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SOME COMMENTS ON A PAPER BY KARADE AND KUMBHARE ENTITLED "ALTERNATIVE APPROACH TO THE SIMPLEX METHOD"

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In this note we point out that the claim of Karade and Kumbhare (1980) that their method to solve linear programming problems never takes more number of iterations than the conventional simplex method is wrong.

1. Introduction

In a recent paper Karade and Kumbhare (1980) consider the linear programming problem

$$\text{minimise } CX$$

$$\text{Subject to } AX = B,\ X \geqslant 0$$

where A is a given $m \times n$ matrix, B is a given $m \times 1$ of column vector and C is a given $1 \times n$ row vector. X is the unknown $n \times 1$ column vector to be determined. We assume that the reader is familiar with the conventional simplex method to solve this problem and also with the paper of Karade and Kumbhare (1980). We use their notations without explaining them. They propose a method and assert that their method never requires more number of iterations to solve the problem than the conventional simplex method. The following counterexample shows that their assertion is wrong.

2. Counterexample and Some Comments

Consider the problem

$$\text{minimise } -3x_1 - 2x_2$$

$$\text{subject to } x_1 - x_2 + x_3 = 1$$

$$2x_1 + 3x_2 + x_4 = 22$$

$$-2x_1 + 5x_2 + x_5 = 18$$

$$-2x_1 + x_2 + x_6 = 2$$

$$x_i \geqslant 0,\ 1 \leqslant i \leqslant 6$$

To solve this the conventional simplex method takes only two iterations. (The third tableau produced is an optimal one). The method of Karade and Kumbhare takes 4 iterations before producing the optimal tableau. The method suggested by Karade and Kumbhare is not unknown in the literature. It is called the method of steepest ascents by Hadley (1978) who devotes more than a page to a discussion of this method. Also, the exercise 4–5 of Hadley (1978, p. 144) is relevant here.

If one is looking for an optimal basis in the sense of one for which $Z_j - c_j \leqslant 0$ for all columns P_j then it is possible that the method of steepest ascents also cycles. Here is an example, which is a modification of an example given by Marshall and Suurballe (1969). In this example we minimize $Z = \Sigma c_i x_i$ with the initial tableau as follows:

C_B	Basic variables	c_i x_0	0 P_1	2 P_2	0 P_3	−1 P_4	7 P_5	1 P_6	0 P_7
0	x_1	1	1	1	0	1	1	1	0
0	x_7	0	0	9	0	1/2	−11/2	−5/2	1
0	x_3	0	0	1	1	1/2	−3/2	−1/2	0
	$Z_j - c_j$	0	0	−2	0	1	−7	−1	0

We resolve ties in the selection of a column to be included in the basis by choosing P_r where

$r = \min \{i$ such that P_i is eligible to be included in the basis under the pivot column choosing rule of the method of steepest ascents$\}$.

Similarly the pivot row is determined by choosing row s where

$s = \min \{i :$ row i is eligible to be pivot row under the minimum ratio rule$\}$.

(Of course it is easy to see that the method of steepest ascents does not cycle at a non-optimal point).

References

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