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ANALYTICAL AND NUMERICAL STUDY OF THREE DIMENSIONAL OSCILLATORY COMPRESSIBLE FLOW PAST YAWED CYLINDERS WITH GIVEN HEAT TRANSFER

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In this paper an analytical and numerical study of three dimensional oscillatory compressible laminar boundary layer flow past infinite yawed cylinders which are kept at a given temperature gradient is made. The temperature gradient at the wall oscillatory in time is assumed to be perturbed about a zero mean. General analysis is given to study the effects of (1) the compressibility, (2) the shape of the cylinder, (3) the orientation of the cylinder, (4) the Prandtl number and (5) the prescribed heat transfer at the wall on the behaviour of the temperature and skin friction components. The equations are first subjected to Stewartson transformation and then linearized as by Lighthill. And finally following Sarma asymptotic and composite solutions are found for small and large frequencies.

Following the general analysis three different problems corresponding to three different shapes of the cylinder are studied numerically. The values of the important quantities are tabulated and the behaviours of the temperature and the skin friction components are illustrated graphically from which conclusions are drawn.

1. INTRODUCTION

The study of incompressible two dimensional boundary layers with small periodic fluctuations in the magnitude of the main stream velocity about a steady mean was initiated by Lighthill (1954). The work was extended by Rott and Rosenzweig (1960), Lam and Rott (1960), Ackerberg and Phillips (1972), Brown and Stewartson (1973a, 1973b) etc. Sarma (1965, 1966), treating the problem as a transient one, studied the two dimensional compressible boundary layer equations.

The present work deals with the three dimensional oscillatory compressible boundary layer flows past yawed cylinders. The assumptions made in this paper are that (1) the yawed cylinder is infinite (2) the main stream velocity is steady with a constant velocity component in the span wise direction (3) the Prandtl number need not be unity (4) the coefficient of viscosity is proportional to the temperature (5) the cylinder is at rest and (6) the temperature gradient at the wall is perturbing about a zero mean and is conveniently assumed (just to suit the inner solution given in section 4.1) as

$$\left(\frac{\partial T}{\partial \bar{Y}}\right)_{\bar{Y}=0} = \epsilon \alpha_1 T_s \bar{X}^{m_1} e^{i\omega t} \qquad \ldots(1.1)$$

where $\bar{Y} = \sqrt{\omega} Y$, $\bar{X} = \dfrac{\omega}{\alpha_0} x^{-(m_0-1)}$ are the variables used in the inner solution (X, Y being the Stewartson variables along and perpendicular to the wall), α_0, α_1, m_0, m_1, ω, T_s, ϵ are all constants, ϵ being small.

In this paper analytical and numerical study of the flow is made. The unsteady compressible boundary layer equations are first subjected to the Stewartson transformation (1949) instead of Howarth transformation (1948) as in Sarma (1965) and later they are linearized as by Lighthill (1954) and Sarma (1964). Then the nonlinear steady equations and the linear unsteady equations are solved analytically by assuming a series solution in powers of Mach number. Following the work of Ackerberg and Phillips (1972), three types of asymptotic solutions are found one for small ω and two for large ω. The two solutions for large ω are the inner and outer solutions satisfying the boundary conditions at the wall and at infinity respectively. Finally, using the inner and outer solutions and following Cole (1968) a composite solution is obtained. No attempt has been made to model out the complicated eigen solutions corresponding to this problem [just as done by Brown and Stewartson (1973a, 1973b) to match the composite solution for small frequencies with that for large]. The terms in the series solutions for small and large frequencies satisfy ordinary differential equations which allow us to study the effects of Mach number, angle of yaw, shape of the cylinder, Prandtl number on the behaviours of the temperature and the skin friction components for which analytical expressions are given here.

As examples, three different problems corresponding to $\beta = 0, .5$ and 1 are studied in detail. Following the work of Cohen and Reshotko (1956) and Reshotko and Beckwith (1957), the ordinary differential equations for small frequencies, are solved numerically. Some of the results are also verified by the other technique given by Moore (1951). The important numerical values are tabulated for different values of the parameters and the various behaviours are represented graphically.

2. Mathematical Formulation of the Problem

The physical problem of periodic compressible boundary layer induced by the heat transfer at the wall and the assumptions involved are explained in the introduction. We shall give below a mathematical formulation of the problem. The equations governing the unsteady three dimensional compressible laminar boundary layer flow past infinite yawed cylinder are

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0 \qquad \ldots(2.1)$$

$$\rho\left(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right) = -\frac{\partial P}{\partial x} + \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) \qquad ...(2.2)$$

$$\rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y}\right) = \frac{\partial}{\partial y}\left(\mu\frac{\partial w}{\partial y}\right) \qquad ...(2.3)$$

$$\frac{\partial P}{\partial y} = 0 \qquad ...(2.4)$$

$$\rho c_p\left(\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y}\right) - \left(\frac{\partial P}{\partial t} + \mu\frac{\partial P}{\partial x}\right)$$
$$= \frac{\partial}{\partial y}\left(\frac{\mu c_p}{\sigma}\frac{\partial T}{\partial y}\right) + \mu\left(\frac{\partial u}{\partial y}\right)^2 + \mu\left(\frac{\partial w}{\partial y}\right)^2 \qquad ...(2.5)$$

$$P = R\rho T \qquad ...(2.6)$$

which, at the edge of the boundary layer, reduce to

$$-\frac{\partial P}{\partial x} = \rho_\infty\left(\frac{\partial U}{\partial t} + U\frac{\partial U}{\partial x}\right) \qquad ...(2.7)$$

$$\frac{\partial W}{\partial t} + U\frac{\partial W}{\partial x} = 0, \ \frac{\rho_\infty}{\rho} = \frac{T}{T_\infty} = \frac{\gamma P}{\rho a_\infty^2} \qquad ...(2.8)$$

$$\frac{\partial P}{\partial t} = \rho_\infty c_p\left[\left(\frac{\partial}{\partial t} + U\frac{\partial}{\partial x}\right)\left(T_\infty + \frac{U^2}{2c_p}\right)\right] \qquad ...(2.9)$$

$$\frac{P}{P_s} = \left(\frac{\rho_\infty}{\rho_s}\right)^\gamma = \left(\frac{T_\infty}{T_s}\right)^{\gamma/(\gamma-1)} \text{(adiabatic condition)} \qquad ...(2.10)$$

and which are to be solved with the boundary conditions

$$u = v = w = 0 \text{ at } y = 0 \qquad ...(2.11)$$

$$\frac{\partial T}{\partial y} = \epsilon\alpha_1 T_s \bar{X}^{m_1} e^{i\omega t}\left(\frac{\partial \bar{Y}}{\partial y}\right)_{y=0} \quad \text{at} \quad y = 0 \qquad ...(2.12)$$

$$u \to U(X),\ w \to W \text{ (a constant)},\ T \to T_\infty(X) \text{ as } y \to \infty \qquad ...(2.13)$$

