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NOTE ON THE FLOW OF A RAREFIED GAS THROUGH A CHANNEL WITH HALL EFFECT

MUKESH GUPTA AND V. P. VARSHNEY

Department of Mathematics, Agra College, Agra

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An attempt has been made to study the effects of Hall current and rarefaction parameter on combined free and forced convective hydromagnetic flow in a parallell plate channel, when there is a uniform axial temperature variation along the channel walls. The expression for induced magnetic field and skin friction are obtained. The profiles for the velocity and induced magnetic field have been shown graphically.

INTRODUCTION

The study of combined free and forced convective flow through a channel has been recognised to be of immense importance for their wide applications in engineering and technological fields. The effect of buoyancy forces on a forced convective flow of an electrically conducting fluid in a horizontal channel with a linear axial temperature variation along the wall under the influence of transverse magnetic field has been investigated by Gupta (1969). In this study, the effects of Hall currents have not been taken into account but these effects have pronounced effect when the strength of magnetic field is very large. Mazumdar *et al.* (1976) studied Gupta's problem by taking Hall effect into account. In all the above studies, the effect of normal density fluid was considered but attempt to analyse the effect of Hall currents in case of rarefied gas does not seem to have attracted any attention. In the present era of high altitude flights, the study of rarefied gas has been recognised to be of immense importance. In the case of rarefied gases, the ordinary continuum approach fails to yield satisfactory results. When the gas is only slightly rarefied results agreeing with the observed physical phenomena can be analysed by solving the usual Navier-Stokes equations together with modified boundary conditions allowing for a velocity slip and temperature jump at the surface. This scheme of theoretical investigation of the flow, the so-called "slip flow regime", is particularly suitable for studying the effects of gas rarefaction on any classical viscous flow problem because for perfectly rarefied gas there is always slip on the boundary. Schaaf has studied the flow of a viscous incompressible fluid past an impulsively moving infinite flat plate in slip flow regime which has been published in the book by Eckert and Drake (1959). Reddy (1964) studied the fluctuating flow past a porous infinite flat plate in slip flow regime.

The purpose of the present investigation is to extend the analysis of Mazumdar *et al.* (1976) in case of a rarefied gas.

FORMULATION AND SOLUTION

We take x^*- and y^*-axes along and transverse to the parallel horizontal plates coinciding with the planes $y^* = \pm L$. A uniform strong magnetic field H_0 is imposed parallel to y^*-axis. Let $(u^*, 0, w^*)$ and (H_x^*, H_y^*, H_z^*) be the components of the velocity $\bar{q}^*$ and the magnetic field $\bar{H}^*$ respectively. At a large distance from the entry section, the flow will be fully developed and in the steady state, all the physical quantities (except pressure) depend on y^* only.

The equations of motion and magnetic field in rationalized M.K.S. unit (Mazumdar *et al.* 1976) are given by

$$\frac{d^2U}{dy^2} + M^2 \frac{dh}{dy} - Gy = -1 \quad \ldots(1)$$

$$\frac{d^2h}{dy^2} = -\frac{1}{(1+im)}\frac{dU}{dy} \quad \ldots(2)$$

where $U = u + iw,\ h = H_x + iH_z,\ m = \omega\tau$ (Hall parameter) ...(3)

with the boundary conditions (Street 1960) :

$$U(-1) = h_1 \frac{dU}{dy},\quad U(+1) = -h_1 \frac{dU}{dy},\quad h(\pm 1) = 0 \quad \ldots(4)$$

where $h_1(= L_2/L)$ is a non-dimensional rarefaction parameter; $L_2 = \frac{2-f_1}{f_1} L_1$; $L_1 = (\pi/2p\rho)^{1/2}$ is the mean free path and is a constant for an incompressible fluid. Consequently L_2 can also be taken as constant; f_1 being the Maxwell's reflection coefficient.

