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FLOW OF A NON-NEWTONIAN SECOND-ORDER FLUID BETWEEN TWO ENCLOSED ROTATING DISCS

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The steady flow of an incompressible second-order fluid between two finite rotating discs (enclosed within a cylindrical casing and flowing with a small mass rate of symmetrical radial outflow) has been discussed. Effects of second-order forces in the flow on the velocity field have been investigated in detail in the regions of no recirculation and recirculation for the cases of radial outflow and inflow and are shown graphically. Moment coefficient and pressure have also been obtained.

1. INTRODUCTION

The problem of the flow of a liquid between two finite enclosed rotating discs (enclosed in a cylindrical casing) or shrouded discs has important engineering applications as its generalisation can be of great help in studies concerning air cooling of turbines and pedestal bearing with central feeding of lubricant, windage losses and leakage flow in a centrifugal pump or compressor. The problem for the flow of a Newtonian fluid over an enclosed rotating disc was first studied by Soo (1958). Sharma (1963), has given an improvement for the velocity profile assumed by Soo (1958). Sharma and Gupta (1964) and Sharma and Sharma (1965) extended the study for elastico-viscous and second-order fluids respectively. The flow of a second-order fluid under an enclosed rotating disc has recently been considered by Sharma and Gupta (1980).

The purpose of the present paper is to investigate the flow of a second-order fluid between two finite rotating discs enclosed in a co-axial cylindrical casing. The symmetrical radial steady outflow has a small mass rate 'm' of radial outflow and $-m$ for radial inflow. The inlet condition is taken as a simple radial source flow along the z-axis starting from radius r_0. The equations of motion have been solved by expanding the velocity function in a series of ascending powers of Reynolds number (assumed small). The second-order effects have been investigated in the region of recirculation and no recirculation for both the cases of radial outflow and inflow and are shown graphically through Figs. 1 to 5. The following three typical cases of the problem have been studied:

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(a) when the discs rotate in the same sense,

(b) when the discs rotate in the opposite sense, and

(c) when one disc rotates and other is at rest.

2. Formulation of the Problem

The constitutive equation of an incompressible second-order fluid as suggested by Coleman and Noll (1959) can be written as

$$t_{ij} = -p\delta_{ij} + 2\mu_1 d_{ij} + 2\mu_2 e_{ij} + 4\mu_3 c_{ij} \qquad \ldots(2.1)$$

where

$$\left.\begin{aligned} d_{ij} &= \tfrac{1}{2}(u_{i,j} + u_{j,i}) \\ e_{ij} &= \tfrac{1}{2}(a_{i,j} + a_{j,i}) + u^{m}_{,i}\, u_{m,j} \\ c_{ij} &= d^{m}_{i}\, d_{mj} \end{aligned}\right\} \qquad \ldots(2.2)$$

and

t_{ij} is the stress tensor; u_i, a_i are the velocity and the acceleration vectors; p the hydrostatic pressure; and μ_1, μ_2, μ_3 represent material constants whose values are given by $\mu_2 = -0.2$ and $\mu_1 = 18.5$, $\mu_3 = 1.0$ (all expressed in C.G.S. system) for a 5.46% solution of polyisobutylene in cetane at 30°C.

Equation (2.1) together with the momentum equation for no extraneous force

$$\rho\left(\frac{\partial u_i}{\partial t} + u^m u_{i,m}\right) = t^{m}_{i,m} \qquad \ldots(2.3)$$

and the equation of continuity for steady flow

$$u^{i}_{,i} = 0, \qquad \ldots(2.4)$$

where ρ is the density of the fluid and comma (,) represents covariant differentiation, form the set of governing equations.

In a three-dimensional cylindrical set of coordinates (r, θ, z) the system consists of two finite rotating discs of radius r_s (coinciding with the planes $z = 0$ and $z = z_0$) rotating with constant angular velocities $s_1\Omega$ and $s_2\Omega$ respectively (where s_1 and s_2 are constant parameters) in an incompressible second-order fluid forming the part of a coaxial cylindrical casing.

