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CURVILINEAR MOVEMENT OF A MICHELL TYPE SHIP

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The movement of a Michell type ship in a circular path has been considered by Bhattacharya (1958, 1962). The movement of a ship of the same type in a parabolic path of low curvature has been considered by Chakraborty (1978) and Chakraborty and Bhattacharya (1978) essentially as an unsteady problem. The curvature of a parabolic path is highest at its vertex. So starting at the vertex if a ship moves along such a trajectory the curvature thereof tends to zero as time tends to infinity. So a steady effect is not unexpected.

This problem has been investigated here for a ship of parabolic cross-section with finite draught, particular attention being is paid to the case of infinite draught.

§1. The problem of wave resistance of submerged bodies moving in circular path was considered by Sretenskii (1946), Havelock (1950), Perzhnyanko (1960) and Kurlovich (1961). The same problem for a thin ship in a non-circular curvilinear path has been considered by Chakraborty and Bhattacharya (1978) and Chakraborty (1978). The actual trajectory considered in Chakraborty and Bhattacharya (1978) and Chakraborty (1978) is a parabolic path of low curvature. The curvature of such a path is highest at the vertex, diminishes away from it and tends to zero ultimately. So if a ship follows this trajectory the possibility of an ultimate steady state wave resistance cannot be ruled out.

This problem has been investigated in this paper for a ship of parabolic cross-section and finite draught. It is found that a ship following such a parabolic trajectory ultimately attains a first order steady wave resistance as $t \to \infty$. Particular attention is given to the case of infinite draught and the result is compared with the rectilinear wave resistance of such a ship.

§2. It has been shown (Chakraborty and Bhattacharya 1978) that the first order effect of curvature in the above problem is contained in R_{w_2}. Equations (15) and (16) of Chakraborty and Bhattacharya (1978) can be written as

$$R_{w_2} = -\frac{g\rho}{\pi^2}\int_0^t \left(\frac{d\theta}{d\tau} + \frac{d\alpha}{d\tau}\right) d\tau \int_0^\infty \cos\left[\sqrt{gk}\,(t-\tau)\right] k\, dk \times$$

(*equation continued on p.* 780)

$$\times \int_{-\pi}^{\pi} \overline{H(k, \psi, t)}\; C(k, \psi, \tau) \exp [ik \{r(\tau) \cos (\theta(\tau) - \psi)$$

$$- r(t) \cos (\theta(t) - \psi)\}]\, d\psi \qquad \ldots(1)$$

$$\overline{H(k, \psi, t)} = \iint \frac{\partial \nu}{\partial \mu} \exp [k \{(\lambda - f) - i\mu \cos (\theta(t) + \alpha(t) - \psi)\}]\, d\lambda\, d\mu \qquad \ldots(2)$$

$$C(k, \psi, \tau) = \iint \mu \exp [k\{(\lambda - f) + i\mu \cos (\theta(\tau) + \alpha(\tau) - \psi)\}]\, d\lambda\, d\mu \quad \ldots(3)$$

where $r = r(\theta)$ is the polar equation of the path traced out by the centroid of the ship with reference to a fixed system of cylindrical coordinates with the origin in the undisturbed free surface of water and z-axis vertically upwards. Further, $\alpha(t)$ is the angle between the radius vector and the tangent to the path through the instantaneous position of the centroid. (μ, λ, ν) are the Cartesian coordinates of any point on the ship with respect to a system of axes fixed in the ship, its origin being at the instantaneous position of the centroid of the ship, the μ-axis being taken along the longitudinal principal axis in the longitudinal plane of symmetry of the ship, λ-axis being vertically upwards and the ν-axis being drawn portwards.

The ship moves in such a way that μ-axis is always tangential to the trajectory.

It has been pointed out in Chakraborty and Bhattacharya (1978) that for a first order approximation in a parabolic path of low curvature

$$\left.\begin{aligned} r(t) &= vt \\ \theta(t) = \alpha(t) &= \frac{vt}{4a} \end{aligned}\right\} \qquad \ldots(4)$$

where v is the instantaneous speed of the ship in the parabolic path which is constant, correct to the first order and $4a$ is the latus rectum of the parabolic path. Correct to the same order of approximation

$$\overline{H(k, \psi, t)} = \bar{H}_0 - \frac{vt}{2a} \frac{\partial \bar{H}_0}{\partial \psi} \qquad \ldots(5)$$

where

$$\bar{H}_0(k, \psi) = \bar{H}(k, \psi, 0)$$

$$= \iint \frac{\partial \nu}{\partial \mu} \exp [k\{(\lambda - f) - i\mu \cos \psi\}]\, d\lambda\, d\mu \qquad \ldots(6)$$

$$C(k, \psi, \tau) = C_0 - \frac{v\tau}{2a} \frac{\partial C_0}{\partial \psi} \qquad \ldots(7)$$

where

$$C_0(k, \psi) = C(k, \psi, 0)$$
$$= \iint \mu \exp [k\{(\lambda - f) + i\mu \cos \psi\}]\, d\lambda\, d\mu. \qquad \text{...(8)}$$

