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SLOW UNSTEADY FLOW OF A VISCOUS INCOMPRESSIBLE FLUID BETWEEN TWO COAXIAL CIRCULAR CYLINDERS WITH AXIAL ROUGHNESS

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Problem of slow unsteady flow of a viscous incompressible fluid between two coaxial circular cylinders with axial roughness, is discussed. The roughness is assumed to be small and the technique of Fourier sine transform is used to solve the equations. Axial and radial velocities are shown graphically for a particular case of sinusoidal roughness.

1. Introduction

Flow of a viscous incompressible fluid between coaxial circular cylinders with exponential pressure gradient has been discussed by Verma (1960) while Hepworth and Rice (1967, 1970) have studied the flow between parallel plates and circular/rectangular tubes with arbitrary time varying pressure gradient. Citron (1962) has discussed the slow viscous flow between two rotating concentric infinite cylinders with axial roughness, while Khamrui (1963) has obtained the expressions for the velocity components and the pressure in case of the slow steady flow of a viscous fluid through a circular tube with axial roughness. Purohit (1965) has extended Khamrui's problem to that of coaxial circular cylinders. Verma and Gaur (1970) have studied the slow unsteady flow through a circular tube with axial roughness.

In this paper, the solution of slow unsteady flow of a viscous incompressible fluid between two coaxial circular cylinders with axial roughness, under the assumption that the roughness is small compared to smooth radii (without roughness) of the cylinders, has been investigated. The technique of Fourier sine transform has been used to obtain the solutions. The radial and axial velocities and pressure have been obtained in terms of modified Bessel functions. For a particular case of sinusoidal roughness, the velocities have been calculated numerically and shown graphically.

2. Problem Formulation

In cylindrical polar coordinates, the Navier-Stokes equations in the non-dimensional form, under the slow flow assumption, are

$$\frac{\partial U}{\partial T} = -\frac{\partial P}{\partial \lambda} + \frac{\partial^2 U}{\partial \lambda^2} + \frac{1}{\lambda}\frac{\partial U}{\partial \lambda} + \frac{\partial^2 U}{\partial Z^2} - \frac{U}{\lambda^2} \qquad ...(2.1)$$

$$\frac{\partial W}{\partial T} = -\frac{\partial P}{\partial Z} + \frac{\partial^2 W}{\partial \lambda^2} + \frac{1}{\lambda}\frac{\partial W}{\partial \lambda} + \frac{\partial^2 W}{\partial Z^2} \qquad ...(2.2)$$

and
$$\frac{1}{\lambda}\frac{\partial (U\lambda)}{\partial \lambda} + \frac{\partial W}{\partial Z} = 0 \qquad ...(2.3)$$

where U and W are the velocities in the radial (λ) and axial (Z) directions, P the pressure and T the time.

In slow flow

$$\nabla^2 P = 0 \qquad ...(2.4)$$

hence (2.2) gives

$$\left[\frac{\partial^2}{\partial \lambda^2} + \frac{1}{\lambda}\frac{\partial}{\partial \lambda} + \frac{\partial^2}{\partial Z^2}\right]^2 W = \frac{\partial}{\partial T}\left[\frac{\partial^2}{\partial \lambda^2} + \frac{1}{\lambda}\frac{\partial}{\partial \lambda} + \frac{\partial^2}{\partial Z^2}\right] W. \qquad ...(2.5)$$

The boundary conditions are

$$\left.\begin{aligned} &U = 0 = W \text{ at } \lambda = 1 + \epsilon\, N_1(Z) \\ \text{and} \quad &\lambda = \sigma + \epsilon\, N_2(Z),\ T > 0 \end{aligned}\right\} \qquad ...(2.6)$$

where $\epsilon \ll 1$ is the roughness parameter, $N_1(Z)$ and $N_2(Z)$ are arbitrary functions of Z, and σ is the ratio of the radius of the outer cylinder to that of the inner cylinder.