where x, y respresent the distance along the chord and normal to the cylinder, u, v, w the velocity components in x, y, z directions, T the temperature, ρ the density, P the pressure, t the time, c_p the specific heat at constant pressure, σ the Prandtl number, μ the coefficient of viscosity, R the gas constant, γ the ratio of the specific heats, a the velocity of sound; U, W, T_∞, ρ_∞, a_∞ the values of u, w, T, ρ, a in the steady main stream and the suffix s indicates the values at a specified point in the main stream. All the quantities describing the flow are assumed to be independent of the distance z along the spanwise direction. This assumption, as explained in Schlichting (1962) on page 194, is true approximately in the case of yawed cylinders and yawed wedge. The corresponding system of equations has no dependence on z and it is this simplified system that is given by (2.1) to (2.13). We are analysing the periodic boundary layers of the stoke's type (1851) induced by the oscillatory heat transfer at the wall. The transient effects are absent and we do not require any initial condition with respect to time as in transient motions of the Rayleigh type (1911). It is assumed just as in Sarma (1965) that the main stream velocity U and the temperature T_∞ are such that,

$$T_\infty + \frac{U^2}{2c_p} = T_s \quad \text{(a constant)} \qquad \ldots(2.14)$$

This assumptions reduces the main stream to steady state. The main stream temperature and the velocity subjected to the condition (2.14) are now given by

$$T_\infty = \frac{a_\infty^2 T_s}{a_s^2} = T_s\left[1 - \frac{(\gamma-1)V^2}{2a_s^2} + O(M^4)\right. \qquad \ldots(2.15)$$

$$U = V\left[1 - \frac{(\gamma-1)V^2}{4a_s^2}\right] + O(M^4) \qquad \ldots(2.16)$$

$$V = \frac{a_s}{a_\infty} U = a_s M \quad (M, \text{ the Mach number}) \qquad \ldots(2.17)$$

V represents the modified main stream and reduces the steady state compressible equations to incompressible form as proved by Stewartson (1949) if the Prandtl number is one. If we introduce the Stewartson variables (1949) X and Y defined by

$$X = \int_0^x \left(\frac{a_\infty}{a_s}\right)^{(3\gamma-1)/(\gamma-1)} dx \qquad \ldots(2.18)$$

$$Y = \frac{a_\infty}{a_s\sqrt{\nu_s}} \int_0^y \frac{\rho}{\rho_s}\, dy \qquad \ldots(2.19)$$

then eqns. (2.1) to (2.5) reduce to

$$u = \frac{\rho_s\sqrt{\nu_s}}{\rho}\frac{\partial\psi}{\partial y} = \frac{a_\infty}{a_s}\frac{\partial\psi}{\partial Y}. \qquad \ldots(2.20)$$

$$v = -\frac{\rho_s\sqrt{\nu_s}}{\rho}\left[\frac{a_s}{a_\infty}\frac{\partial Y}{\partial t} - \frac{a_s^2}{a_\infty^2}\, Y \frac{\partial}{\partial t}\left(\frac{a_\infty}{a_s}\right)\right.$$
$$\left. + \frac{\partial\psi}{\partial X}\frac{\partial X}{\partial x} + \frac{\partial\psi}{\partial Y}\frac{\partial Y}{\partial x}\right] \qquad \ldots(2.21)$$

$$\frac{a_\infty}{a_s}\frac{\partial^2\psi}{\partial Y\partial t}+\frac{a_\infty}{a_s^2}\frac{\partial a_\infty}{\partial X}\left(\frac{\partial\psi}{\partial Y}\right)^2\frac{\partial X}{\partial x}+\frac{a_\infty^2}{a_s^2}\frac{\partial X}{\partial x}$$

$$\times\left[\frac{\partial\psi}{\partial Y}\frac{\partial^2\psi}{\partial X\partial Y}-\frac{\partial\psi}{\partial X}\frac{\partial^2\psi}{\partial Y^2}-\frac{\partial^3\psi}{\partial Y^3}\right]$$

$$=\frac{T}{T_\infty}U\frac{dU}{dX}\frac{\partial X}{\partial x} \qquad \ldots(2.22)$$

$$\frac{\partial w}{\partial t}+\frac{a_\infty}{a_s}\frac{\partial X}{\partial x}\left[\frac{\partial\psi}{\partial Y}\frac{\partial w}{\partial X}-\frac{\partial\psi}{\partial X}\frac{\partial w}{\partial Y}-\frac{\partial^2 w}{\partial Y^2}\right]=0 \qquad \ldots(2.23)$$

$$\frac{\partial T}{\partial t}+\frac{a_\infty}{a_s}\frac{\partial X}{\partial x}\left[\frac{\partial\psi}{\partial Y}\frac{\partial T}{\partial X}-\frac{\partial\psi}{\partial X}\frac{\partial T}{\partial Y}\right]+\frac{Ta_\infty}{c_pT_\infty a_s}\frac{\partial\psi}{\partial Y}U\frac{dU}{dX}\frac{\partial X}{\partial x}$$

$$=\frac{a_\infty}{a_s}\frac{\partial X}{\partial x}\left[\frac{1}{\sigma}\frac{\partial^2 T}{\partial Y^2}+\frac{1}{c_p}\frac{a_\infty^2}{a_s^2}\left(\frac{\partial^2\psi}{\partial Y^2}\right)^2+\frac{1}{c_p}\left(\frac{\partial w}{\partial Y}\right)^2\right]. \qquad \ldots(2.24)$$

As such these equations are complicated to solve. So, we assume just as in Lighthill (1954) that the periodic flow is a perturbed one about a steady mean that is

$$\psi=\psi_0(X,Y)+\epsilon\,\psi_1(X,Y,t) \qquad \ldots(2.25)$$

$$w=w_0(X,Y)+\epsilon\,w_1(X,Y,t) \qquad \ldots(2.26)$$

$$T=T_0(X,Y)+\epsilon\,T_1(X,Y,t). \qquad \ldots(2.27)$$

where the zero suffixed quantities denote the steady state and one suffixed quantities refer to the oscillatory parts.