Assuming (u, v, w) as the velocity components along the cylindrical system of axes the boundary conditions of the problem are:

$$\left.\begin{aligned} z &= 0 : u = 0, \quad v = rs_1\Omega, \quad w = 0 \\ z &= z_0 : u = 0, \quad v = rs_2\Omega, \quad w = 0 \end{aligned}\right\} \qquad \ldots(2.5)$$

where the gap z_0 is assumed small in comparison with the disc radius r_s. The velocity components for the axisymmetric flow compatible with the continuity criterion can be taken as (Sharma and Sharma 1965):

$$\left.\begin{aligned} u &= -r\Omega\, H'(\zeta) + \frac{mM'(\zeta)}{2\pi r\rho z_0} \\ v &= r\Omega\, G(\zeta) + \frac{mL(\zeta)}{2\pi r\rho z_0} \\ w &= 2\,\Omega\, z_0 H(\zeta) \end{aligned}\right\} \qquad \ldots(2.6)$$

where $H(\zeta)$, $M(\zeta)$, $G(\zeta)$ and $L(\zeta)$ are non-dimensional functions of the dimensionless variables $\zeta = z/z_0$ and 'm' the small mass rate of radial outflow is represented by

$$m = 2\pi\rho \int_0^{z_0} ru\, dz \qquad \ldots(2.7)$$

m being positive for net radial outflow and negative for net radial inflow.

The boundary conditions (2.5) transform for H, G, and L into the following form:

$$\left.\begin{aligned} &H(0) = H(1) = 0,\ H'(0) = H'(1) = 0 \\ &G(0) = s_1, \qquad G(1) = s_2, \\ &L(0) = L(1) = 0. \end{aligned}\right\} \qquad \ldots(2.8)$$

The conditions on M on the boundaries are obtainable from the expression (2.7) for m as follows:

$$M(1) - M(0) = 1 \qquad \ldots(2.9)$$

which on choosing the discs as stream lines reduces to

$$M(1) = 1,\ M(0) = 0. \qquad \ldots(2.10)$$

Using (2.1), (2.3) and (2.6) and neglecting the squares and higher powers of R_m/R_s (assumed small) where $R_m (= m/\pi\rho\nu_1 z_0)$ and $R_z\ (= \Omega z_0^2/\nu_1)$ are the Reynolds number based on the radial outflow and the gap length respectively, we have the following equations in dimensionless form:

$$-\frac{1}{\xi}\frac{\partial p}{\partial \xi} = (H'^2 - 2HH'' - G^2) - \frac{R_m}{R_z\xi^2}(GL - HM'')$$
$$+ \frac{1}{R_z}\left(H''' - \frac{R_m}{2R_z\xi^2} M''' \right) - T_1\Big[2(H''^2 - HH^{\text{iv}})$$
$$+ \frac{R_m}{R_z\xi^2}(H'M''' + H'''M' + HM^{\text{iv}} + H''M'' -$$

(*equation continued on p.* 1149)

$$- 2G'L' - 2G''L)\Big] - T_2\Big[(H''^2 - 2H'H''' - G'^2) + \frac{R_m}{R_z\xi^2}(H'M''' + H'''M' + H''M'' - 2G'L' - G''L)\Big], \qquad \ldots(2.11)$$

$$0 = 2(H'G - HG') - \frac{R_m}{R_z\xi^2}(M'G + HL') + \frac{1}{R_z}\left(G'' + \frac{R_m}{R_z\xi^2}L''\right) + T_1\Big[2(HG''' - H''G') + \frac{R_m}{R_z\xi^2}(2M''G' + 2M'G'' + H'''L + HL''' + L'H'')\Big] + T_2\Big[2(H'G'' - H''G') + \frac{R_m}{R_z\xi^2}(2M''G' + M'G'' + H'''L + HL''' + H''L' + H'L'')\Big], \qquad \ldots(2.12)$$

$$-\frac{\partial p}{\partial \zeta} = 4HH' - \frac{2}{R_z}H'' - T_1\Big[4(11H'H'' + HH''') + 4\xi^2(H''H''' + G'G'') - \frac{2R_m}{R_z}(H'''M'' + H''M''' - G''L' - G'L'')\Big] - T_2\Big[28H'H'' + 2\xi^2(H''H''' + G'G'') - \frac{R_m}{R_z}(H'''M'' + H''M''' - G''L' - G'L'')\Big] \qquad \ldots(2.13)$$

where $P = p/\rho\Omega^2 z_0^2$ and $T_1 = \nu_2/z_0^2$, $T_2 = \nu_3/z_0^2$ are two dimensionless parameters.