Let

$$P(\lambda, Z, T) = P_0(Z, T) + P_1(\lambda, Z, T) \qquad ...(2.7)$$

$$U(\lambda, Z, T) = U_1(\lambda, Z, T) \qquad ...(2.8)$$

and
$$W(\lambda, Z, T) = W_0(\lambda, T) + W_1(\lambda, Z, T) \qquad ...(2.9)$$

where P_1, U_1 and W_1 are the variations caused by the roughness and P_0 and W_0 are the known quantities for the coaxial cylinders without roughness, given by

$$\frac{\partial P_0}{\partial \lambda} = 0 \qquad ...(2.10)$$

and
$$\frac{\partial W_0}{\partial T} = -\frac{\partial P_0}{\partial Z} + \frac{\partial^2 W_0}{\partial \lambda^2} + \frac{1}{\lambda}\frac{\partial W_0}{\partial \lambda}. \qquad ...(2.11)$$

Let the applied pressure gradient be harmonic and be given by

$$-\frac{\partial P_0}{\partial Z} = K \cos nT = \text{Real}\,[K\,.\,e^{inT}] \qquad ...(2.12)$$

where K is a constant, $i = \sqrt{-1}$ and let us suppose

$$W_0 = f(\lambda) \cos nT = \text{Real}\,[f(\lambda)\, e^{inT}]. \qquad ...(2.13)$$

Using (2.12) and (2.13), (2.11) reduces to

$$f''(\lambda) + \frac{1}{\lambda} f'(\lambda) - inf(\lambda) = -K \qquad \ldots(2.14)$$

under the boundary conditions $f(1) = f(\sigma) = 0$.

For very slow oscillations the solution of (2.14) gives

$$W_0(\lambda, T) = \frac{K}{4}\left[\frac{(\sigma^2 - 1)\log\lambda - (\lambda^2 - 1)\log\sigma}{\log\sigma}\right]\cos nT$$

$$= \text{Real}\left[\frac{K}{4}\left\{\frac{(\sigma^2 - 1)\log\lambda - (\lambda^2 - 1)\log\sigma}{\log\sigma}\right\} e^{inT}\right]. \qquad \ldots(2.15)$$

Using (2.7) to (2.9), (2.1) to (2.3) reduce to

$$\frac{\partial U_1}{\partial T} = -\frac{\partial P_1}{\partial\lambda} + \frac{\partial^2 U_1}{\partial\lambda^2} + \frac{1}{\lambda}\frac{\partial U_1}{\partial\lambda} + \frac{\partial^2 U_1}{\partial Z^2} - \frac{U_1}{\lambda^2} \qquad \ldots(2.16)$$

$$\frac{\partial W_1}{\partial T} = -\frac{\partial P_1}{\partial Z} + \frac{\partial^2 W_1}{\partial\lambda^2} + \frac{1}{\lambda}\frac{\partial W_1}{\partial\lambda} + \frac{\partial^2 W_1}{\partial Z^2} \qquad \ldots(2.17)$$

and

$$\frac{1}{\lambda}\frac{\partial(U_1\lambda)}{\partial\lambda} + \frac{\partial W_1}{\partial Z} = 0. \qquad \ldots(2.18)$$

Differentiating (2.18) with respect to λ and using (2.16), we get

$$\frac{\partial U_1}{\partial T} = -\frac{\partial P_1}{\partial\lambda} + \frac{\partial^2 U_1}{\partial Z^2} - \frac{\partial^2 W_1}{\partial\lambda\partial Z}. \qquad \ldots(2.19)$$

Now, eqn. (2.5) becomes

$$\left[\frac{\partial^2}{\partial\lambda^2} + \frac{1}{\lambda}\frac{\partial}{\partial\lambda} + \frac{\partial^2}{\partial Z^2}\right]^2 W_1 = \frac{\partial}{\partial T}\left(\frac{\partial^2}{\partial\lambda^2} + \frac{1}{\lambda}\frac{\partial}{\partial\lambda} + \frac{\partial^2}{\partial Z^2}\right) W_1 \qquad \ldots(2.20)$$

under the boundary conditions

$$\left.\begin{array}{l} U_1 = 0,\ W_1 = -W_0 \text{ at } \lambda = 1 + \epsilon N_1(Z) \\ \lambda = \sigma + \epsilon N_2(Z),\ T > 0,\ Z > 0 \\ \text{and} \quad W_1 = 0 \text{ when } \{1 + \epsilon N_1(Z)\} < \lambda < \{\sigma + \epsilon N_2(Z)\},\ T < 0,\ Z = 0. \end{array}\right\} \qquad \ldots(2.21)$$

