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MASS TRANSFER EFFECTS ON THE FLOW PAST AN ACCELERATED VERTICAL INSULATED PLATE

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In the present paper, the mass transfer of an incompressible viscous fluid past an uniformly accelerated vertical plate is investigated by taking into account the viscous dissipative heat. Analytical expressions for the velocity and temperature field have been derived.

NOMENCLATURE

C' = species concentration

C'_w = species concentration at the plate surface

C^1_∞ = species concentration at infinite

c_p = specific heat of the fluid at constant pressure

D = Chemical molecular diffucivity

E = Eckert number

g = gravitational acceleration

Gr = Grashof number

Gc = modified Grashof number

k = thermal conductivity

P = Prandtl number

Sc = Schmidt number

T' = temperature

T'_w = temperature of the plate

T'_∞ = temperature of the fluid at infinity

t' = time

x' = co-ordinate axis along the plate

ψ' = co-ordinate axis normal to the plate

u' = velocity component in x'-direction

β = volumetric coefficient of thermal expansion

β^* = volumetric coefficient of expansion with concentration

ν = kinematic viscosity of the fluid

ρ = density of the fluid.

1. Introduction

Flows arising from differences in concentration or material constitution alone and in conjunction with temperature differences have great significance not only of their own interest but also due to their applications to chemical engineering, geophysics and aeronautics. There are many interesting aspects of such flow, so in recent years analytical solutions to such problems of flow have been presented by many authors. Gebhart and Pera (1971) have studied the laminar flows which arise in fluids due to the interaction of the force of gravity and density differences caused by the simultaneous diffusion of thermal energy and of chemical species. Sparrow *et al.* (1964) have presented an analytical study of the effects of buoyancy in a binary boundary layer into which a foreign gas is injected through a porous surface. Soundalgekar (1976) has studied the mass transfer effects on the free convective steady flow past an infinite vertical porous plate in the presence of suction. Recently Gupta *et al.* (1979) have studied the free convection effects on the flow past an accelerated vertical plate in an incompressible fluid. The purpose of the present paper is to study the mass transfer effects on the flow field of an incompressible, viscous, dissipative fluid past an infinite vertical plate, which is accelerated in its own plane when the wall is perfectly insulated, i.e. there is no heat transfer between the fluid and the wall (adiabatic).

2. Mathematical Analysis

In order to formulate the problem mathematically, we write down the general equations of fluid motion for two-dimensional unsteady flow, in Cartesian co-ordinates, with x'-axis along the vertical plate in the upward direction and the y'-axis normal to it. All the fluid properties are assumed to be independent of temperature, except the influence of the density. Also, the influence of the density variations in other terms of the momentum and energy equations and the variation of the expansion coefficient with temperature, is negligible. At time $t' > 0$ the infinite plate starts moving with a velocity $u' = ct'$ (where c is a positive constant). Under these assumptions, the physical variables are functions of y' and t' only and therefore the equations which govern the problem are:

$$\frac{\partial u'}{\partial t} = g\beta(T' - T'_\infty) + g\beta^*(C' - C'_\infty) + \nu \frac{\partial^2 u'}{\partial \psi'^2}, \qquad \ldots(1)$$

$$\frac{\partial T^1}{\partial t^1} = \frac{k}{\rho c_p} \frac{\partial^2 T^1}{\partial \psi'^2} + \frac{\nu}{c_p} \left(\frac{\partial u^1}{\partial \psi'} \right)^2, \qquad \text{...(2)}$$

$$\frac{\partial C^1}{\partial t^1} = D \frac{\partial^2 C^1}{\partial \psi'^2}. \qquad \text{...(3)}$$

The equation of continuity being identically satisfied by the velocity field $(u'(\psi', t'), 0)$. All the physical variables are defined in Nomenclature. The second term in the right side of (2) represents the viscous dissipative heat and because of the retaining of this term in equation of energy, the problem is governed by the coupled non-linear equations of which the appropriate boundary conditions are:

$$t' > 0: \left\{ \begin{array}{l} u'(0, t') = ct', \ \dfrac{\partial T'(0, t')}{\partial \psi'} = 0, \ C'(0, t') = C'_w \\ u'(\infty, t') = 0, \ T'(\infty, t') = T'_\infty, \ C'(\infty, t') = C'_\infty. \end{array} \right. \qquad \text{...(4)}$$