3. Analysis of Steady State

Before we analyse the oscillatory flow we shall first study the steady compressible flow past yawed cylinder with no heat transfer at the wall. The equations for such a flow are those given by (2.18) to (2.24) with the time derivative terms equal to zero, the proper boundary conditions are

$$\psi_0=\frac{\partial\psi_0}{\partial Y}=0 \text{ at } Y=0 \qquad \ldots(3.1)$$

$$w_0=\frac{\partial T_0}{\partial Y}=0 \text{ at } Y=0 \qquad \ldots(3.2)$$

$$\frac{\partial\psi_0}{\partial Y}\to V(X),\ w_0\to W,\ T_0\to T_\infty(X) \text{ as } Y\to\infty. \qquad \ldots(3.3)$$

It is further assumed that the Mach number of the flow is small so that the compressible flow can be perturbed about an incompressible flow. Thus to solve the system of equations, for steady flow, we assume just as Howarth (1948) the following series in Mach number:

$$\psi_0 = \alpha(X) A(\eta) - \frac{\alpha(X)}{2}\left[A_1(\eta) + \frac{W^2}{V^2} A_2(\eta) \right] \frac{(\gamma-1)}{2} M^2 + O(M^4) \quad ...(3.4)$$

$$T_0 = T_s B(\eta) - T_s \left[B_1(\eta) + \frac{W^2}{V^2} B_2(\eta) \right] \frac{(\gamma-1)M^2}{2} + O(M^4) \quad ...(3.5)$$

$$w_0 = W\, C(\eta) + W \left[C_1(\eta) + \frac{W_2}{V^2} C_2(\eta) \right] \frac{(\gamma-1)\, M^2}{2} + O(M^4) \quad ...(3.6)$$

$$\eta = \left(\frac{\alpha_0(m_0+1)\, X^{m_0-1}}{2} \right)^{1/2} Y, \quad \alpha(X) = \left(\frac{2\alpha_0\, X^{m_0+1}}{m_0+1} \right)^{1/2} \quad ...(3.7)$$

where A's, B's, C's satisfy the ordinary differential equations which can be obtained on substituting (3.4) to (3.7) in eqns. (2.22) to (2.24). The equations for A and C are the same as those given by (210) and (211) of Chapter VII of Rosenhead (1963). The other equations for A's, B's, C's constitute a system of two point boundary value problem. These equations are solved numerically and the important numerical values are recorded in Table I.

TABLE I

β	0	0.5	1.0
$A''(0)$	0.46960	0.92768	1.23258
$C'(0)$	0.46999	0.53952	0.57109

The dimensionless temperature of the wall and skin friction coefficients in steady flow are given by

$$T_0^{(D)} = B(0) - \left[B_1(0) + \frac{W^2}{V^2} B_2(0) \right] \frac{(\gamma-1)}{2} M^2 + O(M^4) \quad ...(3.8)$$

$$\tau_{0u}^{(D)} = A''(0) - \left[A_1''(0) + \frac{W^2}{V^2} A_2''(0) + \frac{2(2\gamma-1)}{(\gamma-1)} A''(0) \right] \times \left(\frac{\gamma-1)}{4} \right) M^2 + O(M^4) \quad ...(3.9)$$

$$\tau_{0w}^{(D)} = C'(0) + \left[C_1'(0) + \frac{W^2}{V^2} C_2'(0) - \frac{(3\gamma-1)}{2(\gamma-1)} C'(0) \right] \times \frac{(\gamma-1)M^2}{2} + O(M^4) \quad ...(3.10)$$

where

$$T_0^{(D)} = \frac{(T_0)_{y=0}}{T_s} \qquad \ldots(3.11)$$

$$\tau_{ou}^{(D)} = \frac{\text{steady part of } \left(\mu \dfrac{\partial u}{\partial y} \right)_{y=0}}{\rho_s \left[\nu_s \; \alpha_0^3 \left(\dfrac{m_0 + 1}{2} \right) X^{(3m_0 - 1)} \right]^{1/2}} \qquad \ldots(3.12)$$

$$\tau_{ow}^{(D)} = \frac{\text{steady part of } \left(\mu \dfrac{\partial w}{\partial y} \right)_{y=0}}{W \rho_s \left[\nu_s \alpha_0 \left(\dfrac{m_0 + 1}{2} \right) X^{(m_0 - 1)} \right]^{1/2}} \qquad \ldots(3.13)$$

Using these formulae we can study the behaviour of the temperature of the wall and skin friction coefficients for various values of γ, β, σ, W/V, M.

4. Analysis of the Oscillatory Part of the Motion

The analysis of the oscillatory part of the motion is done by solving the system of equations for ψ_1, w_1 and T_1. The temperature gradient at the wall is given by (1.1) and is periodic with frequency ω. It is again difficult to give a general analysis valid for all values of the frequency parameter ω. So following Glauert (1956) and Ackerberg and Phillips (1972) we give below two solutions one for large frequencies and the other for small frequencies.

4.1 *Solution for Large Frequencies*

In this case, just as Ackerberg and Phillips we could not find a single solution for large frequencies valid across the whole range of the boundary layer. This is because of the diffusion near the wall and convection near the edge of the layer. The solution satisfying the condition at the wall does not satisfy the condition at infinity and vice versa. So there arises a necessity to develop two asymptotic solutions, one the inner and the other outer which are given below. And finally a composite solution is given.

(A) *Inner Solution* — The proper inner variables defined to describe the inner solution are:

$$\theta = 1/\sqrt{\bar{X}}, \; \bar{X} = \frac{\omega}{\alpha_0} X^{-(m_0 - 1)}, \; \bar{Y} = \sqrt{\omega} Y. \qquad .. (4.1)$$

where ω is the frequency parameter and α_0, m_0 are constants. We assume the following expansions for ψ_1, w_1 and T_1 for the inner solutions

$$\psi_1 = \psi_1^{(i)} = \alpha\alpha_1 \left(\frac{m_0 + 1}{2} \right)^{1/2} e^{i\omega t} \left[\sum_{n=0}^{\infty} \theta^{(n - 2m_1 + 3)} \; f_n^{(i)} (\bar{Y}) + \right.$$

(equation continued on p. 1014)

$$+ \sum_{n=0}^{\infty} \theta^{(n-2m_1)} \left(f_{1,n}^{(i)}(\bar{Y}) + \frac{W^2}{V^2} f_{2,n}^{(i)}(\bar{Y}) \right) \frac{(\gamma - 1)}{2} M^2 \Bigg] + O(M^4) \quad \text{...(4.2)}$$

$$w_1 = w_1^{(i)} = \alpha_1 W \left(\frac{m_0 + 1}{2} \right)^{1/2} e^{i\omega t} \Bigg[\sum_{n=0}^{\infty} \theta^{(n-2m_1+3)} h_n^{(i)}(\bar{Y})$$

$$+ \sum_{n=0}^{\infty} \theta^{(n-2m_1)} \left(h_{1,n}^{(i)}(\bar{Y}) + \frac{W^2}{V^2} h_{2,n}^{(i)}(\bar{Y}) \right) \frac{(\gamma - 1)}{2} M^2 \Bigg] + O(M^4) \quad \text{...(4.3)}$$

$$T_1 = T_1^{(i)} = T_s \alpha_1 \left(\frac{m_0 + 1}{2} \right)^{1/2} e^{i\omega t} \Bigg[\sum_{n=0}^{\infty} \theta^{(n-2m_1+1)} g_n^{(i)}(\bar{Y})$$

$$+ \sum_{n=0}^{\infty} \theta^{(n-2m_1+1)} \left(g_{1,n}^{(i)}(\bar{Y}) + \frac{W^2}{V^2} g_{2,n}^{(i)}(\bar{Y}) \right) \left(\frac{\gamma - 1}{2} \right) M^2 \Bigg] + O(M^4) \quad \text{...(4.4)}$$

where $f^{(i)}$'s, $g^{(i)}$'s and $h^{(i)}$'s satisfy the inner boundary conditions

$$f_n^{(i)}(0) = f_n^{(i)'}(0) = h_n^{(i)}(0) = 0 \quad \text{...(4.5)}$$

$$g_n^{(i)'}(0) = 1 \text{ or } 0 \text{ as } n = 0 \text{ or } \neq 0 \quad \text{...(4.6)}$$

$$f_{1,n}^{(i)}(0) = f_{1,n}^{(i)'}(0) = g_{1,n}^{(i)'}(0) = h_{1,n}^{(i)}(0) = 0 \quad \text{...(4.7)}$$

$$f_{2,n}^{(i)}(0) = f_{2,n}^{(i)'}(0) = g_{2,n}^{(i)'}(0) = h_{2,n}^{(i)}(0) = 0. \quad \text{...(4.8)}$$