3. Method of Solution

Following (2.13), we assume

$$W_1(\lambda, Z, T) = W_1(\lambda, Z)\cos nT = \text{Re}\,[W_1(\lambda, Z)\, e^{inT}]. \qquad \ldots(3.1)$$

Equation (2.20) now reduces to

$$\frac{\partial^2 f}{\partial \lambda^2} + \frac{1}{\lambda}\frac{\partial f}{\partial \lambda} + \frac{\partial^2 f}{\partial Z^2} = inf \qquad \text{...(3.2)}$$

where $$f(\lambda, Z) = \frac{\partial^2 W_1}{\partial \lambda^2} + \frac{1}{\lambda}\frac{\partial W_1}{\partial \lambda} + \frac{\partial^2 W_1}{\partial Z^2}. \qquad \text{...(3.3)}$$

We introduce

$$F(\lambda, \xi) = \sqrt{\frac{2}{\pi}} \int_0^\infty f(\lambda, Z) \sin(\xi Z)\, dZ \qquad \text{...(3.4)}$$

where $F(\lambda, \xi)$ is Fourier sine transform of $f(\lambda, Z)$.

Taking Fourier sine transform of (3.2), we get

$$\frac{d^2 F}{d\lambda^2} + \frac{1}{\lambda}\frac{dF}{d\lambda} - (\xi^2 + in) F = 0 \qquad \text{...(3.5)}$$

the solution of which is

$$F(\lambda, \xi) = C_1(\xi)\, I_0(m\lambda) + D_1(\xi)\, K_0(m\lambda) \qquad \text{...(3.6)}$$

where $m = (\xi^2 + in)^{1/2}$ and $C_1(\xi)$ and $D_1(\xi)$ are constants of integration. Taking Fourier sine transform of (3.3) and using (3.6), we get

$$\frac{d^2 \bar{W}}{d\lambda^2} + \frac{1}{\lambda}\frac{d\bar{W}}{d\lambda} - \xi^2 \bar{W} = C_1(\xi)\, I_0(m\lambda) + D_1(\xi)\, K_0(m\lambda) \qquad \text{...(3.7)}$$

where $\bar{W}$ is the Fourier sine transform of W_1.

The solution of (3.7) is given by

$$\bar{W}(\lambda, \xi) = A(\xi)\, I_0(\xi\lambda) + B(\xi)\, K_0(\xi\lambda) + C(\xi)\, I_0(m\lambda) + D(\xi)\, K_0(m\lambda) \qquad \text{...(3.8)}$$

where $A(\xi)$, $B(\xi)$, $C(\xi)$ and $D(\xi)$ are constants of integration.

Taking inverse Fourier sine transform of $\bar{W}$, we get

$$W_1(\lambda, Z, T) = \text{Re}\left[\sqrt{\frac{2}{\pi}} \int_0^\infty [A(\xi)\, I_0(\xi\lambda) + B(\xi)\, K_0(\xi\lambda) + C(\xi)\, I_0(m\lambda) + D(\xi)\, K_0(m\lambda)] \sin(\xi Z)\, e^{inT}\, d\xi\right]. \qquad \text{...(3.9)}$$

Using (3.9), eqn. (2.18) gives

$$U_1(\lambda, Z, T) = \mathrm{Re}\Big[-\sqrt{\frac{2}{\pi}} \int_0^\infty \Big[A(\xi)\, I_1(\xi\lambda) - B(\xi)\, K_1(\xi\lambda) + \frac{\xi}{m} C(\xi)\, I_1(m\lambda) - \frac{\xi}{m} D(\xi)\, K_1(m\lambda) \Big] \times \cos(\xi Z)\, e^{inT}\, d\xi \Big]. \quad \ldots(3.10)$$

Using (3.9) and (3.10), the equation (2.17) and (2.19) give

$$P_1(\lambda, Z, T) = \mathrm{Re}\Big[\sqrt{\frac{2}{\pi}}\, in \int_0^\infty \frac{1}{\xi} [A(\xi)\, I_0(\xi\lambda) + B(\xi)\, K_0(\xi\lambda)] \times \cos(\xi Z)\, e^{inT}\, d\xi \Big] + C \quad \ldots(3.11)$$

where C is the constant of integration.