We introduce the following non-dimensional quantities:

$$\left. \begin{array}{l} \psi = \psi'\left(\dfrac{c}{\nu^2}\right)^{1/3}, \quad E = \dfrac{(\nu c)^{2/3}}{c_p(T'_w - T'_\infty)}, \quad t = t'\left(\dfrac{c^2}{\nu}\right)^{1/3}, \\ Gr = \dfrac{1}{c} g\beta(T'_w - T'_\infty), \quad u = \dfrac{u'}{(\nu c)^{1/3}}, \ Gc = \dfrac{1}{C} g\beta^*(C'_w - C'_\infty), \\ T = \dfrac{(T' - T'_\infty)}{(T'_w - T'_\infty)}, \ Sc = \dfrac{\nu}{D}, \ C = \dfrac{C' - C'_\infty}{C'_w - C'_\infty}, \ P = \dfrac{\rho \nu c_p}{k}. \end{array} \right\} \qquad \text{...(5)}$$

On introducing the non-dimensional parameters (4) into equations (1), (2) and (3), we get:

$$\frac{\partial u}{\partial t} = GrT + GcC + \frac{\partial^2 u}{\partial \psi^2} \qquad \text{...(6)}$$

$$P \frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial \psi^2} + PE\left(\frac{\partial u}{\partial \psi}\right)^2 \qquad \text{...(7)}$$

$$Sc \frac{\partial C}{\partial t} = \frac{\partial^2 C}{\partial \psi^2}. \qquad \text{...(8)}$$

The corresponding boundary conditions become:

$$t > 0: \left\{ \begin{array}{l} u(0, t) = t, \ \dfrac{\partial T(0, t)}{\partial \psi} = 0, \quad C(0, t) = 1 \\ u(\infty, t) = 0, \quad T(\infty, t) = 0, \ C(\infty, t) = 0. \end{array} \right. \qquad \text{...(9)}$$

Since E is very small for incompressible fluids, we expand the velocity field and temperature field as

$$\left.\begin{aligned} u(\psi, t) &= u_0(\psi, t) + Eu_1(\psi, t) + \dots \\ T(\psi, t) &= T_0(\psi, t) + ET_1(\psi, t) + \dots \end{aligned}\right\} \quad \dots(10)$$

and substituting (10) into (6) and (7), following standard procedure, we get

$$\frac{\partial u_0}{\partial t} = GrT_0 + GcC + \frac{\partial^2 u_0}{\partial \psi^2} \quad \dots(11)$$

$$P\,\frac{\partial T_0}{\partial t} = \frac{\partial^2 T_0}{\partial \psi^2} \quad \dots(12)$$

$$\frac{\partial u_1}{\partial t} = GrT_1 + \frac{\partial^2 u_1}{\partial \psi^2} \quad \dots(13)$$

$$P\,\frac{\partial T_1}{\partial t} = \frac{\partial^2 T_1}{\partial \psi^2} + P\left(\frac{\partial u_0}{\partial \psi}\right)^2. \quad \dots(14)$$

The boundary conditions (9) for the velocity field and temperature field, become

$$t > 0: \left.\begin{aligned} &u_0(0, t) = t,\ u_1(0, t) = 0,\ u_0(\infty, t) = 0,\ u_1(\infty, t) = 0 \\ &\frac{\partial T_0}{\partial t}(0, t) = 0,\ \frac{\partial T_1}{\partial t}(0, t) = 0,\ T_0(\infty, t) = 0,\ T_1(\infty, t) = 0. \end{aligned}\right\} \quad \dots(15)$$