Substituting (4.1) to (4.4) in eqns. (2.22) to (2.24) we obtain the ordinary differential equations which are to be solved with the inner boundary conditions (4.5) to (4.8), using the expansions of steady state functions in powers of η just as in Sarma (1964), and Ackerberg and Phillips (1972). There solutions being lengthy are not given here.

(B) *Outer Solution* — In this case the proper outer variables defined are θ and η. As an outer solution, we assume the expansion for ψ_1, w_1 and T_1 as

$$\psi_1 = \psi_1^{(0)} = \alpha\alpha_1 \left(\frac{m_0 + 1}{2} \right)^{1/2} \theta^{-(2m_1+1)} e^{i\omega t} \Bigg[\sum_{n=0}^{\infty} \theta^n f_n^{(0)}(\eta) +$$

(*equation continued on p.* 1015)

$$+ \sum_{n=0}^{\infty} \theta^n \Big(f_{1,n}^{(0)}(\eta) + \frac{W^2}{V^2} f_{2,n}^{(0)}(\eta) \Big) \Big(\frac{\gamma - 1}{2} \Big) M^2 \Big] + O(M^4)$$

...(4.9)

$$w_1 = w_1^{(0)} = \alpha_1 W \left(\frac{m_0 + 1}{2} \right)^{1/2} \theta^{-(2m_1+1)} e^{i\omega t} \Big[\sum_{n=0}^{\infty} \theta^n h_n^{(0)}(\eta)$$

$$+ \sum_{n=0}^{\infty} \theta^n \Big(h_{1,n}^{(0)}(\eta) + \frac{W^2}{V^2} h_{2,n}^{(0)}(\eta) \Big) \Big(\frac{\gamma - 1}{2} \Big) M^2 \Big] + O(M^4)$$

...(4.10)

$$T_1 = \alpha_1 T_s \left(\frac{m_0 + 1}{2} \right)^{1/2} \theta^{-(2m_1+1)} e^{i\omega t} \Big[\sum_{n=0}^{\infty} \theta^n g_n^{(0)}(\eta)$$

$$+ \sum_{n=0}^{\infty} \theta^n \Big(g_{1,n}^{(0)}(\eta) + \frac{W^2}{V^2} g_{2,n}^{(0)}(\eta) \Big) \Big(\frac{\gamma - 1}{2} \Big) M^2 \Big] + O(M^4)$$

...(4.11)

where $f^{(0)}$'s, $g^{(0)}$'s, $h^{(0)}$'s satisfy the outer boundary conditions

$$f_n^{(0)\prime}(\infty) = g_n^{(0)}(\infty) = h_n^{(0)}(\infty) = 0 \qquad \text{...(4.12)}$$

$$f_{1,n}^{(0)\prime}(\infty) = g_{1,n}^{(0)}(\infty) = h_{1,n}^{(0)}(\infty) = 0 \qquad \text{...(4.13)}$$

$$f_{2,n}^{(0)\prime}(\infty) = g_{2,n}^{(0)}(\infty) = h_{2,n}^{(0)}(\infty) = 0. \qquad \text{...(4.14)}$$

Substituting (4.9) to (4.11) in eqns. (2.22) to (2.24) we obtain the ordinary differential equations in η. Solutions are obtained just as Ackerberg and Phillips (1972). The actual outer solutions are not given here as they are lengthy.

Although each term in the inner expansion is completely determined, unknow constants are there in the outer expansion as integration constants at each order of θ. These constants are determined according to the matching principle, such as

$$\lim_{\bar{Y} \to \infty} \psi_1^{(i)}(\theta, \bar{Y}) = \lim_{\eta \to 0} \psi_1^{(0)}(\theta, \eta). \qquad \text{...(4.15)}$$

On the left the limit is carried out neglecting exponentially small terms in it. On the right the outer variable η is written in terms of the inner variable $\bar{Y}$. Then comparison of each term on both the sides results in determination of these constants.

Using these two inner and outer solutions and following Cole (1968), we get the composite solution as

Composite solution = inner solution + outer solution − common part.

The outer solution when expanded in powers of $\bar{Y}$ gets cancelled with the common part. Thus the composite solutions are

$$\psi_1 = \psi_1^{(c)} = \psi_1^{(i)} \qquad ...(4.16)$$

$$w_1 = w_1^{(c)} = w_1^{(i)} \qquad ...(4.17)$$

$$T_1 = T_1^{(c)} = T_1^{(i)} . \qquad ...(4.18)$$

Now we get ultimately the expressions for the dimensionless temperature and skin friction coefficients which are physically important and given by

$$T_1^{(D)} = \sum_{n=0}^{\infty} \bar{X}^{(2m_1-n-1)/2}\, g_n^{(c)}(0) + \frac{(\gamma-1)}{2} M^2 \left[\sum_{n=0}^{\infty} \bar{X}^{(2m_1-n-1)/2} g_{1,n}^{(c)}(0) + \frac{W^2}{V^2} \sum_{n=0}^{\infty} \bar{X}^{(2m_1-n-1)/2}\, g_{2,n}^{(c)}(0) \right] \qquad ...(4.19)$$

$$\tau_{1,u}^{(D)} = \left(\frac{m_0+1}{2}\right)^{1/2} \Bigg[\sum_{n=0}^{\infty} \bar{X}^{(2m_1-n-3)/2}\, f_n^{(c)''}(0) + \left(\frac{\gamma-1}{2}\right) M^2 \left\{ \sum_{n=0}^{\infty} \bar{X}^{(2m_1-n)/2} \left(f_{1,n}^{(c)''}(0) + \frac{W^2}{V^2} f_{2,n}^{(c)''}(0) \right) - \left(\frac{2\gamma-1}{\gamma-1}\right) \sum_{n=0}^{\infty} \bar{X}^{(2m_1-n-3)/2}\, f_n^{(c)''}(0) \right\} \Bigg], \qquad ...(4.20)$$

$$\tau_{1w}^{(D)} = \left(\frac{m_0+1}{2}\right)^{1/2} \Bigg[\sum_{n=0}^{\infty} \bar{X}^{(2m_1-n-3)/2}\, h_n^{(c)'}(0) +$$