Solution of eqns. (1) and (2) satisfying the boundary conditions (4) are

$$U(y) = \frac{\cosh M_1 - \cosh M_1 y + h_1 M_1 \sinh M_1}{M_1 \sinh M_1}$$
$$+ G\left\{\frac{(1+h_1)\sinh M_1 y - y(\sinh M_1 + h_1 M_1 \cosh M_1)}{M_1^2(\sinh M_1 + h_1 M_1 \cosh M_1)}\right\} \quad \ldots(5)$$

and

$$M^2 h = G\left\{\tfrac{1}{2}(y^2-1) + \frac{(1+h_1)(\cosh M_1 - \cosh M_1 y)}{M_1(\sinh M_1 + h_1 M_1 \cosh M_1)}\right\}$$
$$+ \frac{\sinh M_1 y - y \sinh M_1}{\sinh M_1} \quad \ldots(6)$$

where $M_1^2 = M^2/(1+im)$.

The dimensionless shear stress for the primary flow at the upper and lower plates respectively, are given by

$$\left(\frac{du}{dy}\right)_{y=1} = -1 + F(\alpha, \beta) \qquad \ldots(7)$$

$$\left(\frac{du}{dy}\right)_{y=-1} = 1 + F(\alpha, \beta) \qquad \ldots(8)$$

where
$$F(\alpha, \beta) = \frac{G}{(a_5^2 + a_6^2)} [a_5 \{(1 + h_1)(\alpha \cosh \alpha \cos \beta - \beta \sinh \alpha \sin \beta) - a_3\} + a_6 \{(1 + h_1)(\alpha \sinh \alpha \sin \beta + \beta \cosh \alpha \cos \beta) - a_4\}]. \qquad \ldots(9)$$

The dimensionless shear stress for the cross-flow at the upper plate is the same as that at the lower plate and is given by

$$\left(\frac{dw}{dy}\right)_{y=\pm 1} = \frac{G}{(a_5^2 + a_6^2)} [a_5 \{(1 + h_1)(\alpha \sinh \alpha \sin \beta + \beta \cosh \alpha \cos \beta) - a_4\} - a_6 \{(1 + h_1)(\alpha \cosh \alpha \cos \beta - \beta \sinh \alpha \sin \beta) - a_3\}]. \qquad \ldots(10)$$

Particular Case

When rarefaction parameter $h_1 \to 0$, the problem reduces to that considered by Mazumdar *et al.* (1976).

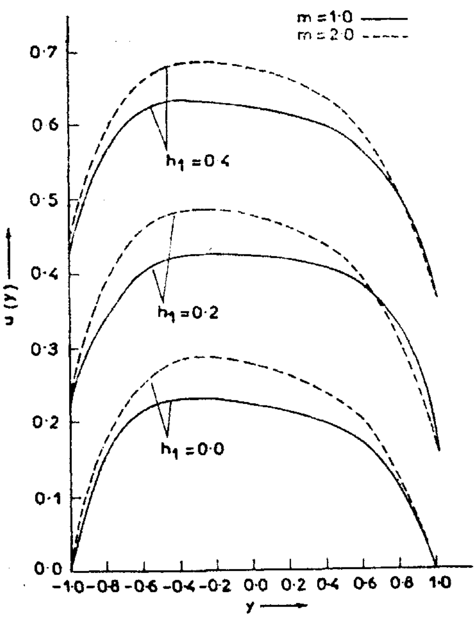

FIG. 1. Profiles of non-dimensional primary velocity $u(y)$ for $G = 1$ and $M = 5$.

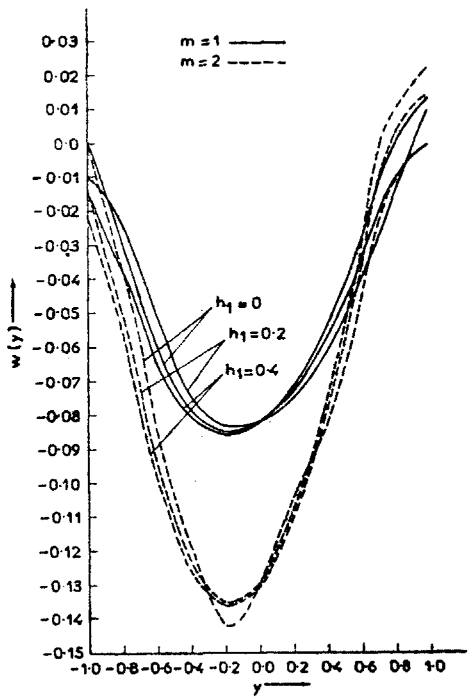

FIG. 2. Profiles of non-dimensional secondary velocity $w(y)$ for $G = 1$ and $M = 5$.