Equation (2.11) suggests the following form for the pressure:

$$P = P_0(\zeta) + \xi^2 P_1(\zeta) + \log\xi \,.\, P_2(\zeta). \qquad \ldots(2.14)$$

Substituting (2.14) in (2.11) and (2.13) and equating the terms independent of ξ and coefficients of similar powers of ξ we have

$$2P_1 = -(H'^2 - 2HH'' - G^2) - \frac{H'''}{R_z} + 2T_1(H''^2 - HH^{iv}) + T_2(H''^2 - 2H'H''' - G'^2),$$

$$P_2 = \frac{R_m}{R_z}(GL - HM'') + \frac{R_m}{2R_z^2}M''' + T_1\frac{R_m}{R_z}(H'M''' + H'''M' + HM^{iv} + H''M'' - 2G'L' - 2G''L) + T_2\frac{R_m}{R_z}(H'M''' + H'''M' + H''M'' - 2G'L' - G''L),$$

$$P_0' = 4HH' + \frac{2}{R_z} H'' + T_1 \Big[4(11H'H'' + HH''') - \frac{2R_m}{R_z} (H'''M''$$
$$+ H''M'' - G''L' - G'L'') \Big] + T_2 \Big[28H'M'' - \frac{R_m}{R_z}(H'''M''$$
$$+ H''M''' - G''L' - G'L'') \Big],$$

$$P_1' = 2(2T_1 + T_2)(H''H''' + G'G''),$$

$$P_2' = 0. \qquad \ldots(2.15)$$

The unknowns involved in the velocity field are to be determined from the following set of equations:

$$H^{\text{iv}} = 2R_z(HH''' + GG') - 2T_1R_z(2HH''' + 4G'G'' + HH^{\text{v}} + H'H^{\text{iv}}) - 2T_2R_z(H'H^{\text{iv}} + 2H''H''' + 3G'G''), \qquad \ldots(2.16)$$

$$M^{\text{iv}} = 2R_z(H'M'' + HM''' - G'L - GL') - 2T_1R_z(2H''M''' + 2H'''M'' + 2H'M^{\text{iv}} + HM^{\text{v}} + H^{\text{iv}}M' - 4G''L' - 2G'L'' - 2G'''L) - 2T_2R_z(2H''M''' + 2H'''M'' + H'M^{\text{iv}} + H^{\text{iv}}M' - 3G''L' - 2G'L'' - G'''L) \qquad \ldots(2.17)$$

$$G'' = 2R_z(HG' - H'G) + 2T_1R_z(H''G' - HG''') + 2T_2R_z(H''G' - H'G'') \qquad \ldots(2.18)$$

$$L'' = 2R_z(M'G + HL') - 2T_1R_z(2M''G' + 2M'G'' + H'''L + HL''' + H''L' + H'L'') - 2T_2R_z(2M''G' + M'G'' + H'''L + H''L' + H'L'' + HL''') \qquad \ldots(2.19)$$

where (2.16) and (2.17) are obtained by eliminating P_1 and P_2 from (2.15) while (2.18) and (2.19) are obtained by equating the coefficients of $1/\xi^2$ and terms independent of ξ.

By their very formulation $T_1, T_2 < 1$ will imply that the inertial effects dominate the second-order effects, the reverse being the implication of $T_1, T_2 > 1$.

3. Solution of the Problem

From small values of Reynolds number R_z, a solution of (2.16) – (2.19) is sought by expanding H, M, G and L in powers of R_z. Substituting the series

$$\left.\begin{aligned} H &= \sum_{n=0}^{\infty} R_z^n H_n, \qquad M = \sum_{n=0}^{\infty} R_z^n M_n \\ G &= \sum_{n=0}^{\infty} R_z^n G_n, \qquad L = \sum_{n=0}^{\infty} R_z^n L_n \end{aligned}\right\} \qquad \ldots(3.1)$$

into (2.16) – (2.19) and equating the terms independent of R_z and coefficients of R_z and R_z^2 respectively, three sets of equation are obtained. Solving them under the boundary conditions:

$$H_n(0) = H_n(1) = 0, \qquad H_n'(0) = H_n'(1) = 0,$$

$$G_0(0) = s_1,\ G_0(1) = s_2,\ G_{n+1}(1) = 0 = G_{n+1}(0),$$
$$(\text{for } n = 0, 1, 2, 3 \ldots)$$

$$L_n(0) = L_n(1) = 0,$$

$$M_n(0) = 0,\ M_0(1) = 1,\ M_{n+1}(1) = 0,\ M_n'(0) = M_n'(1) = 0, \qquad \ldots(3.2)$$

and using (2.6) the dimensionless velocity components $\bar{U}$, $\bar{V}$ and $\bar{W}$ correct to $O(R_z^2)$ are

