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ON THE ABSOLUTE SUMMABILITY OF WEIGHTED FOURIER SERIES BY NÖRLUND MEANS

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In this paper a general result on the absolute Nörlund summability factors of a Fourier series with a weight has been obtained.

§1. Let $\{p_n\}$ be a sequence of real constants and let

$$P_n = \sum_{\nu=0}^{n} p_\nu \neq 0, \; P_{-1} = p_{-1} = 0.$$

The series $\Sigma\, a_n$ is said to be summable $|\, N, p_n \,|$ if

$$\sum_{n=1}^{\infty} |\, t_n - t_{n-1} \,| < \infty$$

where

$$t_n = \frac{1}{P_n} \sum_{\nu=0}^{n} p_{n-\nu} S_\nu,$$

S_n being the nth partial sum of the series $\Sigma\, a_n$.

Let $f(t)$ be a periodic function with period 2π and integrable in the Lebesgue sense in $(-\pi, \pi)$. We write

$$\phi(t) = \tfrac{1}{2}\,\{f(x+t) + f(x-t)\},$$

$$\tau = \left[\frac{1}{t}\right], \; m = \left[\frac{n}{2}\right]$$

and

$$\Delta\, \epsilon_n = \epsilon_n - \epsilon_{n+1}.$$

C stands for an absolute constant not necessarily the same at each occurrence.

§2. The absolute Nörlund summability factors of Fourier series have been studied by one of the authors (see Lal 1963a, 1963b, 1969, 1974, 1976), Singh (1967), Kishore (1967), Bhatt (1969), Kanno (1969), Izumi and Izumi (1970), Okuyama (1975), Dikshit (1976) and others. The results obtained by these authors are either for special values of p_n or for a sequence $\{p_n\}$ which is nonnegative and non-increasing. In this paper we study the $| N, p_n |$ summability factors of a Fourier series, with a 'weight', for a nonnegative sequence $\{p_n\}$ and establish the following

Theorem — Let $\{p_n\}$ be a sequence of nonnegative numbers such that (1) $\{\chi_n\} = \left\{\frac{(n+1)\,p_n}{P_n}\right\}$ is of bounded variation and (2) $\{p_n/P_n\}$ is monotonic non-increasing. Let $\epsilon(t)$ be a positive, monotonic non-decreasing function and $\lambda(t)$ a function of bounded variation such that $\epsilon(n)/P_n$ and $\epsilon(n)\,\lambda(n)/n$ are monotonic non-increasing and

$$\sum_{\nu=n}^{\infty} \frac{\epsilon(\nu)\,\lambda(\nu)}{\nu P_{\nu-1}} = O\left(\frac{\epsilon(n)}{P_n}\right) \qquad \ldots(2.1)$$

$$\sum_{\nu=n}^{\infty} \frac{\epsilon(\nu)}{\nu^2} = O\left(\frac{\epsilon(n)}{n}\right) \qquad \ldots(2.2)$$

$$\sum_{\nu=n}^{\infty} \frac{\epsilon(\nu)\,|\,p_\nu - p_{\nu+1}\,|}{P_{\nu+1}} = O\left(\frac{\epsilon(n)}{n}\right). \qquad \ldots(2.3)$$

If

$$\int_0^{\pi} \epsilon(K/t)\,|\,d\phi(t)\,| < \infty, \qquad \ldots(2.4)$$

K being a suitable constant, then the series $\Sigma\,\epsilon(n)\,\lambda(n)\,A_n(t)$ is summable $| N, p_n |$ at $t = x$.

It is interesting to note that the particular case

$$p_n = \frac{1}{n+1}, \quad \epsilon(n) = 1 \quad \text{and} \quad \lambda(n) = \frac{1}{\log(n+1)}$$

of the above theorem is an earlier result due to Varshney (1959) which is an improvement upon a theorem due to Mohanty (1951) in view of the fact that the summability $\left| N, \frac{1}{n+1} \right|$ is weaker than the method $| R, e^{n^\alpha}, 1 |$ (c. f. Das 1969).

§3. The following lemma is pertinent to the proof of our theorem.