We take the Prandtl number P and Schmidt number Sc equal to one. This is a plausible assumption since P is a measure of the relative importance of viscosity and heat conductivity, in the fluid. For most gases P is of unit order, so that the velocity and the thermal boundary layers will be the same order of thickness (Houghton and Boswell 1969). Also, the concentration and velocity profiles will have the same shape when $\nu = D$ or $Sc = 1$. Thus the Schmidt number in convection and mass transfer problems plays a role similar to that of the Prandtl number in convection heat-transfer problems. With this assumption (P, $Sc = 1$) the solution of the problem, for small t, can be obtained in the form

$$u_0 = tf_0(\eta),\ T_0 = g_0(\eta),\ u_1 = t^3 f_1(\eta),\ T_1 = t^2 g_1(\eta) \quad \dots(16)$$

where $$\eta = \frac{\psi}{2t^{1/2}}\,.$$

Substituting (16) into (11) – (14), we get

$$\ddot{f}_0 + 2\eta \dot{f}_0 - 4f_0 = -4Grg_0 - 4GcC \quad \dots(17)$$

$$\ddot{f}_1 + 2\eta \dot{f}_1 - 12 f_1 = -4 Gr g_1 \quad ...(18)$$

$$\ddot{g}_0 + 2\eta \dot{g}_0 = 0 \quad ...(19)$$

$$\ddot{g}_1 + 2\eta \dot{g}_1 - 8 g_1 = -\dot{f}_0^2 \quad ...(20)$$

while equation (8) can be written now as

$$\ddot{C} + 2\eta \dot{C} = 0 \quad ...(21)$$

where a dot denotes differentiation with respect to η. The corresponding boundary conditions become

$$f_0(0) = 1,\ \dot{g}_0(0) = 0,\ C(0) = 1,\ f_0(\infty) = 0,\ g_0(\infty) = 0,\ C(\infty) = 0$$

$$f_1(0) = 0,\ \dot{g}_1(0) = 0,\ f_1(\infty) = 0,\ g_1(\infty) = 0. \quad ...(22)$$

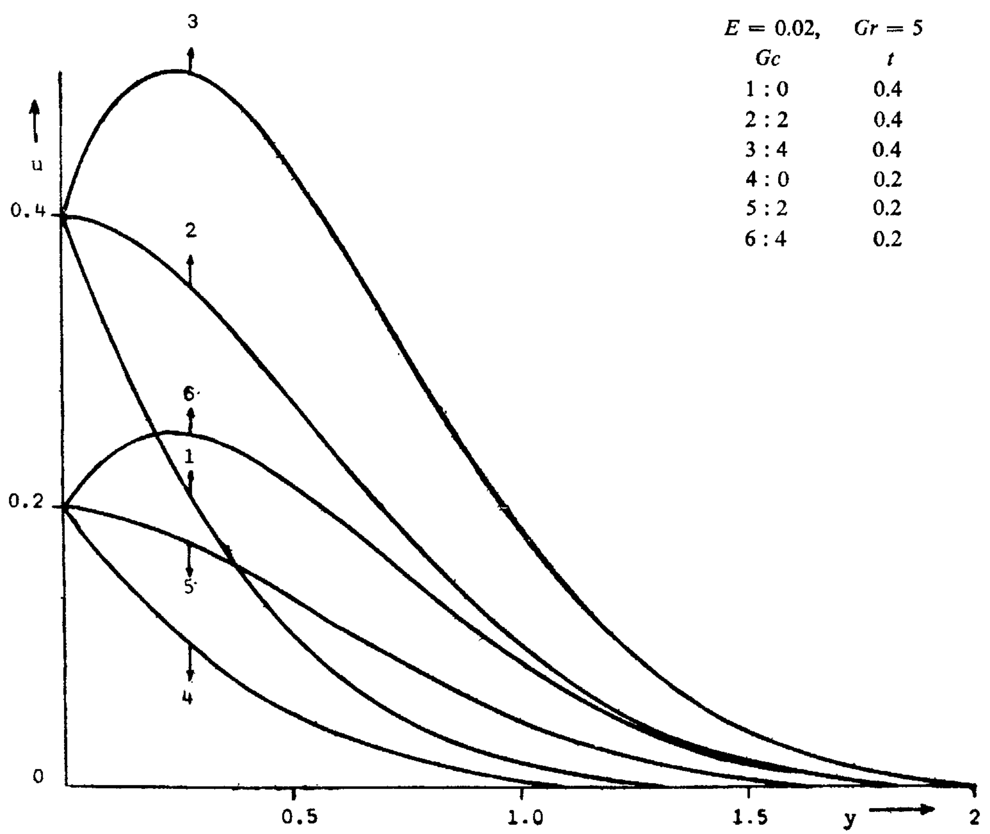

FIG. 1. Velocity profiles.