$$\bar{U} = \left(\frac{u}{\Omega z_0}\right) = -\frac{\xi R_z}{60}\Big[s_1^2(5\zeta^4 - 20\zeta^3 + 21\zeta^2 - 6\zeta) - 2s_1s_2(5\zeta^4 - 10\zeta^3 + 6\zeta^2 - \zeta) + s_2^2(5\zeta^4 - 9\zeta^2 + 4\zeta)\Big] + \frac{3R_m}{\xi R_z}(\zeta - \zeta^2)$$
$$+ \frac{R_mR_z}{8400\xi}\Big[s_1^2(60\zeta^8 - 300\zeta^7 + 588\zeta^6 - 294\zeta^5 - 630\zeta^4 + 840\zeta^3 - 263\zeta^2 - \zeta) + s_2^2(60\zeta^8 - 180\zeta^7 + 168\zeta^6 - 294\zeta^5 + 420\zeta^4 - 263\zeta^2 + 89\zeta) - 2s_1s_2(60\zeta^8 - 240\zeta^7 + 238\zeta^6 + 126\zeta^5 - 105\zeta^4 - 280\zeta^3 + 247\zeta^2 - 46\zeta)\Big]$$
$$- \frac{TR_mR_z}{1050\xi}\Big[s_1^2(105\zeta^6 - 595\zeta^4 + 1190\zeta^3 - 939\zeta^2 + 239\zeta) + s_2^2(105\zeta^6 - 630\zeta^5 + 980\zeta^4 - 910\zeta^3 + 636\zeta^2 - 181\zeta) - s_1s_2(210\zeta^6 - 630\zeta^5 + 385\zeta^4 + 280\zeta^3 - 303\zeta^2 + 58\zeta)\Big] + \frac{4T^2R_mR_z}{5\zeta}(s_2 - s_1)^2$$
$$\times (5\zeta^4 - 10\zeta^3 + 6\zeta^2 - \zeta) \qquad \ldots(3.3)$$

$$\bar{V} = \left(\frac{v}{\Omega z_0}\right) = \xi\,[s_1 + (s_2 - s_1)\,\zeta] + \frac{\xi R_z^2}{6300}[s_1^3(20\zeta^7 - 140\zeta^6 + 357\zeta^5 - 420\zeta^4 + 210\zeta^3 - 27\zeta) - s_2^3(20\zeta^7 - 63\zeta^5 +$$

(*equations continued on p.* 1152)

$$+ 35\zeta^4 + 8\zeta) - s_1^2 s_2 (60\zeta^7 - 280\zeta^6 + 441\zeta^5 - 280\zeta^4$$
$$+ 70\zeta^3 - 11\zeta) + s_2^2 s_1 (60\zeta^7 - 140\zeta^6 + 21\zeta^5 + 175\zeta^4$$
$$- 140\zeta^3 + 24\zeta)] + \frac{T\xi R_z^2}{30} [s_2^3 (\zeta^5 - 3\zeta^3 + 2\zeta^2)$$
$$- s_1^3 (\zeta^5 - 5\zeta^4 + 7\zeta^3 - 3\zeta^2) + s_1^2 s_2 (3\zeta^5 - 10\zeta^4$$
$$+ 11\zeta^3 - 4\zeta^2) - s_2^2 s_1 (3\zeta^5 - 5\zeta^4 + \zeta^3 + \zeta^2)]$$
$$+ \frac{R_m}{10\xi} [s_1(3\zeta^5 - 10\zeta^4 + 10\zeta^3 - 3\zeta) - s_2(3\zeta^5 - 5\zeta^4$$
$$+ 2\zeta)] + \frac{2TR_m}{\xi} (s_2 - s_1) . (2\zeta^3 - 3\zeta^2 + \zeta) \quad \ldots(3.4)$$

$$\bar{W} = \left(\frac{W}{\Omega_{z0}}\right) = \frac{R_z}{30} s_1^2 (\zeta^5 - 5\zeta^4 + 7\zeta^3 - 3\zeta^2) + s_2^2 (\zeta^5 - 3\zeta^3 + 2\zeta^2)$$
$$- s_1 s_2 (2\zeta^5 - 5\zeta^4 + 4\zeta^3 - \zeta^2)]. \quad \ldots(3.5)$$