Lemma (Kishore 1967) — If a sequence $\{\alpha_n\}$ is of bounded variation, then

$$\sum_{n=1}^{\infty} \frac{1}{n+1} \sum_{\nu=m}^{n} |\,\Delta \alpha_\nu\,| < \infty.$$

PROOF OF THE THEOREM : Before proceeding to prove the theorem we observe that if $\{p_n\}$ is a sequence of nonnegative numbers such that the conditions (2.1) and (2.2) hold, then $P_{2n} = O(P_n)$. Indeed,

$$\frac{P_{2n}}{P_n} = \left(\frac{P_n}{P_{n+1}}\right)^{-1}\left(\frac{P_{n+1}}{P_{n+2}}\right)^{-1}\left(\frac{P_{n+2}}{P_{n+3}}\right)^{-1}\cdots\left(\frac{P_{2n-1}}{P_{2n}}\right)^{-1}$$

$$= \left(1-\frac{p_{n+1}}{P_{n+1}}\right)^{-1}\left(1-\frac{p_{n+2}}{P_{n+2}}\right)^{-1}\cdots\left(1-\frac{p_{2n}}{P_{2n}}\right)^{-1}$$

$$\leqslant \left(1-\frac{C}{n+1}\right)^{-n}$$

and since

$$\lim_{n\to\infty}\frac{(1-C(n+1)^{-1})}{((1-C(n+1)^{-1})^{-(n+1)/C})^{-C}} = e^C$$

it follows that $P_{2n} = O(P_n)$ as $n \to \infty$.

If t_n denotes the (N, p_n) transformation of the series $\Sigma\, u_n$ where

$$u_n = \lambda(n)\,\epsilon(n)\,A_n(t),$$

then

$$t_n - t_{n-1} = \frac{1}{P_nP_{n-1}}\sum_{k=0}^{n-1}(P_np_k - P_kp_n)\,u_{n-k}$$

$$= \frac{1}{(n+1)\,P_{n-1}}\sum_{k=0}^{n-1}p_k(n-k)\,u_{n-k}$$

$$+\frac{1}{(n+1)\,P_{n-1}}\sum_{k=0}^{n-1}P_k(\chi_k-\chi_n)\,u_{n-k}.$$

Since

$$A_n(t) = \frac{2}{\pi}\int_0^\pi \phi(t)\cos nt\,dt$$

$$= -\frac{2}{\pi}\int_0^\pi \frac{\sin nt}{n}\,d\phi(t)$$

we have

$$\sum_{n=1}^{\infty} | t_n - t_{n-1} |$$

$$\leqslant \frac{2}{\pi} \int_0^{\pi} | d\phi(t) | \Big[\sum_{n=1}^{2\tau} \frac{1}{P_n P_{n-1}} \Big| \sum_{k=0}^{n-1} (P_n p_k - P_k p_n)$$

$$\times \frac{\lambda(n-k)\, \epsilon(n-k)}{(n-k)} \sin (n-k)\, t \Big|$$

$$+ \sum_{n=2\tau+1}^{\infty} \frac{1}{(n+1)\, P_{n-1}} \Big| \sum_{k=0}^{n-1} p_k \lambda(n-k)\, \epsilon(n-k) \sin (n-k)\, t \Big|$$

$$+ \sum_{n=2\tau+1}^{\infty} \frac{1}{(n+1)\, P_{n-1}} \Big| \sum_{k=0}^{n-1} P_k (\chi_k - \chi_n)$$

$$\times \frac{\lambda(n-k)\, \epsilon(n-k)}{(n-k)} \sin (n-k)\, t \Big| \Big]$$

$$= \frac{2}{\pi} \int_0^{\pi} | d\phi(t) | \Big[\sum\nolimits_1 + \sum\nolimits_2 + \sum\nolimits_3 \Big], \text{ say.} \qquad \ldots(3.1)$$

Now

$$\sum\nolimits_1 \leqslant Ct \sum_{n=1}^{2\tau} \frac{\epsilon(n)}{P_{n-1}} \sum_{k=0}^{n-1} p_k$$

$$= O(\epsilon(k/t)) \qquad \ldots(3.2)$$

using the hypotheses that $\epsilon(n)$ is non-decreasing and $\lambda(n)$ is bounded.