We assume

$$A(\xi) = A_0(\xi) + \epsilon\, A_1(\xi) + \ldots$$

and similar expressions for $B(\xi)$, $C(\xi)$ and $D(\xi)$. ...(3.12)

Using (2.21) and (3.12) in (3.9) and (3.10), we equate the coefficients of like powers of ϵ^0 and ϵ. Inverting the resulting equations by Fourier sine and consine integral theorems and solving, we get

$$A_0(Z) = B_0(Z) = C_0(Z) = D_0(Z) = 0 \quad \ldots(3.13)$$

and

$$\left.\begin{aligned} A_1(Z) &= \tfrac{1}{4} K\,[\Delta A_1(Z)\, \bar{N}_1(Z) - \Delta A_2(Z)\, \bar{N}_2(Z)] \\ B_1(Z) &= \tfrac{1}{4} K\,[-\Delta B_1(Z)\, \bar{N}_1(Z) + \Delta B_2(Z)\, \bar{N}_2(Z)] \\ C_1(Z) &= \tfrac{1}{4} K\,[\Delta C_1(Z)\, \bar{N}_1(Z) - \Delta C_2(Z)\, \bar{N}_2(Z)] \\ D_1(Z) &= \tfrac{1}{4} K\,[-\Delta D_1(Z)\, \bar{N}_1(Z) + \Delta D_2(Z)\, \bar{N}_2(Z)] \end{aligned}\right\} \quad \ldots(3.14)$$

where $\Delta A_1(Z)$, $\Delta B_1(Z)$, $\Delta C_1(Z)$ and $\Delta D_1(Z)$ are respectively the minors of $I_0(Z)$, $K_0(Z)$, $I_0(m')$ and $K_0(m')$ in Δ, each multiplied by X/Δ. Similarly $\Delta A_2(Z)$, $\Delta B_2(Z)$, $\Delta C_2(Z)$ and $\Delta D_2(Z)$ are respectively the minors of $I_0(\sigma Z)$, $K_0(\sigma Z)$, $I_0(\sigma m')$ and $K_0(\sigma m')$ in Δ, each multiplied by Y/Δ, where

$$X = \left[2 - \frac{\sigma^2 - 1}{\log \sigma} \right], \quad Y = \left[2\sigma - \frac{\sigma^2 - 1}{\sigma \log \sigma} \right],$$

$$\Delta = \begin{vmatrix} I_0(Z) & K_0(Z) & I_0(m') & K_0(m') \\ I_0(\sigma Z) & K_0(\sigma Z) & I_0(\sigma m') & K_0(\sigma m') \\ I_1(Z) & -K_1(Z) & \frac{Z}{m'} I_1(m') & -\frac{Z}{m'} K_1(m') \\ I_1(\sigma Z) & -K_1(\sigma Z) & \frac{Z}{m'} I_1(\sigma m') & -\frac{Z}{m'} K_1(\sigma m') \end{vmatrix}$$

and $m' = (Z^2 + in)^{1/2}$. $\bar{N}_1$ and $\bar{N}_2$ are Fourier sine transforms of N_1 and N_2 respectively.

Using (3.13) and (3.14) the complete expressions for the axial velocity, radial velocity and pressure are

$$\begin{aligned} W(\lambda, Z, T) &= W_0(\lambda, T) + W_1(\lambda, Z, T) \\ &= \frac{K}{4}\left[(\sigma^2 - 1) \frac{\log \lambda}{\log \sigma} - \lambda^2 + 1 \right] \cos nT + \frac{K}{4} \epsilon \sqrt{\frac{2}{\pi}} \\ &\quad \times \mathrm{Re} \int_0^\infty [\{\Delta A_1(\xi) I_0(\xi\lambda) - \Delta B_1(\xi) K_0(\xi\lambda) \\ &\quad + \Delta C_1(\xi) I_0(m\lambda) - \Delta D_1(\xi) K_0(m\lambda)\} \bar{N}_1(\xi) \\ &\quad - \{\Delta A_2(\xi) I_0(\xi\lambda) - \Delta B_2(\xi) K_0(\xi\lambda) + \Delta C_2(\xi) I_0(m\lambda) \\ &\quad - \Delta D_2(\xi) K_0(m\lambda)\} \bar{N}_2(\xi)] \sin (\xi Z) e^{inT} d\xi \end{aligned} \qquad \ldots(3.15)$$

