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THERMAL DEFLECTIONS OF THICK ELASTIC TRIANGULAR PLATE RESTING ON ELASTIC FOUNDATION

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The present investigation is aimed to determine the deflections of a thick right-angled isosceles triangular plate placed on elastic foundation including the effect of a steady temperature distribution which is obtained as the solution of the three-dimensional heat conduction equation. Some numerical results have been presented showing the variation of the coefficient of the maximum non-dimensional deflection with the foundation parameter.

1. INTRODUCTION

Although a fairly good number of papers have been devoted to the analysis of deflections and stresses of thick plates without thermal loading Iyengar *et al.* 1974; Chandrasekhara and Mathana (1977*a*, *b*, 1978, 1979) but little attention has been paid to the analysis of thick plates with thermal loading. Considering the effect of transverse shear deformation and stresses normal to the middle surface of thick elastic-plates subjected to stationary temperature distribution, Ariman (1965) derived the governing differential equation for deflection of the plate when placed on an elastic foundation of the Winkler type. His analysis was confined to only a rectangular plate. The state of stresses and displacements in a thick circular plate due to non-stationary temperature distribution has been obtained by Derski (1959) by a different method. Flexural vibrations of a thick rectangular laminated plate with variation of temperature across the thickness has been investigated by Srinivas and Rao (1972). In this paper thermal deflections for a thick right-angled isosceles triangular plate resting on an elastic foundation have been determined. The variations of the coefficient of the maximum non-dimensional deflection for different values of the foundation parameter have been presented graphically.

2. GOVERNING EQUATION

For a thick elastic plate of thickness h placed on an elastic foundation of the Winkler type of foundation modulus K, and subjected to a steady temperature distribution $T(x, y, z)$, the differential equation for deflection ω is given by (Ariman 1965)

$$D\nabla^4\omega - \frac{h^2}{10}\,\frac{2-\nu}{1-\nu}\,K\nabla^2\omega + K\omega + D\alpha_t(1+\nu)\,\frac{12}{h^3}\,\nabla^2 M_T = 0 \qquad \ldots(1)$$

where ν is the Poisson's ratio, α_t the coefficient of linear thermal expansion, D the flexural rigidity, ∇^2 the two-dimensional Laplacian operator, and

$$M_T = \int_{-h/2}^{h/2} zT(x, y, z)\, dz. \qquad \ldots(2)$$

3. SOLUTION FOR SIMPLY-SUPPORTED RIGHT-ANGLED ISOSCELES TRIANGULAR PLATE

Let us consider a right-angled isosceles triangular plate simply-supported along the edges. The equal sides of the triangle are of length a and are taken along the x and y axes, z-axis being normal to the plane of the plate. Origin of the co-ordinate system is at the middle surface.

Temperatures on the upper and lower faces of the plate are T_1 and T_2 and temperatures at all the other faces are zero. The boundary conditions are given in the following form:

$$\omega = 0 = \frac{\partial^2 \omega}{\partial x^2} + \frac{\alpha_t E M_T}{D(1-\nu)}, \quad \Omega_y = 0 \text{ for } x = 0$$

$$\omega = 0 = \frac{\partial^2 \omega}{\partial y^2} + \frac{\alpha_t E M_T}{D(1-\nu)}, \quad \Omega_x = 0 \text{ for } y = 0$$

$$\omega = 0 = \frac{\partial^2 \omega}{\partial \xi^2} + \frac{\alpha_t E M_T}{D(1-\nu)} \text{ for } x + y = a \qquad \ldots(3)$$

where

$$\frac{\partial}{\partial \xi} \equiv \frac{1}{\sqrt{2}} \left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} \right)$$

and Ω_x, Ω_y are the rotational resultants given by the expressions (Ariman 1965)

$$\Omega_x = -\frac{\partial \omega}{\partial x} + \frac{6}{5h} \frac{Q_x}{G}; \quad \Omega_y = -\frac{\partial \omega}{\partial y} + \frac{6}{5h} \frac{Q_y}{G} \qquad \ldots(4)$$

where

$$Q_x = -D \frac{\partial}{\partial x} (\nabla^2 \omega) + \frac{h^2}{10} \nabla^2 Q_x + \frac{h^2}{10(1-\nu)} \frac{\partial}{\partial x} (K\omega)$$
$$- D\alpha_t (1+\nu) \frac{12}{h^3} \frac{\partial}{\partial x} M_T \qquad \ldots(5)$$

$$Q_y = -D \frac{\partial}{\partial y} (\nabla^2 \omega) + \frac{h^2}{10} \nabla^2 Q_y + \frac{h^2}{10(1-\nu)} \frac{\partial}{\partial y} (K\omega)$$
$$- D(1+\nu) \alpha_t \frac{12}{h^3} \frac{\partial}{\partial y} M_T. \qquad \ldots(6)$$