(*equation continued on p.* 1017)

$$+\left(\frac{\gamma-1}{2}\right)M^2\left\{\sum_{n=0}^{\infty}\bar{X}^{(2m_1-n)/2}\left(h_{1,n}^{(c)'}(0)+\frac{W^2}{V^2}\,h_{2,n}^{(c)'}(0)\right)\right.$$

$$\left.-\left(\frac{3\gamma-1}{\gamma-1}\right)\sum_{n=0}^{\infty}\bar{X}^{(2m_1-n-3)/2}\,h_n^{(c)'}(0)\right\}\Bigg] \qquad \ldots(4.21)$$

where the superscript c stands for the composite solution and

$$T_1^{(D)}=\frac{T_1e^{-i\omega t}}{\alpha_1T_s}\,, \qquad \ldots(4.22)$$

$$\tau_{1u}^{(D)}=\frac{\text{unsteady part of the }\left(\mu\dfrac{\partial u}{\partial y}\right)_{y=0}}{\alpha_1\rho_s\left[\nu_s\alpha_0^3\left(\dfrac{m_0+1}{2}\right)X^{(3m_0-1)}\right]^{1/2}}, \qquad \ldots(4.23)$$

$$\tau_{1w}^{(D)}=\frac{\text{unsteady part of the }\left(\mu\dfrac{\partial w}{\partial y}\right)_{y=0}}{\alpha_1W\rho_s\left[\nu_s\alpha_0\left(\dfrac{m_0+1}{2}\right)X^{(m_0-1)}\right]^{1/2}}. \qquad \ldots(4.24)$$

The magnitudes and arguments of the above quantities are studied graphically for various values of γ, β, σ, m_1, M, W/V in the later sections of the paper.

4.2 *Solution for Small Frequencies*

As explained already, to find a solution for small frequencies of the system of eqns. (2.22) to (2.24) when the temperature gradient at the wall is periodic in time. We use the variables $\bar{X}$ and η. We assume the following asymptotic series for ψ_1, w_1, T_1 in terms of the dimensionless frequency parameter $\bar{X}$.

$$\psi_1=\alpha\alpha_1\left(\frac{m_0+1}{2}\right)^{1/2}\bar{X}^{(2m_1+1)/2}e^{i\omega t}\left[\sum_{n=0}^{\infty}\left\{p_n(\eta)+\left(p_{1,n}(\eta)\right.\right.\right.$$

$$\left.\left.\left.+\frac{W^2}{V^2}p_{2,n}(\eta)\right)\left(\frac{\gamma-1}{2}\right)M^2\right\}(2i\bar{x})^n\right]+O(M^4) \qquad \ldots(4.25)$$

$$w_1=\alpha_1W\left(\frac{m_0+1}{2}\right)^{1/2}\bar{X}^{(2m_1+1)/2}e^{i\omega t}\left[\sum_{n=0}^{\infty}\left\{r_n(\eta)+\left(r_{1,n}(\eta)\right.\right.\right.$$

$$\left.\left.\left.+\frac{W^2}{V^2}r_{2,n}(\eta)\right)\left(\frac{\gamma-1}{2}\right)M^2\right\}(2i\bar{x})^n\right]+O(M^4) \qquad \ldots(4.26)$$

$$T_1 = \alpha_1 T_s \left(\frac{m_0+1}{2}\right)^{-1/2} \bar{X}^{(2m_1+1)/2} e^{i\omega t}\Big[\sum_{n=0}^{\infty}\Big\{q_n(\eta) + \Big(q_{1,n}(\eta) + \frac{W^2}{V^2} q_{2,n}(\eta)\Big)\Big(\frac{\gamma-1}{2}\Big) M^2\Big\}(2i\bar{x})^n\Big] + O(M^4) \quad ...(4.27)$$

with the boundary conditions

$$p'_n = q_n = r_n = p'_{1,n} = q_{1,n} = r_{1,n} = p'_{2,n} = q_{2,n} = r_{2,n} = 0 \text{ if } n < 0 \quad ...(4.28)$$

$$q'_n(0) = 1 \text{ or } 0 \text{ according as } n = 0 \text{ or } \neq 0 \quad ...(4.29)$$

$$p_{1,n}(0) = p'_{1,n}(0) = p'_{1,n}(\infty) = q'_{1,n}(0) = q_{1,n}(\infty) = r_{1,n}(0) = r_{1,n}(\infty) = 0 \quad ...(4.30)$$

$$p_{2,n}(0) = p'_{2,n}(0) = p'_{2,n}(\infty) = q'_{2,n}(0) = q_{2,n}(\infty) = r_{2,n}(0) = r_{2,n}(\infty) = 0. \quad ...(4.31)$$

When we substitute (4.25) to (4.28) in the system of eqns. (2.22) to (2.24) we get third and second order ordinary differential equations. These equations are solved following Cohen and Reshotko (1956) and Reshotko and Beckwith (1957) by an iterative procedure using fourth order Runge-Kutta method. The numerical results are verified with the other technique by reducing boundary value problem, following Moore (1951), into an initial value problem and then solving by Runge-Kutta method. The important numerical values are recorded in Tables II to VIII. The dimensionless temperature at the wall and the skin friction coefficient for small frequencies as defined earlier, are given by

$$\begin{aligned} T_1^{(D)} = \left(\frac{m_0+1}{2}\right)^{-1/2} \bar{X}^{(2m_1+1)/2} \Big[\Big\{ & q_0(0) + \frac{(\gamma-1)M^2}{2}\Big(q_{1,0}(0) + \frac{W^2}{V^2} q_{2,0}(0)\Big) - 4\bar{X}^2\Big(q_2(0) + \frac{(\gamma-1)M^2}{2}\Big(q_{1,2}(0) \\ & + \frac{W^2}{V^2} q_{2,2}(0)\Big)\Big)\Big\} + i\Big\{2\bar{X}\Big(q_1(0) + \frac{(\gamma-1)M^2}{2}\Big(q_{1,1}(0) \\ & + \frac{W^2}{V^2} q_{2,1}(0)\Big)\Big) - 8\bar{X}^3\Big(q_3(0) + \frac{(\gamma-1)M^2}{2}\Big(q_{1,3}(0) \\ & + \frac{W^2}{V^2} q_{2,3}(0)\Big)\Big)\Big\}\Big] + O(\bar{X}^{(2m_1+9)/2}) \end{aligned} \quad ...(4.32)$$

$$\tau_{1u}^{(D)} = \left(\frac{m_0+1}{2}\right)^{-1/2} \bar{X}^{(2m_1+1)/2}\Big[\Big\{ p_0''(0) + \frac{(\gamma-1)M^2}{2}\Big(p_{1,0}''(0)$$

$$+ \frac{W^2}{V^2}\, p_{2,0}''(0) - \frac{(2\gamma-1)}{(\gamma-1)}\, p_0''(0)\Big) - 4\bar{X}^2\Big(p_2''(0)$$

$$+ \frac{(\gamma-1)M^2}{2}\Big(p_{1,2}''(0) + \frac{W^2}{V^2}\, p_{2,2}''(0)$$

$$- \left(\frac{2\gamma-1}{\gamma-1}\right) p_2''(0)\Big)\Big)\Big\} + i\,\Big\{2\bar{X}\,(p_1''(0)$$