$$\begin{aligned} U(\lambda, Z, T) &= U_1(\lambda, Z, T) \\ &= -\frac{K}{4} \epsilon \sqrt{\frac{2}{\pi}} \mathrm{Re} \int_0^\infty \Big[\Big\{ \Delta A_1(\xi) I_1(\xi\lambda) + \Delta B_1(\xi) K_1(\xi\lambda) \\ &\quad + \frac{\xi}{m} [\Delta C_1(\xi) I_1(m\lambda) + \Delta D_1(\xi) K_1(m\lambda)] \Big\} \bar{N}_1(\xi) \\ &\quad - \Big\{ \Delta A_2(\xi) I_1(\xi\lambda) + \Delta B_2(\xi) K_1(\xi\lambda) \\ &\quad + \frac{\xi}{m} [\Delta C_2(\xi) I_1(m\lambda) + \Delta D_2(\xi) K_1(m\lambda)] \Big\} \bar{N}_2(\xi) \Big] \\ &\quad \times \cos (\xi Z) e^{inT} d\xi \end{aligned} \qquad (3.16)$$

and $$P(\lambda, Z, T) = P_0(Z, T) + P_1(\lambda, Z, T) = C - KZ \cos nT + \frac{K}{4} \epsilon \sqrt{\frac{2}{\pi}} \times$$

(*equation continued on p.* 1166)

$$\times \operatorname{Re} \int_0^\infty in \, \frac{1}{\xi} [\{\Delta A_1(\xi)\, I_0(\xi\lambda) - \Delta B_1(\xi)\, K_0(\xi\lambda)\} \bar{N}_1(\xi)$$

$$- \{\Delta A_2(\xi)\, I_0(\xi\lambda) - \Delta B_2(\xi)\, K_0(\xi\lambda)\} \bar{N}_2(\xi)] \cos(\xi Z)\, e^{inT} d\xi \quad \ldots(3.17)$$

where C denotes a constant.

4. Particular Case (Sinusoidal Roughness)

For this case we take the roughness function at the walls

$$N_1(Z) = N_2(Z) = \sin \left\{\frac{Z}{l}\right\} \quad \ldots(4.1)$$

where $2\pi l$ is the wavelength of roughness at both walls.

We may formally write

$$\bar{N}_1(\xi) = \bar{N}_2(\xi) = \sqrt{\frac{\pi}{2}}\, \delta\left(\xi - \frac{1}{l}\right) \quad \ldots(4.2)$$

where $\bar{N}_1(\xi)$ and $\bar{N}_2(\xi)$ are Fourier sine transforms of $N_1(Z)$ and $N_2(Z)$ respectively and δ is the Dirac delta function.

Substituting $\bar{N}_1(\xi)$ and $\bar{N}_2(\xi)$ from (4.2) in (3.15) to (3.17) and using the property of Dirac delta function (Sneddon 1951), we get

$$W(\lambda, Z, T) = \frac{K}{4}\left[(\sigma^2 - 1)\frac{\log \lambda}{\log \sigma} - \lambda^2 + 1\right] \cos nT$$

$$+ \frac{K}{4}\, \epsilon \operatorname{Re}\left[\left\{\Delta A_1\left(\frac{1}{l}\right) - \Delta A_2\left(\frac{1}{l}\right)\right\} I_0\left(\frac{\lambda}{l}\right)\right.$$

$$- \left\{\Delta B_1\left(\frac{1}{l}\right) - \Delta B_2\left(\frac{1}{l}\right)\right\} K_0\left(\frac{\lambda}{l}\right) + \left\{\Delta C_1\left(\frac{1}{l}\right)\right.$$

$$\left. - \Delta C_2\left(\frac{1}{l}\right)\right\} I_0\left(\frac{\lambda}{l'}\right) - \left\{\Delta D_1\left(\frac{1}{l}\right)\right.$$

$$\left.\left. - \Delta D_2\left(\frac{1}{l}\right)\right\} K_0\left(\frac{\lambda}{l'}\right)\right] \sin(Z/l)\, e^{inT}$$

$$U(\lambda, Z, T) = -\frac{K}{4}\, \epsilon \operatorname{Re}\left[\left\{\Delta A_1\left(\frac{1}{l}\right) - \Delta A_2\left(\frac{1}{l}\right)\right\} I_1\left(\frac{\lambda}{l}\right) + \right.$$