The above expressions for velocities show that for the present case and the approximation introduced, the cross-viscous effects and elastico-viscous effects are additive. Dimensionless form of radii at which there is no recirculation for the cases of net radial outflow ($m > 0$) and net radial inflow ($m < 0$) respectively satisfy the following conditions :

$$\text{(i)}\ R_m > 0;\ \left(\frac{\partial \bar{U}}{\partial \zeta}\right)_{\zeta=0} \geqslant 0,\ \left(\frac{\partial \bar{U}}{\partial \zeta}\right)_{\zeta=1} \leqslant 0,$$

$$\text{(ii)}\ R_m(= - R_n) < 0;\ \left(\frac{\partial \bar{U}}{\partial \zeta}\right)_{\zeta=0} \leqslant 0,\ \left(\frac{\partial \bar{U}}{\partial \zeta}\right)_{\zeta=1} \geqslant 0, \quad \ldots(3.6)$$

which are equivalent to:

(i) $R_m > 0$

$$\frac{\xi^2}{R_m} \leqslant \frac{25200 - R_z^2 [6720 (s_1 - s_2)^2 T^2 - 8T(181 s_1^2 - 239 s_2^2 + 58 s_1 s_2) - 89 s_1^2 + s_2^2 - 92 s_1 s_2]}{280 R_z^2 (2s_1^2 - 3s_2^2 + s_1 s_2)},$$

$$\frac{\xi^2}{R_m} + \frac{25200 - R_z^2 [6720 T^2 (s_1 - s_2)^2 - 8T(181 s_2^2 - 239 s_1^2 + 58 s_1 s_2) - 89 s_2^2 + s_1^2 - 92 s_1 s_2]}{280 R_z^2 (3s_1^2 - 2s_2^2 - s_1 s_2)} \geqslant 0.$$

$$\ldots(3.7)$$

(ii) $R_m < 0$

$$\frac{\xi^2}{R_n} \leqslant \frac{25200 - R_z^2\,[6720T^2(s_1-s_2)^2 - 8T(181s_2^2 - 239s_1^2 + 58s_1s_2) - 89s_2^2 + s_1^2 - 92s_1s_2]}{280R_z^2\,(3s_1^2 - 2s_2^2 - s_1s_2)},$$

$$\frac{\xi^2}{R_n} + \frac{25200 - R_z^2\,[6720T^2(s_1-s_2)^2 - 8T(181s_1^2 - 239s_2^2 + 58s_1s_2) - 89s_1^2 + s_2^2 - 92s_1s_2]}{280R_z^2\,(2s_1^2 - 3s_2^2 + s_1s_2)} \geqslant 0.$$

...(3.8)

The maximum values ξ_1 and ξ_2 of these radii which make a certain region for no recirculation can easily be found out for different values of s_1, s_2 and T.

4. Numerical Calculations and Discussion of Results

The numerical calculations have been carried out for the case $s_1 = 1$ and $s_2 = -1$. Setting Reynolds number $R_z = 0.5$, conditions (3.7) and (3.8) gives restriction on T which is assumed positive (considering that the cross-viscous effects dominate elastico-viscous effects as in the case of 5.46% solution of polyisobutylene in cetane at 30°C) to be greater than 1.91 for the case $R_m > 0$ and to be less than 1.91 for the case $R_m < 0$. The values of maximum radii ξ_1 and ξ_2 for which there is no recirculation with varying T are given in the following Table I and the formula is found to be as follows:

$$\left.\begin{aligned} \frac{\xi_1^2}{R_m} &= \frac{R_z^2\,(6720T^2 + 232T + 1) - 6300}{140R_z^2} \text{(for } R_m > 0) \\ \frac{\xi_2^2}{R_n} &= \frac{6300 - R_z^2\,(6720T^2 + 232T + 1)}{140R_z^2} \text{ (for } R_m < 0). \end{aligned}\right\} \quad \ldots(4.1)$$

TABLE I

$\xi/\sqrt{R_m}$ \ T	0	1	2	3
$\xi_1/\sqrt{R_m}$	—	—	3.64984	16.03054
$\xi_2/\sqrt{R_n}$	13.41613	11.41647	—	—

The Table I shows that $\xi_1(T)$ increases with an increase in T while $\xi_2(T)$ decreases with an increase in T. The values of

$$U^{(+)}_{\xi_1(T)} = [\bar{U}\,(R_z/R_m)^{1/2}]_{\xi_1(T)},\quad U^{(-)}_{\xi_2(T)} = [(\bar{U}\,(R_z/R_n)^{1/2}]_{\xi_2(T)},$$

$$V^{(+)}_{\xi_1(T)} = [\bar{V}\,(R_z/R_m)^{1/2}]_{\xi_1(T)},\quad V^{(-)}_{\xi_2(T)} = [\bar{V}(R_z/R_n)^{1/2}]_{\xi_2(T)},$$

for different values of T and for maximum and other values of ξ_1 and ξ_2 have been calculated and drawn in Figs. 1 and 2.