$$+ \frac{(\gamma-1)M^2}{2}\Big(p_{1,1}''(0) + \frac{W^2}{V^2}\, p_{2,1}''(0)$$

$$- \left(\frac{2\gamma-1}{\gamma-1}\right) p_1''(0)\Big)\Big) - 8\bar{X}^3\Big(p_3''(0)$$

$$+ \frac{(\gamma-1)M^2}{2}\Big(p_{1,3}''(0) + \frac{W^2}{V^2}\, p_{2,3}''(0)$$

$$- \frac{(2\gamma-1)}{(\gamma-1)}\, p_3''(0)\Big)\Big)\Big\}\Big] + O(\bar{X}^{(2m_1+9)/2}) \qquad \ldots(4.33)$$

$$\tau_{1w}^{(D)} = \left(\frac{m_0+1}{2}\right)^{-1/2} \bar{X}^{(2m_1+1)/2}\Big[r_0'(0) + \frac{(\gamma-1)M^2}{2}\Big(r_{1,0}'(0)$$

$$+ \frac{W^2}{V^2}\, r_{2,0}'(0) - \frac{(3\gamma-1)}{2(\gamma-1)}\, r_0'(0)\Big) - 4\bar{X}^2\Big(r_2'(0)$$

$$+ \frac{(\gamma-1)M^2}{2}\Big(r_{1,2}'(0) + \frac{W^2}{V^2}\, r_{2,2}'(0)$$

$$- \frac{(3\gamma-1)}{2(\gamma-1)}\, r_2'(0)\Big)\Big) + i\,2\bar{X}\Big(r_1'(0)$$

$$+ \frac{(\gamma-1)M^2}{2}\Big(r_{1,1}'(0) + \frac{W^2}{V^2}\, r_{2,1}'(0)$$

$$- \frac{(3\gamma-1)}{2(\gamma-1)}\, r_1'(0)\Big)\Big) - 8\bar{X}^3\Big(r_3'(0)$$

$$+ \frac{(\gamma-1)M^2}{2}\Big(r_{1,3}'(0) + \frac{W^2}{V^2}\, r_{2,3}'(0)$$

$$- \frac{(3\gamma-1)}{2(\gamma-1)}\, r_3'(0)\Big)\Big)\Big] + O(\bar{X}^{(2m_1+9)/2}). \qquad \ldots(4.34)$$

Using the values given in Tables II to VIII, the behaviour of the skin friction and temperature at the wall represented graphically for various values of γ, β, σ, m_1.

TABLE II

$\gamma = 1.4$

β	0.0	0.0	0.0	0.0
σ	0.76	0.76	1.0	1.0
m_1	0.0	0.5	0.0	0.5
$q_0(0)$	−1.69166	−1.42958	−1.53887	−1.30165
$q_1(0)$	0.92722	0.63894	0.91908	0.63464
$q_2(0)$	−0.46240	−0.28020	−0.49983	−0.30372
$q_3(0)$	0.18892	0.10431	0.22286	0.122344

TABLE III

$\gamma = 1.4$

β	0.5	0.5	0.5	0.5
σ	0.76	0.76	1.0	1.0
m_1	0.0	0.5	0.0	0.5
$q_0(0)$	−1.66705	−1.45401	−1.50486	−1.31430
$q_1(0)$	0.72966	0.53099	0.70599	0.51544
$q_2(0)$	−0.30387	−0.19569	−0.31537	−0.20404
$q_3(0)$	0.10516	0.06178	0.11721	0.06925

TABLE IV

$\gamma = 1.4$

β	0.5	0.5	0.5	0.5
σ	0.76	0.76	1.0	1.0
m_1	0.0	0.5	0.0	0.5
$p_0''(0)$	−0.50489	−0.40270	−0.43130	−0.34464
$p_1''(0)$	0.33285	0.23043	0.29854	0.20725
$p_2''(0)$	−0.15414	−0.09626	−0.14557	−0.09121
$p_3''(0)$	0.05603	0.03227	0.05586	0.03229

TABLE V

$\gamma = 1.4$

β	0.5	0.5	0.5	0.5
σ	0.76	0.76	1.0	1.0
m_1	0.0	0.5	0.0	0.5
$r_0'(0)$	−0.06794	−0.05634	−0.05566	−0.04629
$r_1'(0)$	0.08306	0.05893	0.07058	0.05022
$r_2'(0)$	−0.05294	−0.03361	−0.04676	−0.02977
$r_3'(0)$	0.02340	0.01363	0.02155	0.01259

TABLE VI

$\gamma = 1.4$

β	σ	$q_0(0)$	$q_1(0)$	$q_2(0)$	$q_3(0)$
1.0	0.76	−1.95037	1.10388	−0.80739	0.60958
1.0	1.0	−1.75102	1.04632	−0.80600	0.64037

TABLE VII

$\gamma = 1.4$

β	σ	$p_0''(0)$	$p_1''(0)$	$p_2''(0)$	$p_3''(0)$
1.0	0.76	−1.09486	0.86190	−0.66144	0.50462
1.0	1.0	−0.93641	0.77421	−0.62461	0.50115

TABLE VIII

$\gamma = 1.4$

β	σ	$r_0'(0)$	$r_1'(0)$	$r_2'(0)$	$r_3'(0)$
1.0	0.76	−0.11163	0.14208	−0.13132	0.10854
1.0	1.0	−0.09126	0.12020	−0.11550	0.09965

5. EXAMPLES AND CONCLUSIONS

Till now we have made a general study of a three dimensional compressible boundary layers. We shall now study three problems in detail as examples corresponding to $\beta = 0$, 0.5 and 1.

(A) *Yawed Flat Plate* ($\beta = 0$, $M = 0$)

This particular example corresponding to the flow past a flat plate ($\beta = 0$) in the X, Y-plane. It is found from the numerical study of the analytical formulae (4.32) and (4.19) that the perturbation of temperature gradient at the wall about zero mean has no effect on the skin frictions. The effects of compressibility and the effects due to other parameters on temperature distribution are illustrated in Fig. 1 as functions of the dimensionless frequency $\bar{X}$. We infer from them that the magnitude of temperature at the wall for small $\bar{X}$ decreases with σ, m_1 and increases with $\bar{X}$ whereas for large $\bar{X}$ decreases with σ, m_1, $\bar{X}(m_1 = 0)$ and increases with $\bar{X}(m_1 = 0.5)$. The tangent of the argument of the temperature at the wall for small $\bar{X}$ decreases with $\bar{X}$, σ and increases with m_1 whereas for large $\bar{X}$ decreases with σ, m_1, $\bar{X}(m_1 = 0)$ and increases with $\bar{X}(m_1 = 0.5)$. The curves for small $\bar{X}$ show the tendency to join with those for large $\bar{X}$.