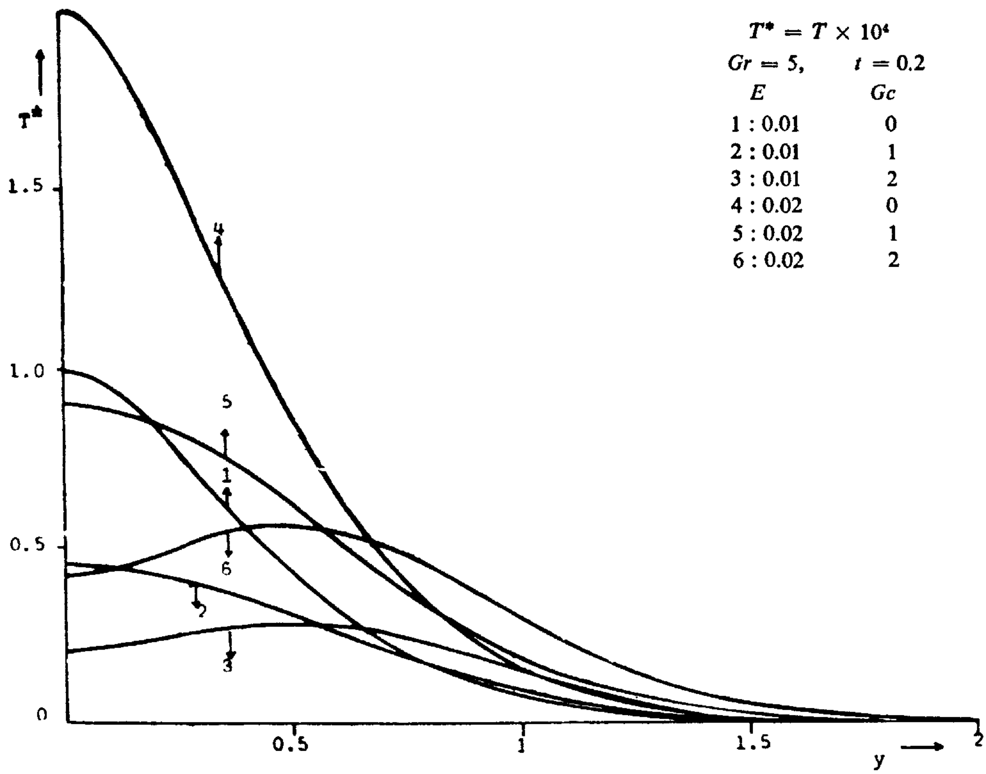

FIG. 2. Temperature profiles.

The solutions of (17) – (21), under the boundary conditions (22) are

$$g_0(\eta) = 0$$

$$C(\eta) = \sqrt{\frac{2}{\pi}}\, Hh_0(\sqrt{2}\eta) \qquad \ldots(23)$$

$$f_0(\eta) = \sqrt{\frac{2}{\pi}}\,[2(1 - Gc)Hh_2\,(\sqrt{2}\eta) + GcHh_0(\sqrt{2}\eta)] \qquad \ldots(24)$$

$$\begin{aligned} g_1(\eta) = {} & \left(-\frac{Gc^2}{\pi}\right)(Hh_0^2\,(\sqrt{2}\eta) + 2Hh_1^2\,(\sqrt{2}\eta)) + \frac{4}{\pi}\,Gc(Gc - 1) \\ & \times (Hh_0\,(\sqrt{2}\eta).\ Hh_2(\sqrt{2}\eta) + Hh_1(\sqrt{2}\eta)Hh_3(\sqrt{2}\eta)) \\ & + \left(\frac{-\,6Gc^2 + 8Gc - 4}{\pi}\right) Hh_2^2\,(\sqrt{2}\eta) + \frac{2}{9\pi}\sqrt{\frac{\pi}{2}} \\ & \times (3Gc^2 + 2Gc + 6)\,Hh_4(\sqrt{2}\eta) \end{aligned} \qquad \ldots(25)$$