This shows that if T increases, the radial component of velocity for maximum or fixed radii decreases near the discs in both the cases of radial outflow and radial inflow. Figure 1 representing the behaviour of the radial velocity at maximum radii $\xi = \xi_1(T)$ for $T = 2, 3$ in the case $R_m > 0$ and $\xi = \xi_2(T)$ for $T = 0,\ 1$ in the case $R_m < 0$, exhibit the associated phenomenon of no recirculation. Figure 2 represents the behaviour of the radial velocity at fixed radius $\xi_2(0)$ for $R_m > 0$ and $R_m < 0$ respectively. It is found that there is no recirculation. The transverse velocity for $R_m > 0$ at fixed radii decreases near the lower disc and increases near the upper disc with an increase in T. Reverse is the case if $R_m < 0$. Thus it is clear that increase of T produces more and more recirculation, however there is no recirculation for the viscous case. The values of H', M', G and L have also been calculated and are illustrated in Figs. 3–5.

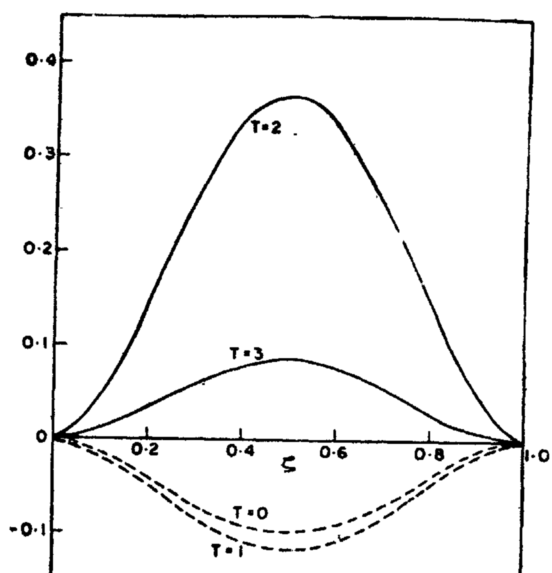

FIG. 1. Variation of radial velocity at maximum radii for no recirculation with the case (i) $m > 0 : U^{(+)}_{\xi_1(T)}$ ———, (ii) $m < 0 : U^{(-)}_{\xi_2(T)}$ - - - -.

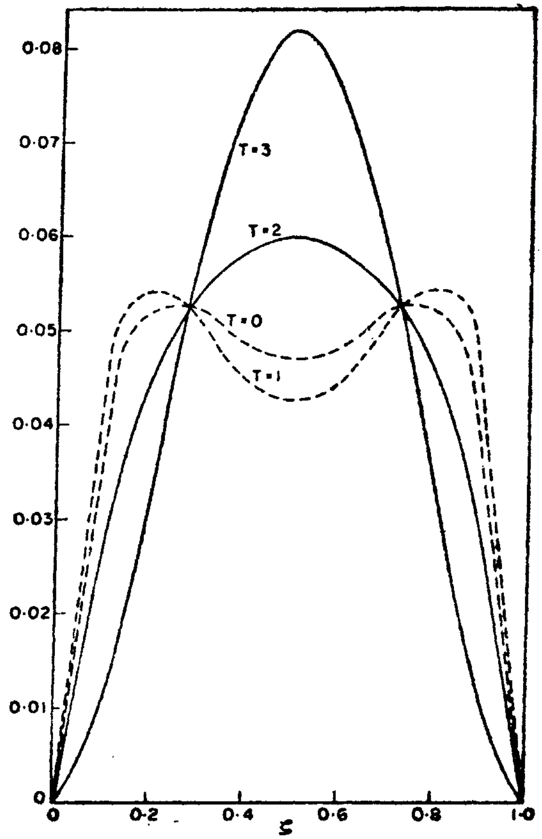

FIG. 2. Variation of radial velocity at fixed radii for the case (i) $m > 0 : U^{(+)}_{\xi_2(0)}$ ——, (ii) $m < 0 : U^{(-)}_{\xi_2(0)}$ - - - - .

If $s_1 = s_2 = 1$ (both discs rotate in the same directions with equal angular velocity), eqns. (3.7) and (3.8) show that there is no recirculation for any T in both the cases of radial outflow and inflow as it is almost a solid rotation. For other values of s_1 and s_2 recirculation exists. If $s_1 = 1$, $s_2 = 0$ (upper disc is at rest and lower disc rotates), our results are in good agreement with those obtained by Sharma and Sharma (1965) and when $s_1 = 0$, $s_2 = 1$ (lower disc is at rest and upper disc rotates), the results are same with those obtained by Sharma and Gupta (1980).