And

$$\sum\nolimits_2 \leqslant \sum_{n=2\tau+1}^{\infty} \frac{1}{(n+1)\, P_{n-1}} \Big| \sum_{k=0}^{n-m-1} p_k \lambda(n-k)\, \epsilon(n-k) \sin (n-k)\, t \Big|$$

$$+ \sum_{n=2\tau+1}^{\infty} \frac{1}{(n+1)\, P_{n-1}} \Big| \sum_{k=n-m}^{n-1} p_k \lambda(n-k)\, \epsilon(n-k) \sin (n-k)\, t \Big|$$

$$= \Sigma_{2,1} + \Sigma_{2,2}, \text{ say.} \qquad \ldots(3.3)$$

Now

$$\sum_{2,1} \leqslant \sum_{n=2\tau+1}^{\infty} \frac{1}{(n+1)\,P_{n-1}} \left| \sum_{k=0}^{\tau} p_k \lambda(n-k)\,\epsilon(n-k)\sin(n-k)\,t \right|$$

$$+ \sum_{n=2\tau+1}^{\infty} \frac{1}{(n+1)\,P_{n-1}} \left| \sum_{k=\tau+1}^{n-m-1} p_k \lambda(n-k)\,\epsilon(n-k)\sin(n-k)\,t \right|$$

$$\leqslant \sum_{n=2\tau+1}^{\infty} \frac{1}{(n+1)\,P_{n-1}} \sum_{k=0}^{\tau} p_k \frac{\lambda(n-k)}{(n-k)}\,\epsilon(n-k)\,(n-k)$$

$$+ \sum_{n=2\tau+1}^{\infty} \frac{1}{(n+1)\,P_{n-1}} \Bigg| \sum_{k=\tau+1}^{n-m-2} \Delta_k(\lambda(n-k)\,p_k)$$

$$\times \sum_{r=\tau+1}^{k} \epsilon(n-r)\sin(n-r)\,t$$

$$+ \lambda(m+1)\,p_{n-m-1} \sum_{r=\tau+1}^{n-m-1} \epsilon(n-r)\sin(n-r)\,t \Bigg|$$

$$= O(P_\tau) \sum_{n=2\tau+1}^{\infty} \frac{\epsilon(n-\tau)\,(n-\tau)}{P_{n-1}(n-\tau)}$$

$$+ O(\tau) \sum_{n=2\tau+1}^{\infty} \frac{\epsilon(n)}{(n+1)\,P_{n-1}} \sum_{k=\tau+1}^{m} \left| \Delta_k(\lambda(n-k)\,p_k) \right|$$

$$+ O(\tau) \sum_{n=2\tau+1}^{\infty} \frac{\epsilon(n)\,\lambda(m+1)}{(n+1)\,P_{n-1}}\,p_{n-m-1}$$

$$= O(P_\tau) \sum_{\nu=\tau+1}^{\infty} \frac{\epsilon(\nu)\,\lambda(\nu)}{\tau P_\nu} + O(\tau) \sum_{n=2\tau+1}^{\infty} \frac{\epsilon(n)}{(n+1)\,P_{n-1}}$$

$$\times \sum_{k=\tau+1}^{m} \frac{\lambda(n-k)}{(n-k)}\,(n-k)\,|\,\Delta p_k\,| +$$

(equations continued on p. 355)

$$+ O(\tau) \sum_{n=2\tau+1}^{\infty} \frac{\epsilon(n)}{(n+1) P_{n-1}}$$

$$\times \sum_{k=\tau+1}^{m} | \Delta p_{k+1} | \sum_{r=\tau+1}^{k} | \Delta\lambda(n-r) |$$

$$+ O(\tau) \sum_{n=2\tau+1}^{\infty} \frac{\epsilon(n)}{(n+1) P_{n-1}} p_{m+1} \sum_{r=\tau+1}^{m} | \Delta\lambda(n-r) |$$

$$+ O(\tau) \sum_{n=\tau}^{\infty} \frac{\epsilon(n)\, \lambda(n)}{n^2}$$

$$= O(\epsilon(K/t)) + O(\tau) \sum_{n=2\tau+1}^{\infty} \frac{\epsilon(m)\, \lambda(m)}{mP_{m-1}} \sum_{k=\tau+1}^{m} | \Delta p_k |$$