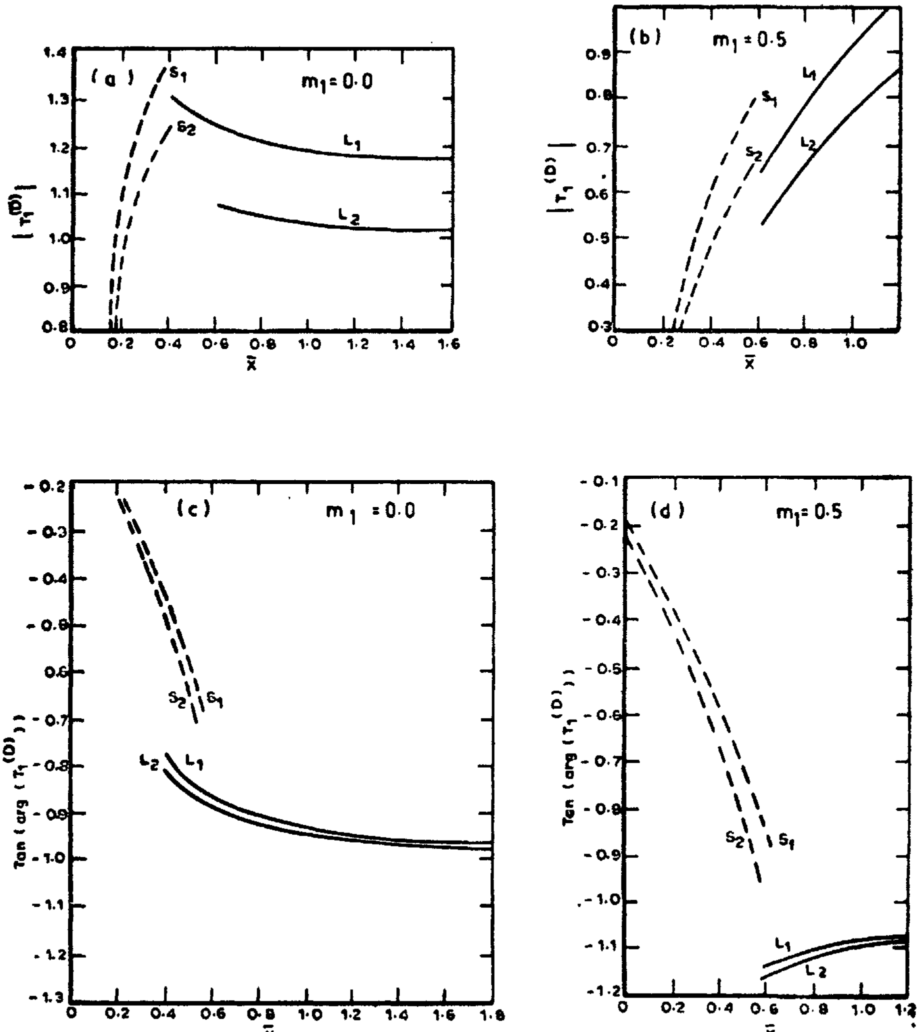

FIG. 1. Temperature variation at the wall with frequency $\bar{X}$ when $\beta = 0$, $M = 0$, $L_n(\sigma) = L_1(.76)$, $L_2(1)$ represent the curves for large $\bar{X}$ from (4.19) and $S_n(\sigma) = S_1(.76)$, $S_2(1)$, represent the curves for small $\bar{X}$ from (4.32).

(B) *Yawed Wedge* ($\beta = 0.5$, $M = 0$)

This example corresponds to the flow past a yawed infinite wedge with an included angle equal to a right angle. Figures (2a) to (2d) give the behaviour of the temperature as predicted by the analytical formulae (4.32) and (4.19). The magnitudes of temperature at the wall for small $\bar{X}$ decrease with m_1, σ and increase with $\bar{X}$ whereas for large $\bar{X}$ increase with $\bar{X}(m_1 = 0.5)$ and decrease with $\bar{X}(m_1 = 0)$, σ. The tangents