The transverse shearing stress at the upper disc is given by

$$(t_{\theta z})_{\zeta=1} = \mu_1\Omega\xi\left[s_2 - s_1 + \frac{R_s^2}{6300}(8s_1^3 + 27s_2^3 - 24s_1^2 s_2 - 11s_2^2 s_1)\right] + \frac{\mu_1\Omega R_m}{2\xi}\left[\frac{2s_1 + 3s_2}{5} + 4T(s_2 - s_1)\right] \quad ...(4.2)$$

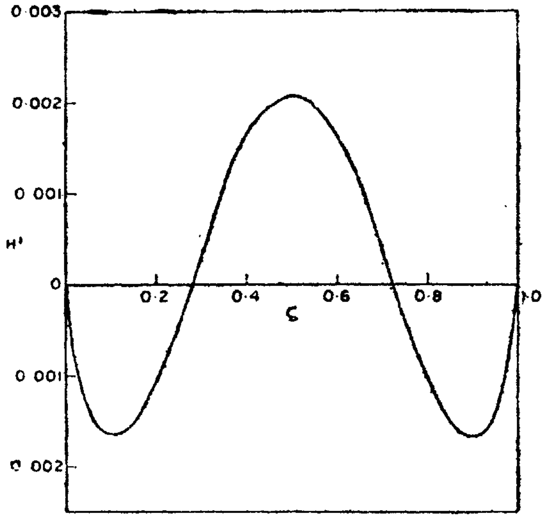

FIG. 3. Variation of H' for all T

so that the moment on one side

$$M^* = \int_0^{\xi_s} 2\pi\xi^2 \, (t_{\theta z})_{\zeta=1} \, d\xi$$

$$= \frac{\mu_1\Omega\pi\xi_s^4}{2}\left[s_2 - s_1 + \frac{R_z^2}{6300}\left(8s_1^3 + 27s_2^3 - 24s_1^2s_2 - 11s_2^2s_1\right)\right]$$

$$+ \frac{\mu_1\Omega\pi R_m\xi_s^2}{2}\left[\frac{2s_1 + 3s_2}{5} + 4T(s_2 - s_1)\right]. \qquad \ldots(4.3)$$

The expression (4.3) shows that an increase in second-order effects increases the moment on the upper disc.

The radial pressure variation between any radii ξ and ξ_0 at lower disc is obtained from (2.14) and (2.15) in the following form:

$$P - P_0 = \frac{\xi^2 - \xi_0^2}{20}\left[(3s_1^2 + 3s_2^2 + 4s_1s_2) - 10T_2(s_2 - s_1)^2\right]$$

$$- \frac{R_m}{R_z^2}\log\left(\frac{\xi}{\xi_0}\right)\left[6 + \frac{R_z^2}{4200}(263s_1^2 + 263s_2^2 + 494s_1s_2)\right.$$

$$\left. + \frac{2TR_z^2}{175}(s_1^2 + s_2^2 - 2s_1s_2) - \frac{8T^2R_z^2}{5}(s_2 - s_1)^2\right]. \qquad \ldots(4.4)$$