$$+ O(\tau) \sum_{n=\tau}^{\infty} \frac{\epsilon(n)\, \lambda(n)}{n^2} \left(\text{since } \sum_{r=0}^{m} | \Delta\lambda(n-r) | = O(\lambda(m))\right)$$

$$= O(\epsilon(K/t)) + O(\tau) \sum_{k=\tau}^{\infty} | \Delta p_k | \sum_{n=2k}^{\infty} \frac{\epsilon(m)\, \lambda(m)}{mP_{m-1}}$$

$$= O(\epsilon(K/t)) + O(\tau) \sum_{k=\tau}^{\infty} \left| \Delta \left(\frac{p_k}{P_k} P_k\right)\right| \frac{\epsilon(k)}{P_k}$$

$$= O(\epsilon(K/t)) + O(\tau) \sum_{k=\tau}^{\infty} \left| \Delta \left(\frac{p_k}{P_k}\right)\right| P_k \frac{\epsilon(k)}{P_k}$$

$$+ O(\tau) \sum_{k=\tau}^{\infty} \frac{p_{k+1}^2\, \epsilon(k)}{P_{k+1} P_k}$$

$$= O(\epsilon(K/t)) + O(\tau) \sum_{k=\tau}^{\infty} \frac{\epsilon(k)\, | p_k - p_{k+1} |}{P_{k+1}} + O(\tau) \sum_{k=\tau}^{\infty} \frac{\epsilon(k)}{k^2}$$

$$= O(\epsilon(K/t)) \qquad \ldots(3.4)$$

using the hypotheses of the theorem.

And

$$\sum_{2,2} = \sum_{n=2\tau+1}^{\infty} \frac{1}{(n+1)\,P_{n-1}} \Big| \sum_{k=n-m}^{n-2} \Delta p_k$$

$$\times \sum_{r=n-m}^{k} \epsilon(n-r)\,\lambda(n-r)\sin(n-r)\,t$$

$$+ p_{n-1} \sum_{r=n-m}^{n-1} \epsilon(n-r)\,\lambda(n-r)\sin(n-r)\,t \Big|$$

$$= O(\tau) \sum_{n=2\tau+1}^{\infty} \frac{\epsilon(n)}{(n+1)\,P_{n-1}} \sum_{k=n-m}^{n-2} \left| \Delta \left(\frac{p_k}{P_k} P_k \right) \right|$$

$$+ O(\tau) \sum_{n=2\tau+1}^{\infty} \frac{\epsilon(n)\,p_{n-1}}{(n+1)\,P_{n-1}}$$

$$= O(\tau) \sum_{n=2\tau+1}^{\infty} \frac{\epsilon(n)}{(n+1)\,P_{n-1}} \sum_{k=n-m}^{n-2} \left| P_k \Delta \left(\frac{p_k}{P_k} \right) - \frac{p_{k+1}^2}{P_{k+1}} \right|$$

$$+ O(\tau) \sum_{n=\tau}^{\infty} \frac{\epsilon(n)}{n^2}$$

$$= O(\tau) \sum_{n=2\tau+1}^{\infty} \frac{\epsilon(n)}{n+1} \frac{p_m}{P_m} + O(\epsilon(K/t))$$

$$= O(\epsilon(K/t)) \qquad \ldots(3.5)$$

by the hypotheses of the theorem.

$$\sum_{3} \leqslant \sum_{n=2\tau+1}^{\infty} \frac{1}{(n+1)\,P_{n-1}}$$

$$\times \left| \sum_{k=0}^{m} P_k(\chi_k - \chi_n) \frac{\lambda(n-k)\,\epsilon(n-k)}{(n-k)} \sin(n-k)\,t \right|$$

$$+ \sum_{n=2\tau+1}^{\infty} \frac{1}{(n+1)\,P_{n-1}} \times$$

(*equation continued on p. 357*)

$$\times \left| \sum_{k=m+1}^{n-\tau} P_k(\chi_k - \chi_n) \frac{\lambda(n-k)\,\epsilon(n-k)}{(n-k)} \sin(n-k)t \right|$$

$$+ \sum_{n=2\tau+1}^{\infty} \frac{1}{(n+1)P_{n-1}}$$

$$\times \left| \sum_{k=n-\tau+1}^{n-1} P_k(\chi_k - \chi_n) \frac{\lambda(n-k)\,\epsilon(n-k)}{(n-k)} \sin(n-k)t \right|$$

$$= \Sigma_{3,1} + \Sigma_{3,2} + \Sigma_{3,3}, \text{ say.} \qquad \ldots(3.6)$$