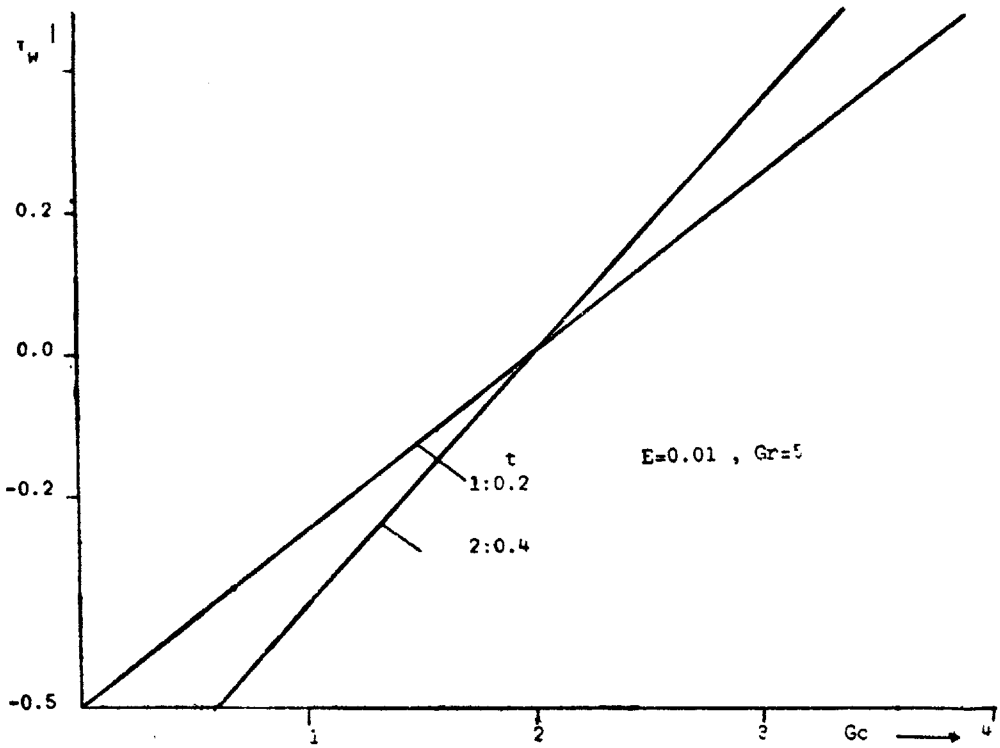

FIG. 3. Skin friction profiles.

$$f_1(\eta) = Gr\Bigg[\frac{2}{9\pi}\sqrt{\frac{\pi}{2}}\,(3Gc^2 + 2Gc + 6)\,Hh_4\,(\sqrt{2}\eta)$$

$$+\left(\frac{6Gc^2 - 8Gc + 4}{\pi}\right) Hh_3^2\,(\sqrt{2}\eta) + \frac{Gc^2}{\pi}\,Hh_1^2\,(\sqrt{2}\eta)$$

$$+\frac{4Gc^2}{\pi}\,(Hh_2^2\,(\sqrt{2}\eta) + Hh_3^2\,(\sqrt{2}\eta)) + \frac{4}{\pi}\,Gc(1 - Gc)$$

$$\times\, Hh_1(\sqrt{2}\eta)\;\; Hh_3(\sqrt{2}\eta) + \frac{8}{\pi}\,Gc(1 - Gc)$$