In eqns. (5) and (7) the functions $\overline{H(k, \psi, t)}$ and $C(k, \psi, \tau)$ have been split up into their finite (zero) and first order parts respectively. In the same way the exponential expression in eqn. (1) can be split into two similar parts:

$$\exp [ik\{r(\tau) \cos (\theta(\tau) - \psi) - r(t) \cos (\theta(t) - \psi)\}]$$
$$= \exp \{-ikv(t - \tau) \cos \psi\} \left[1 - \frac{ikv^2}{4a} (t^2 - \tau^2) \sin \psi \right]. \qquad \text{...(9)}$$

Also

$$\frac{d\theta}{d\tau} + \frac{d\alpha}{d\tau} = \frac{v}{2a} \text{ (to first order).}$$

So to get a result correct to first order in curvature, it is enough to take only the zero order terms of (5), (7) and (9) and substitute them into (1). Thus, correct to the first order

$$R^{(1)}_{w_2} = - \frac{g\rho v}{\pi^2 a} \int_0^t d\tau \int_0^\infty \cos [\sqrt{gk}\, (t - \tau)]\, k\, dk$$
$$\times \int_0^\pi \bar{H}_0 C_0 \exp [- ikv(t - \tau) \cos \psi]\, d\psi. \qquad \text{...(10)}$$

§3. Now changing the range of ψ-integration from $(-\pi, \pi)$ to $(0, \pi/2)$ the ψ-integral in (10)

$$= \int_0^\pi \bar{H}_0\, C_0 \exp [- ikv(t - \tau) \cos \psi]\, d\psi$$
$$= 2 \int_0^{\pi/2} [(H_1 C_1 + H_2 C_2) \cos \{kv(t - \tau) \cos \psi\}$$
$$+ (H_1 C_2 - H_2 C_1) \sin \{kv(t - \tau) \cos \psi\}]\, d\psi \qquad \text{...(11)}$$

where H_1, H_2 are the real and imaginary parts of

$$H = \iint \frac{\partial \nu}{\partial \mu} \exp [k\{(\lambda - f) + i\mu \cos \psi\}]\, d\lambda\, d\mu$$
$$= \iint \frac{\partial y}{\partial x} \exp [k\{z + ix \cos \psi\}]\, dx\, dz \qquad \text{...(12)}$$

after the transformations $\lambda - f = z$, $\nu = y$, $\mu = x$ and C_1, C_2 are the real and imaginary parts of

$$C = \iint \mu \exp\left[k\{\lambda - f) + i\mu \cos\psi\}\right] d\lambda\, d\mu$$
$$= \iint x \exp\left[k\{z + ix\cos\psi\}\right] dx\, dz. \qquad ...(13)$$

At this stage it will be convenient to introduce the equation of a suitable ship form. We shall take a symmetrical cylindrical ship with a parabolic cross-section given by

$$y = \frac{B}{2}\left(1 - \frac{4x^2}{L^2}\right) \qquad ...(14)$$

where L and B are the length and width of the ship respectively. For such a ship it is easy to see that

$$H_1 = C_1 = 0. \qquad ...(15)$$

Further, we have

$$H_2 = -\frac{8B}{L^2}\int_{-T}^{0} e^{kz}dz \int_{0}^{L/2} x \sin(kx\cos\psi)\, dx \qquad ...(16)$$

and

$$C_2 = 2\int_{-T}^{0} e^{kz}dz \int_{0}^{L/2} x\sin(kx\cos\psi)\, dx \qquad ...(17)$$

where T is the draught of the ship.

Substituting the results in (11), (15), (16) and (17) into equation (10) and proceeding to the limit as $t \to \infty$, for a steady state we get

$$R^{(1)}_{w_2} = -\frac{g\rho v}{\pi^2 a}\int_0^{\pi/2} d\psi \lim_{t\to\infty}\int_0^{\infty} H_2C_2\left[\frac{\sin(\sqrt{gk} + kv\cos\psi)t}{\sqrt{gk} + kv\cos\psi}\right.$$
$$\left. + \frac{\sin(\sqrt{gk} - kv\cos\psi)t}{\sqrt{gk} - kv\cos\psi}\right] k\, dk. \qquad ...(18)$$

For the simultaneous evaluation of the limit and the k-integral in (18) we may use the formula quoted in Doctors and Sharma (1970) and reproduced below:

$$\lim_{t\to\infty}\int_a^b f(x)\frac{\sin g(x)t}{g(x)} = \pi\sum \frac{f(x_i)}{|g'(x_i)|}, \qquad ...(19)$$

where x_i are the zeros of $g(n)$, assuming that $a < x_i < b$ and $g'(x_i) \neq 0$