By Abel's transformation

$$\sum_{k=0}^{m} P_k(\chi_k - \chi_n) \frac{\lambda(n-k)\,\epsilon(n-k)}{(n-k)} \sin(n-k)t$$

$$= \sum_{k=0}^{m-1} \Delta_k \left\{ (\chi_k - \chi_n) \frac{\lambda(n-k)\,\epsilon(n-k)}{(n-k)} \right\} \sum_{\nu=0}^{k} P_\nu \sin(n-\nu)t$$

$$+ (\chi_m - \chi_n) \frac{\lambda(n-m)\,\epsilon(n-m)}{(n-m)} \sum_{\nu=0}^{m} P_\nu \sin(n-\nu)t$$

$$= O(\tau) \sum_{k=0}^{m-1} P_k \,|\, \Delta\chi_k \,|\, \frac{\lambda(n-k)\,\epsilon(n-k)}{(n-k)}$$

$$+ O(\tau) \sum_{k=0}^{m-1} P_k \,|\, \chi_{k+1} - \chi_n \,| \left| \Delta_k \left(\frac{\lambda(n-k)\,\epsilon(n-k)}{n-k} \right) \right|$$

$$+ O\left(\frac{P_m \lambda(n-m)\,\epsilon(n-m)}{(n-m)\,t} \right)$$

$$= O\left(\frac{\lambda(n-m+1)\,\epsilon(n-m+1)\,P_{m-1}}{t(n-m+1)} \right) \sum_{k=0}^{m-1} |\,\Delta\chi_k\,|$$

$$+ O(\tau) \sum_{k=0}^{m-1} P_k \,|\, \chi_{k+1} - \chi_n \,| \left\{ \frac{\lambda(n-k-1)\,\epsilon(n-k-1)}{n-k-1} \right.$$

$$\left. - \frac{\lambda(n-k)\,\epsilon(n-k)}{n-k} \right\} + O\left(\frac{P_m \lambda(n-m)\,\epsilon(n-m)}{(n-m)\,t} \right)$$

$$= O\left(\frac{P_m \lambda(m)\,\epsilon(m)}{mt} \right)$$

by the hypotheses of the theorem, and therefore

$$\sum\nolimits_{3,1} = O(\tau) \sum_{n=2\tau}^{\infty} \frac{\lambda(m)\,\epsilon(m)}{m^2}$$

$$= O(\epsilon(K/t)) \qquad \ldots(3.7)$$

using the fact that $\lambda(n)$ is bounded and the condition (2.4) of the theorem.

Again by Abel's transformation

$$\sum_{k=m+1}^{n-\tau} P_k(\chi_k - \chi_n) \frac{\lambda(n-k)\,\epsilon(n-k)}{(n-k)} \sin(n-k)\,t$$

$$= \sum_{k=m+1}^{n-\tau-1} \Delta_k \left\{P_k(\chi_k - \chi_n) \frac{\lambda(n-k)\,\epsilon(n-k)}{(n-k)}\right\} \sum_{\nu=m+1}^{k} \sin(n-\nu)\,t$$

$$+ P_{n-\tau}(\chi_{n-\tau} - \chi_n) \frac{\lambda(\tau)\,\epsilon(\tau)}{\tau} \sum_{k=m+1}^{n-\tau} \sin(n-k)\,t$$

$$= O(\tau) \sum_{k=m+1}^{n-\tau-1} p_{k+1} \mid \chi_k - \chi_n \mid \frac{\lambda(n-k)\,\epsilon(n-k)}{(n-k)}$$

$$+ O(\tau) \sum_{k=m+1}^{n-\tau-1} P_{k+1} \mid \Delta\chi_k \mid \frac{\lambda(n-k)\epsilon\,(n-k)}{(n-k)}$$