$$\times\, Hh_2(\sqrt{2}\eta)\;\; Hh_4(\sqrt{(2}\eta) - 2\sqrt{\frac{2}{\pi}}\,(7Gc^2 + \frac{20}{3}\,Gc + 2$$

$$+\frac{86}{3\pi}\,Gc^2 + \frac{32}{3\pi}\,Gc + \frac{32}{3\pi}\Big)\,.\; Hh_6(\sqrt{2}\eta)\Bigg] \qquad \ldots(26)$$

where the function $Hh_n(\sqrt{2}\eta)$ is defined in Jeffreys and Jeffreys (1972) and is related with the complementary error function (erfc(η)) as in Appendix. Now we can calculate the velocity temperature and skin friction from (24) – (26) which are given in the dimensionless form by the expressions:

$$u = tf_0(\eta) + Et^3 f_1(\eta) \qquad \text{...(27)}$$

$$T = Et^2 g_1(\eta) \qquad \text{...(28)}$$

$$\tau_w = \tfrac{1}{2}\, t^{1/2}(\dot{f}_0(0) + Et^2 \dot{f}_1(0)). \qquad \text{...(29)}$$

3. Results

In Fig. 1, the velocity profiles have been drawn for different values of Gc and t by taking $E = 0.02$, $Gr = 5$. We observe that when the Grashof number Cc increases the velocity increases and this phenomenon is more evident for higher values of time t. The temperature profiles are shown in Fig. 2 when $Gr = 5$, $t = 0.2$ and for different values of E and Gc. In this case, Grashof number causes the formation of a temperature field in the boundary layer. Also we observe that, the temperature increases due to greater dissipative heat. Finally, from Fig. 3, we conclude that when the Gc increases, the skin friction τ_w increases.

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Appendix

$$Hh_0(\sqrt{2}\eta) = \sqrt{\pi/2}\,\text{erfc}\,(\eta)$$

$$Hh_1(\sqrt{2}\eta) = e^{-\eta^2} - \sqrt{\pi}\eta\,\text{erfc}\,(\eta)$$

$$Hh_2(\sqrt{2}\eta) = 0.5[\sqrt{\pi/2}\,(1 + 2\eta^2)\,\text{erfc}\,(\eta) - \sqrt{2}\eta\, e^{-\eta^2}]$$

$$Hh_3(\sqrt{2}\eta) = \tfrac{1}{6}[2(1 + \eta^2)\, e^{-\eta^2} - \sqrt{\pi}\eta \cdot (3 + 2\eta^2)\,\text{erfc}\,(\eta)]$$

$$Hh_4(\sqrt{2}\eta) = \frac{1}{24}\,[\sqrt{\pi/2}\,.\,(3 + 12\eta^2 + 4\eta^4)\,\text{erfc}\,(\eta) - \sqrt{2}\eta(5 + 2\eta^2)e^{-\eta^2}]$$

$$Hh_5(\sqrt{2}\eta) = \frac{1}{720}\,[\sqrt{\pi/2}\,(15 + 90\eta^2 + 60\eta^4 + 8\eta^6)\,\text{erfc}\,(\eta)$$

$$- \sqrt{2}\eta\,.\,(33 + 28\eta^2 + 4\eta^4)\, e^{-\eta^2}\,].$$

ERRATUM

Correction to

GRAPHS WHOSE SQUARES ARE CHORDAL

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The statement of the theorem should read as : If G has no induced subgraph isomorphic to $K_{1,3}$ or cycle $C_n(n \geqslant 6)$ or $P_5 + a$, where P_5 is a path on five points and a is the edge that joins the two non-adjacent internal vertices of P_5, then G^2 is chordal.

The additional hypothesis introduced takes care of the case when u_i is adjacent to u_{i+1} or u_{i-1} in the proof of the theorem.

Replace the counter example in the remark by the wheel W_7 with a pendant edge attached to each of any two consecutive vertices of the rim of the wheel.

In consonance with the change in the statement of the theorem, the Corollary should read as : If G is chordal and free from $K_{1,3}$ and $P_5 + a$ as induced subgraphs, then G^2 is chordal.