It is easy to see that the first term in the k-integral involving $(\sqrt{gk} + kv\cos\psi)$ does not contribute anything to the limiting value of the integral. So we have from (18)

$$R_{w_2}^{(1)} = -\frac{g\rho v}{\pi^2 a}\int_0^{\pi/2} d\psi \lim_{t\to\infty}\int_0^\infty H_2 C_2 \frac{\sin(\sqrt{gk} - kv\cos\psi)t}{\sqrt{gk} - kv\cos\psi}\, k\, dk. \qquad \ldots(20)$$

§4. Evaluating H_2 and C_2 with a finite draught (T) of the ship and also substituting them into (20) we get

$$R_{w_2}^{(1)} = \frac{16\, Bg\rho v}{\pi^2 L^2 a}\int_0^{\pi/2} \sec^4\psi\, d\psi \lim_{t\to\infty}\int_0^\infty \frac{(1-e^{-kT})^2}{k^5}$$
$$\times\left[\sin\left(\frac{kL}{2}\cos\psi\right) - \frac{kL}{2}\cos\psi\cos\left(\frac{kL}{2}\cos\psi\right)\right]^2$$
$$\times\frac{\sin(\sqrt{gk} - kv\cos\psi)t}{\sqrt{gk} - kv\cos\psi}\, dk, \qquad \ldots(21)$$

where

$$H_2 = \frac{8B\sec^2\psi}{L^2k^3}(e^{-kT} - 1)\left[\sin\left(\frac{kL}{2}\cos\psi\right)\right.$$
$$\left. - \frac{kL}{2}\cos\psi\cos\left(\frac{kL}{2}\cos\psi\right)\right]$$

and

$$C_2 = -\frac{2\sec^2\psi}{k^3}(e^{-kT} - 1)\left[\sin\left(\frac{kL}{2}\cos\psi\right)\right.$$
$$\left. - \frac{kL}{2}\cos\psi\cos\left(\frac{kL}{2}\cos\psi\right)\right].$$

Now the k-integral and the limit in eqn. (21) can be evaluated simultaneously with the help of eqn. (19).

Here,

$$f(k) = \frac{(1-e^{-kT})^2}{k^5}\left[\sin\left(\frac{kL}{2}\cos\psi\right)\right.$$
$$\left. - \frac{kL}{2}\cos\psi\cos\left(\frac{kL}{2}\cos\psi\right)\right]^2$$

and

$$g(k) = \sqrt{gk} - kv\cos\psi.$$

Therefore applying the formula quoted in eqn. (19) and after some calculations we have

$$R^{(1)}_{w_2} = \frac{32Bg\rho}{\pi L^2 a k_0^5} \int_0^{\pi/2} \cos^5 \psi \,[1 - \exp(-k_0 T \sec^2 \psi)]^2 \left[\sin\left(\frac{k_0 L}{2} \sec\psi\right) - \left(\frac{k_0 L}{2}\sec\psi\right)\cos\left(\frac{k_0 L}{2}\sec\psi\right)\right]^2 d\psi$$

$$= \frac{32Bg\rho}{\pi L^2 a k_0^5} \int_0^{\pi/2} \cos^3 \psi \,[1 - \exp(-k_0 T \sec^2 \psi)]^2 \times \left[\tfrac{1}{2} k_0 L \cos\left(\frac{k_0 L}{2}\sec\psi\right) - \cos\psi \sin\left(\frac{k_0 L}{2}\sec\psi\right)\right]^2 d\psi \quad \ldots(22)$$

This represents the first order steady effect of curvature on the wave resistance of a thin wall-sided ship of a parabolic cross-section and finite draught T.

§5. The case of same ship with infinite draught is of special interest. For such a ship the curvature effect is obtained from (22) by making $T \to \infty$ in the form,

$$R^{(1)}_{w_2} = \frac{32Bg\rho}{\pi L^2 a k_0^5} \int_0^{\pi/2} \cos^3 \psi \left[\tfrac{1}{2} k_0 L \cos\left(\frac{k_0 L}{2}\sec\psi\right) - \cos\psi \sin\left(\frac{k_0 L}{2}\sec\psi\right)\right]^2 d\psi. \quad \ldots(23)$$

The most interesting aspect of this result is that the integral here is exactly identical with the integral given in formula (26) of Havelock (1949, p. 388) for rectilinear wave resistance of the same ship. This fact enables us to make a simple comparison between the first order curvature effect and the rectilinear effect represented by Havelock's formula and hereafter denoted by $R^{(0)}_{w_1}$. Comparing (23) with Havelock's formula referred to above we get

$$\frac{R^{(1)}_{w_2}}{R^{(0)}_{w_1}} = \frac{1}{4F_r^2}\left(\frac{L}{2a}\right). \quad \ldots(24)$$

This formula again becomes completely identical with the corresponding result for a circular path of large radius obtained by Bhattacharya and Gangopadhyay (1979).

This shows that the ultimate steady first order effect of curvature is independent of the path followed by the ship.

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