In the steady state the heat conduction equation is

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0. \qquad \ldots(7)$$

The solution of this equation for the boundary conditions $T = 0$ at $x = 0$, $y = 0$ and $x + y = a$ can be taken as (Hildebrand 1976)

$$T(x, y, z) = \frac{32}{3\pi^2} \sum_{m=1,3,\ldots}^{\infty} \frac{T_1 \sinh(\frac{1}{2} lh - lz) + T_2 \sinh(\frac{1}{2} lh + lz)}{m^2 \sinh(lh)} \times \left(\sin\frac{2m\pi x}{a} \sin\frac{m\pi y}{a} + \sin\frac{m\pi x}{a} \sin\frac{2m\pi y}{a} \right) \qquad \ldots(8)$$

where $l^2 = 5m^2\pi^2/a^2$.

From (2) one gets

$$M_T = \frac{16h}{3\pi^2} \sum_{m=1,3,\ldots}^{\infty} \psi_m \left[\frac{T_1 - T_2}{l^2 h \sinh(lh)} \{2 \sinh(lh) - (lh)(1 + \cosh(lh))\} \right] \qquad \ldots(9)$$

where

$$\psi_m = \sin\frac{2m\pi x}{a} \sin\frac{m\pi y}{a} + \sin\frac{m\pi x}{a} \sin\frac{2m\pi y}{a} . \qquad \ldots(10)$$

Compatible with the boundary conditions (3) the deflection ω is chosen in the form

$$\omega = \sum_{m=1,3,\ldots}^{\infty} A_m \psi_m. \qquad \ldots(11)$$

Substituting (9) and (11) into eqn. (1) one gets

$$A_m = \frac{64(1 + \nu) h(T_1 - T_2) \alpha_t \{2 \sinh(lh) - (lh)(1 + \cosh(lh))\}}{\bar{h}^1 \pi^2 \sinh(lh) \left\{ 25m^4\pi^4 + 32 \bar{h}^2 \frac{2 - \nu}{1 - \nu} m^2\pi^2\beta^4 + 64\beta^4 \right\}} \qquad \ldots(12)$$

where

$$\bar{h} = \frac{h}{a}, \quad \beta = \frac{a}{2} \sqrt[4]{\frac{K}{4D}} .$$

To solve the differential equations (5) and (6) we express Q_x and Q_y as

$$Q_x = \sum F_m \left(\frac{2m\pi}{a} \cos\frac{2m\pi x}{a} \sin\frac{m\pi y}{a} + \frac{m\pi}{a} \cos\frac{m\pi x}{a} \sin\frac{2m\pi y}{a} \right) \qquad \ldots(13)$$

$$Q_y = \sum G_m \left(\frac{m\pi}{a} \sin \frac{2m\pi x}{a} \cos \frac{m\pi y}{a} + \frac{2m\pi}{a} \sin \frac{m\pi x}{a} \cos \frac{2m\pi y}{a} \right). \quad ...(14)$$

Substituting (13) and (14) along with (9) and (11) into eqns. (5) and (6) one gets

$$F_m = G_m$$

$$= \left\{ \frac{5m^2\pi^2}{a^2} D + \frac{h^2}{10(1-\nu)} K - D\alpha_t(1+\nu) \frac{12}{h^3} \lambda_m \right\} \Big/ \left(1 + \frac{h^2}{10} \cdot \frac{5m^2\pi^2}{a^2} \right)$$

where

$$\lambda_m = \frac{16h}{3\pi^2} \{(T_1 - T_2)/l^2 h \sinh (lh)\}. \{2 \sinh (lh) - (lh)(1 + \cosh (lh))\}. \quad ...(15)$$

4. Numerical Results

Maximum deflection occurs at the centroid of the plate and is given by

$$\omega_{max} = \sum_{m=1,3,\ldots}^{\infty} 2A_m \sin \frac{2m\pi}{3} \sin \frac{m\pi}{3}. \quad ...(16)$$