Discussions

We have plotted $u(y)$, $w(y)$, $H_x(y)$ and $H_z(y)$ for different values of Hall parameter (m) and rarefaction parameter (h_1) with $G = 1$ and $M = 5$.

In Fig. 1, the primary flow is shown for different values of rarefaction parameter h_1 ($=0, 0.2, 0.4$) and Hall parameter m ($=1, 2$). It is clear from the figure that the velocity increases with an increase in the value of h_1. It it also evident that an increase in 'm' increases the velocity of fluid which is in agreement with the results of Mazumdar *et al.* (1976).

The profiles for the secondary flow have been displayed in Fig. 2. The effect of the Hall parameter is to sharpen the velocity profiles and the rarefaction parameter is favourable to the backward flow.

The effect of m and h_1 on the induced magnetic fields H_x and H_z have been shown in Fig. 3 and Fig. 4 respectively. The value of H_x decreases with an increase

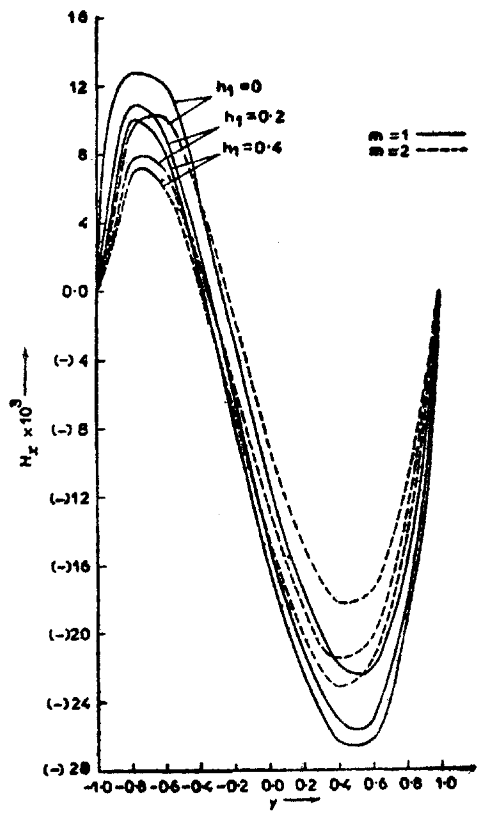

FIG. 3. Profiles of the non-dimensional induced magnetic fieid H_x for $M = 5$ and $G = 1$.

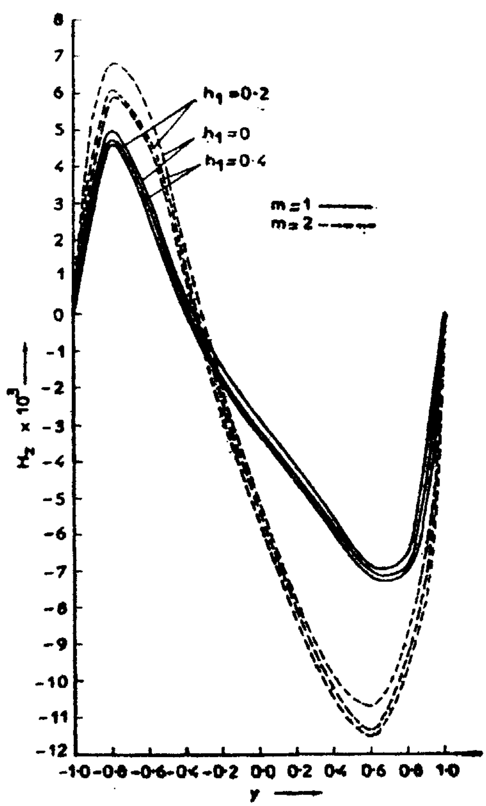

FIG. 4. Profiles of non-dimensional induced magnetic field H_z for $M = 5$ and $G = 1$.

in h_1 while H_z first decreases and then increases for increasing value of h_1. On the other hand the value of H_x decreases numerically with the increase of the Hall parameter (m) while a reverse effect is seen in case of H_z.

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