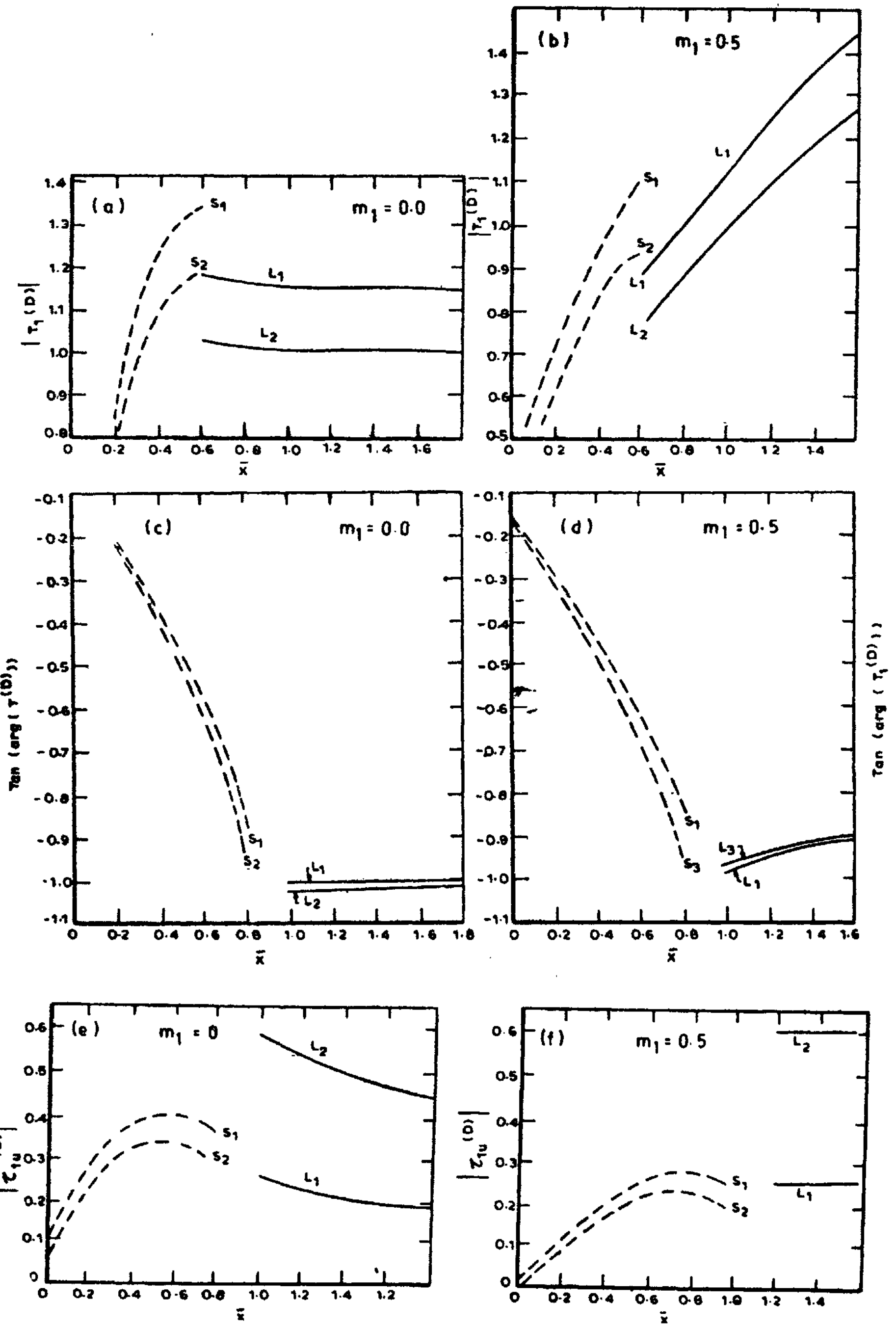

FIG 2. Variation of temperature at the wall and skin friction coefficient with frequency $\bar{X}$ when $\beta = 0.5$, $M = 0$, $L_n(\sigma)$, $S_u(\sigma)$, $(n = 1, 2)$ as explained in the legend under Fig. 1, represent the curves for large frequencies from (4.19), (4.20) and for small frequencies from (4.32), (4.33) respectively. (In Figure 2d, S_3, L_3 should be read as S_2, L_2).

of the argument of temperature at the wall for small $\bar{X}$ decrease with $\bar{X}$, σ and increase with $\bar{X}$. The curves for small $\bar{X}$ show the tendency to join with those for large $\bar{X}$.

Figures (2e) and (2f) give the behaviour of the skin friction in u-direction as predicted by (4.33) and (4.20). The magnitude of the skin friction in u-direction for small values of $\bar{X}$ increase with $\bar{X}$ and decreases with σ, m_1 while for large values of $\bar{X}$ increase with σ, m_1 and decreases with $\bar{X}$ (to an almost constant value when $m_1 = 0$) and becomes independent of $\bar{X}$ ($m_1 = 0.5$). From the numerical study of (4.33) and (4.20) we infer that the tangent of the argument of the skin friction in u-direction for small values of $\bar{X}$ increases with m_1 and decreases with $\bar{X}$, σ while for large values of $\bar{X}$ decreases with σ and becomes independent of m_1. It is found that the joining of the curves for small $\bar{X}$ with those for large $\bar{X}$ is not so good.

From the numerical study of (4.34) and (4.21) we infer, that the magnitude of the skin friction in w-direction for small values of $\bar{X}$ increases with $\bar{X}$, σ and decreases with m_1 while for large values of $\bar{X}$ increases with m_1 and becomes independent of $\bar{X}$, σ. The tangent of the argument of skin friction in w-direction for small $\bar{X}$ decreases with $\bar{X}$, $\sigma(m_1 = 0.5)$ and increases with $\sigma(m_1 = 0)$, m_1 while for large $\bar{X}$ increases with $\sigma(m_1 = 0)$ and becomes independent of $\bar{X}$, m_1. It is found that the joining of the curves for small $\bar{X}$ with those for large $\bar{X}$ is not so good.

(C) *Yawed Flat Plate with a Stagnation Point* ($\beta = 1$, $M = 0$)

This example corresponds to the stagnation flow past a yawed flat plate. Figures (3a) and (3b) give the behaviour of the temperature as predicted by (4.32) and (4.19) when $\beta = 1$ and $M = 0$. The magnitude of the temperature at the wall

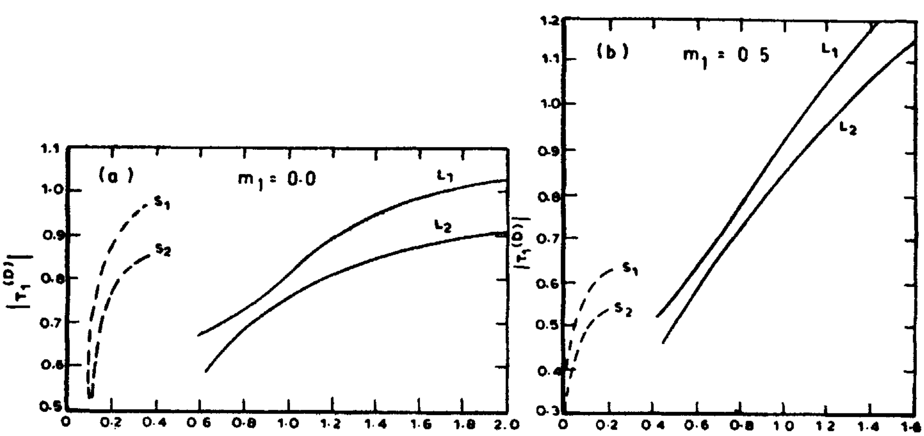

FIG. 3. Variations of temperature at the wall [as predicted by (4.32) and (4.19)] with frequency $\bar{X}$ when $\beta = 1$, $M = 0$. $L_n(\sigma)$, $S_n(\sigma)$ ($n = 1, 2$) stand for the same meaning as in the legend under Fig. 2.

for small $\bar{X}$ increases with $\bar{X}$ and decreases with σ, m_1 while for large values of $\bar{X}$ increases with $\bar{X}$, m_1 and decreases with σ. From the numerical study of the analytical formulae (4.32) and (4.19) we infer when $\beta = 1$, that the tangent of the argument of the temperature at the wall for small $\bar{X}$ decreases with σ, $\bar{X}$ while for large $\bar{X}$ increases with $\bar{X}$, σ and for both small and large values of $\bar{X}$ it becomes independent of m_1. Figures show a good joining of the curves for small and large values of $\bar{X}$.

From the numerical study of (4.33) and (4.20) we infer that the magnitude of the skin friction in u-direction for small values of $\bar{X}$ increases with $\bar{X}$ and decreases with σ, m_1 while for large values of $\bar{X}$ increases with σ, m_1 and decreases with $\bar{X}$(when $m_1 = 0$) and becomes independent of $\bar{X}$(when $m_1 = 0.5$). The tangent of the argument of skin friction in u-direction for small $\bar{X}$ decreases with $\bar{X}$, σ and becomes independent of m_1. while for large $\bar{X}$ decreases with σ and becomes independent of $\bar{X}$, m_1. It is found that the joining of the curves for small $\bar{X}$ with those for large $\bar{X}$ is not so good.

From the numerical study of the formulae (4.34) and (4.21) we infer that the magnitude of the skin friction in w-direction for small $\bar{X}$ increases with $\bar{X}$, $\sigma(m_1 = 0)$ and decreases with $\sigma(m_1 = 0.5)$, m_1 while for large $\bar{X}$ increases with m_1 and is zero when $M^2 = 0$. The tangent of the argument of the skin friction in w-direction for small $\bar{X}$ increase with σ and decreases with $\bar{X}$, $m_1(\sigma = 1)$ and becomes independent of $m_1(\sigma = 0.76)$ while for large values of $\bar{X}$ increase with σ and becomes independent of $\bar{X}$, m_1. It is found that the joining of the curves for small $\bar{X}$ with those for large $\bar{X}$ is not so good.

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