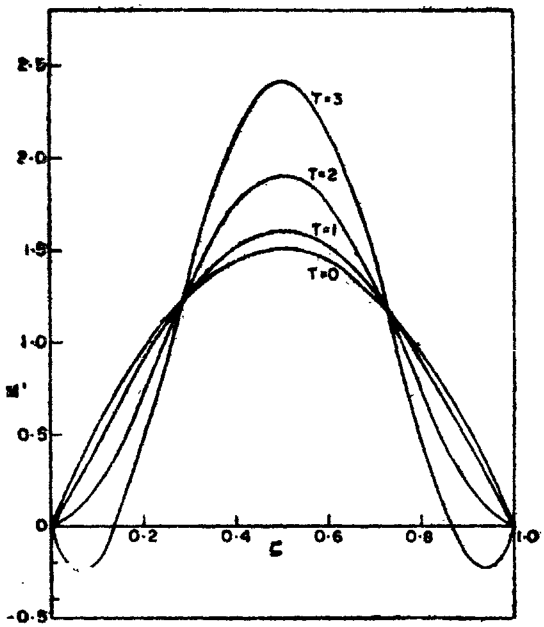

Fig. 4. Variation of M' with T

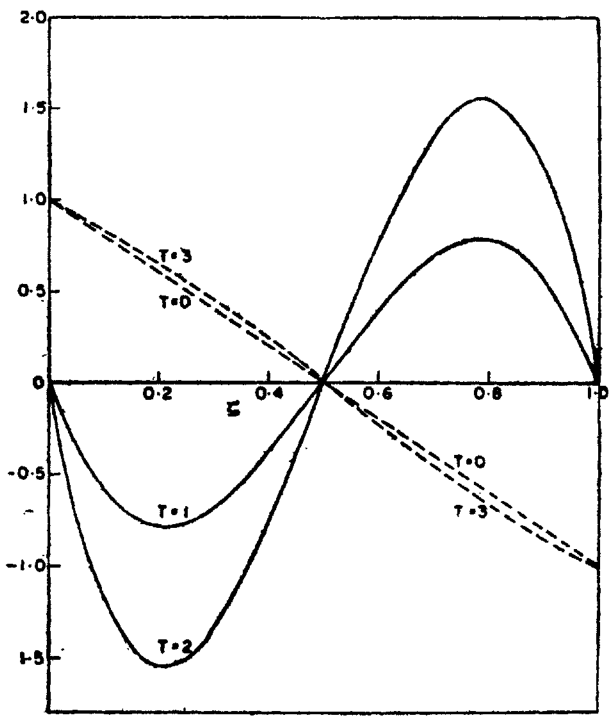

Fig. 5. Variation of G with T - - - - and of L with T ——

The expression (4.4) for the case $s_1 = 1$ and $s_2 = -1$ is given by

$$= \frac{\xi^2 - \xi_0^2}{10}(1 - 20T_2) - \frac{2R_m}{R_z^2}\log\left(\frac{\xi}{\xi_0}\right)\left[3 + \frac{2R_z^2}{525}(1 + 6T - 840T^2)\right] \quad \ldots(4.5)$$

where P_0 is the pressure at $\xi = \xi_0$.

The average normal force on the lower disc upto a radius ξ_s is therefore

$$\frac{1}{\pi\xi_s^2}\int_0^{\xi_s} 2\pi\xi(t_{zz})_{\zeta=0}\, d\xi$$

$$= \rho\Omega^2 z_0^2\Big[-P_0 + 4T(T + T_1) R_m(s_2 - s_1)^2 - \frac{2R_m}{5}(2s_2^2 - 3s_1^2$$

$$+ s_1 s_2)(T + T_1) + \frac{\xi_s^2}{2}(s_2 - s_1)^2(T + T_1) - \frac{1}{40}(\xi_s^2 - 2\xi_0^2)(3s_1^2$$

$$+ 3s_2^2 + 4s_1 s_2) + \frac{T_2}{4}(\xi_s^2 - 2\xi_0^2)(s_2 - s_1)^2\Big]$$

$$+ \frac{\rho\Omega^2 z_0^2 R_m}{R_z^2}\left(\log\frac{\xi}{\xi_0} - \frac{1}{2}\right)\Big[6 + \frac{R_z^2}{4200}(263s_1^2 + 263s_2^2 + 494s_1 s_2)$$

$$+ \frac{2TR_z^2}{175}(s_2 - s_1)^2 - \frac{8T^2R_z^2}{5}(s_2 - s_1)^2\Big]. \quad \ldots(4.6)$$

The expression (4.6) for the case $s_1 = 1$ and $s_2 = -1$ is given by

$$\rho\Omega^2 z_0^2\Big[-P_0 + 4R_m(T + T_1)\left(4T + \frac{1}{5}\right) + 2\xi_s^2(T + T_1)$$

$$- \frac{1}{20}(\xi_s^2 - 2\xi_0^2)(1 - 20T_2)\Big] + \frac{\Omega^2 z_0^2 R_m}{R_z^2}\left(\log\frac{\xi}{\xi_0} - \frac{1}{2}\right)$$

$$\times\left[6 + R_z^2\left(\frac{4}{525} + \frac{8T}{175} - \frac{32T^2}{5}\right)\right]. \quad \ldots(4.7)$$

Thus the concerned disc experiences suction or thrust according as the above expression (4.6) or (4.7) is negative or positive.

Putting $T_1 = T_2 = 0$ (then T also equal to zero) in the above result, the average normal force in the Newtonian case (with flow rate $m = 0$) becomes

$$-\rho\Omega^2 z_0^2 \left[P_0 + \frac{1}{20} \quad \xi_s^2 \right]. \qquad \ldots(4.8)$$

The expression (4.8) is always negative which implies the well known result that the disc experiences suction in the Newtonian case.

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