Equation (16) may be expressed in the form

$$\frac{\omega_{max}}{h} = \phi_R \, 10^2 \alpha_t (T_2 - T_1) \quad ...(17)$$

where ϕ_R is given by

$$\phi_R = \sum_{m=1,3,\ldots}^{\infty} \frac{128 \times 10^{-2}(1+\nu)\{(lh)(1+\cosh (lh)) - 2 \sinh (lh)\}}{\bar{h}^4 \pi^2 \sinh (lh) \left\{ 25m^4\pi^4 + 32\bar{h}^4 \frac{2-\nu}{1-\nu} m^2\pi^2\beta^4 + 64\beta^4 \right\}}$$

$$\times \sin \frac{2m\pi}{3} \sin \frac{m\pi}{3}. \quad ...(18)$$

Figure 1 shows the variations of the coefficient ϕ_R of the maximum non-dimensional deflection with the foundation parameter β and $\bar{h}$.

Therefore once ϕ_R is known for different values of β and $\bar{h}$ eqn. (17) determines w_{max}/h for different variations of the temperature parameter $10^2\alpha_t(T_2 - T_1)$.

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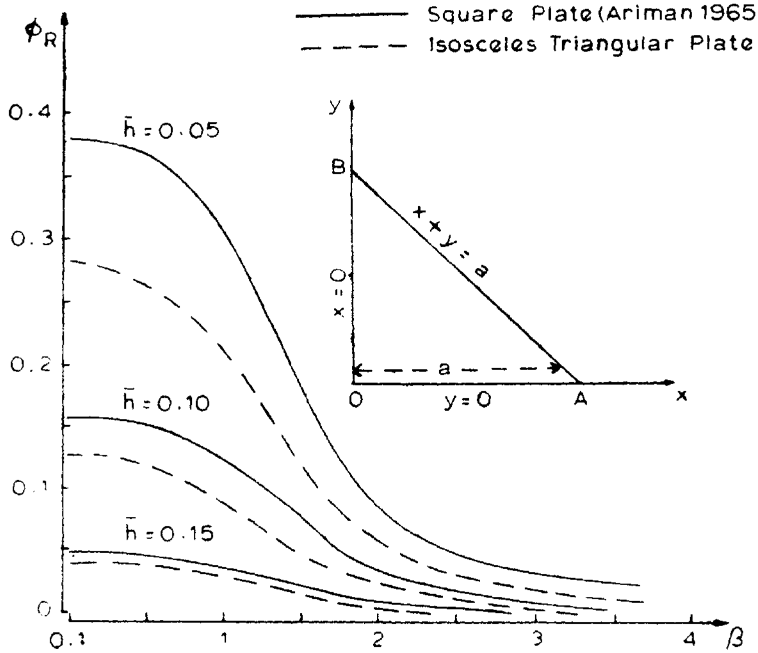

FIG. 1. Variation of coefficient of non-dimensional deflection with parameter β.
Inset : Plate geometry.

REFERENCES

Iyengar, K. T. S. R., Chandrashekhara, K., and Sebastian, V. K. (1974). On the analysis of thick rectangular plates. *ZAMM*, **54**, 589–91.

Chandrashekhara, K., and Mathana, S. K. (1977). Stresses in a thick plate with a circular hole under axisymmetric loading. *Int. J. Engng Sci.*, **15**, 135–46.

————(1977). Axisymmetric stress distribution in a transversely isotropic thick plate with a circular hole. *Acta Mech.*, **28**, 65–76.

————(1978). Stress analysis of a thick plate having a circular hole under axisymmetric radial load. *Ing. Archiv*, **47**, 1–9.

————(1979). Analysis of a thick plate with a circular hole resting on a smooth rigid bed and subjected to axisymmetric normal load. *Acta Mech.*, **33**, 33–44.

Ariman, T. (1965). Recent Advances in Engineering Sciences, edited by A. C. Eringen. Gordon & Breach Co., London.

Derski, W. (1959). The state of stress and displacement in a thick circular plate due to a non-stationary temperature field. *Bull. Acad Pol. Sci. Ser. Sci. Tech.*, **7**, 53–64.

Srinivas, S., and Rao, A. K. (1972). A note on flexure of thick rectangular plates and laminates with variation of temperature across thickness, *Bull. Acad. Pol. Sci. Ser. Sci. Tech.*, **20**, 123–28.

Hildebrand, F. B. (1976). Advanced Calculus for Applications, 2nd edition. Prentice Hall Inc., Englewood Cliffs, N. J.