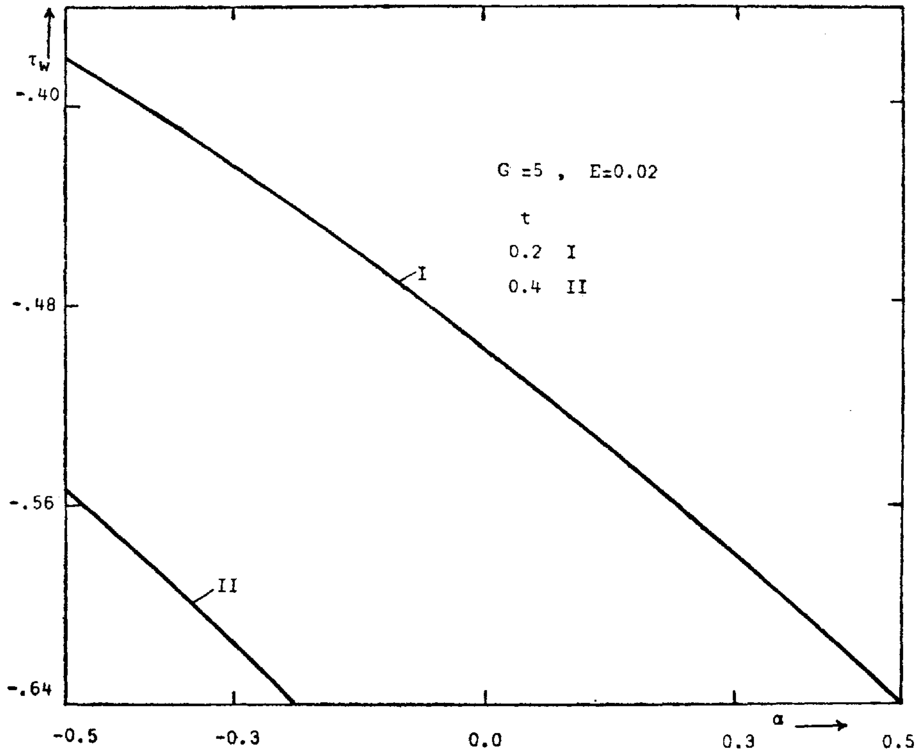

FIG. 3. Variation of the skin friction.

Also we see that more heat is produced inside the thermal boundary layer in the case of suction than injection.

Finally from Fig. 3 we conclude that application of suction does not help in reducing the frictional drag from the plate.

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Appendix

$$Hh_0(\sqrt{2}\,\xi) = \sqrt{\pi/2}\ \text{erfc}\,(\xi),$$

$$Hh_1(\sqrt{2}\,\xi) = e^{-\xi^2} - \sqrt{\pi}\xi\ \text{erfc}\,(\xi),$$

$$Hh_2(\sqrt{2}\,\xi) = 0.5\,[\sqrt{\pi/2}\,(1 + 2\xi^2)\ \text{erfc}\,(\xi) - \sqrt{2}\,\xi e^{-\xi^2}],$$

$$Hh_3(\sqrt{2}\,\xi) = \frac{1}{6}\,[2(1 + \xi^2)\,e^{-\xi^2} - \sqrt{\pi}\,\xi(3 + 2\xi^2)\ \text{erfc}\,(\xi)],$$

$$Hh_4(\sqrt{2}\,\xi) = \frac{1}{24}\,[\sqrt{\pi/2}\,(3 + 12\xi^2 + 4\xi^4)\ \text{erfc}\,(\xi) - \sqrt{2}\,\xi(5 + 2\xi^2)\,e^{-\xi^2}],$$

$$Hh_5(\sqrt{2}\,\xi) = \frac{1}{120}\,[(8 + 18\xi^2 + 4\xi^4)\,e^{-\xi^2} - \sqrt{\pi}\,\xi(15 + 20\xi^2 + 4\xi^4)\ \text{erfc}\,(\xi)],$$

$$Hh_6(\sqrt{2}\,\xi) = \frac{1}{720}\,[\sqrt{\pi/2}\,(15 + 90\xi^2 + 60\xi^4 + 8\xi^6)\ \text{erfc}\,(\xi) - \sqrt{2}\,\xi(33 + 28\xi^2 + 4\xi^4)\,e^{-\xi^2}].$$