$$+ O(\tau) \sum_{k=m+1}^{n-\tau-1} P_{k+1} \mid \chi_{k+1} - \chi_n \mid \left| \Delta_k \left(\frac{\lambda(n-k)\,\epsilon(n-k)}{(n-k)}\right)\right|$$

$$+ O(P_{n-\tau} \mid \chi_{n-\tau} - \chi_n \mid \lambda(\tau)\,\epsilon(\tau))$$

$$= O(\lambda(\tau)\,\epsilon(\tau)) \sum_{k=m+1}^{n-1} p_{k+1} \sum_{\nu=k}^{n-1} \mid \Delta\chi_\nu \mid$$

$$+ O(\lambda(\tau)\,\epsilon(\tau)) \sum_{k=m+1}^{n-1} P_{k+1} \mid \Delta\chi_k \mid +$$

(*equation continued on p.* 359)

$$+ O(\tau P_{n-\tau}) \sum_{k=m+1}^{n-\tau-1} \left(\frac{\lambda(n-k-1)\,\epsilon(n-k-1)}{(n-k-1)} - \frac{\lambda(n-k)\,\epsilon(n-k)}{k-n} \right)$$

$$\times \sum_{\nu=k+1}^{n-1} |\Delta \chi_\nu| + O(P_{n-\tau}\lambda(\tau)\,\epsilon(\tau)) \left| \sum_{k=1}^{\tau} (\chi_{n-k} - \chi_{n-k+1}) \right|$$

$$= O\left(\epsilon(\tau)\, P_{n-1} \sum_{k=m+1}^{n} |\Delta \chi_k| \right)$$

$$+ O\left(P_{n-1}\,\epsilon(\tau) \sum_{k=1}^{\tau+1} |\chi_{n-k} - \chi_{n-k+1}| \right)$$

using the hypotheses of the theorem, and therefore

$$\sum\nolimits_{3,2} = O(\epsilon(K/t)) \sum_{n=1}^{\infty} \frac{1}{n+1} \sum_{k=m}^{n} |\Delta \chi_k|$$

$$+ O(\epsilon(/Kt)) \sum_{n=\tau+1}^{\infty} \frac{1}{n+1} \sum_{k=1}^{\tau+1} |\chi_{n-k} - \chi_{n-k+1}|$$

$$= O(\epsilon(K/t)) + O(\epsilon(K/t)) \sum_{k=1}^{\tau+1} \sum_{n=\tau+1}^{\infty} \frac{|\chi_{n-k} - \chi_{n-k+1}|}{(n+1)}$$

$$= O(\epsilon(K/t)) \qquad \ldots(3.8)$$

using the condition (2.1) of the theorem and the lemma.

Again we note that

$$\left| \sum_{k=n-\tau+1}^{n-1} P_k(\chi_k - \chi_n) \frac{\lambda(n-k)\,\epsilon(n-k)}{(n-k)} \sin(n-k)t \right|$$

$$\leqslant Ct \sum_{k=n-\tau+1}^{n-1} P_k\, |\chi_k - \chi_n|\, \epsilon(n-k)$$

$$= O(t\epsilon(K/t)) \sum_{k=n-\tau+1}^{n-1} P_k \sum_{\nu=k}^{n-1} |\Delta \chi_\nu|$$

$$= O(t\epsilon(K/t)) \sum_{\nu=n-\tau+1}^{n-1} |\triangle \chi_\nu| \sum_{k=n-\tau+1}^{n-1} P_k$$

$$= O\left(\epsilon(K/t)\, P_{n-1} \sum_{\nu=n-\tau+1}^{n} |\triangle \chi_\nu|\right)$$

and therefore

$$\sum\nolimits_{3,3} = O(\epsilon(K/t)) \sum_{n=1}^{\infty} \frac{1}{n+1} \sum_{\nu=m}^{n} |\triangle \chi_\nu|$$

$$= O(\epsilon(K/t)) \qquad \ldots(3.9)$$

by an application of the lemma.

Combining the estimates in (3.2) through (3.9), in view of the inequality in (3.1), it follows that

$$\sum_{n=1}^{\infty} |t_n - t_{n-1}| \leqslant C \int_0^{\pi} \epsilon(K/t)\, |d\phi(t)| < \infty.$$

Hence the theorem.

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