(equation continued on p. 1167)

$$+\left\{\Delta B_1\left(\frac{1}{l}\right)-\Delta B_2\left(\frac{1}{l}\right)\right\}K_1\left(\frac{\lambda}{l}\right)$$
$$+\frac{l'}{l}\left\{\Delta C_1\left(\frac{1}{l}\right)-\Delta C_2\left(\frac{1}{l}\right)\right\}I_1\left(\frac{\lambda}{l'}\right)$$
$$+\frac{l'}{l}\left\{\Delta D_1\left(\frac{1}{l}\right)-\Delta D_2\left(\frac{1}{l}\right)\right\}K_1\left(\frac{\lambda}{l'}\right)\Bigg]$$
$$\times\cos(Z/l)\,e^{inT}$$

and
$$P(\lambda, Z, T) = C - KZ\cos nT + \frac{K}{4}\epsilon\,\mathrm{Re}\Bigg[\,inl\,\Bigg[\left\{\Delta A_1\left(\frac{1}{l}\right)\right.$$
$$\left.-\Delta A_2\left(\frac{1}{l}\right)\right\}I_0\left(\frac{\lambda}{l}\right)-\left\{\Delta B_1\left(\frac{1}{l}\right)-\Delta B_2\left(\frac{1}{l}\right)\right\}$$
$$\times K_0\left(\frac{\lambda}{l}\right)\Bigg]\cos(Z/l)\,e^{inT}\Bigg]$$

where
$$\frac{1}{l'} = \left(\frac{1}{l^2} + in\right)^{1/2}.$$

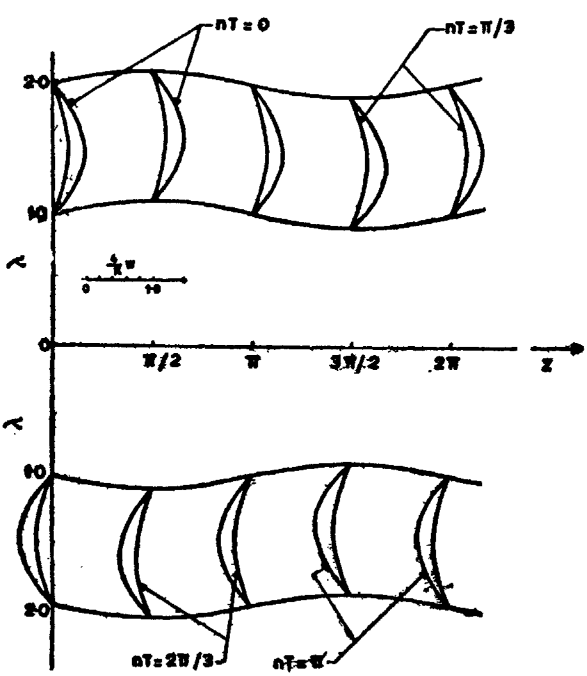

FIG. 1. The axial velocity profiles at different sections of a roughness wave for $\sigma = 2$, $\epsilon = 0.1$ and $nT = 0, \pi/3, 2\pi/3$ and π.

The values of $A_1(Z)$, $B_1(Z)$, $C_1(Z)$, $D_1(Z)$, for $Z = 1$, $\sigma = 2$, $l = 1$ are calculated and are given by

$$A_1(1) = \tfrac{1}{4}K\left[-2.22145 + \frac{i}{n} 2.46058 \right]$$

$$B_1(1) = \tfrac{1}{4}K\left[6.02216 - \frac{i}{n} 6.48534 \right]$$

$$C_1(1) = \tfrac{1}{4}K\left[1.37516 - \frac{i}{n} 2.46058 \right]$$

and

$$D_1(1) = \tfrac{1}{4}K\left[-15.29324 + \frac{i}{n} 6.48534 \right].$$

5. Numerical Discussions

Axial velocity profiles for particular values of $\sigma = 2$, $l = 1$ and for $\epsilon = 0.1$ and $\epsilon = 0.2$ have been shown in Figs. 1 and 2 at different cross-sections of the tube

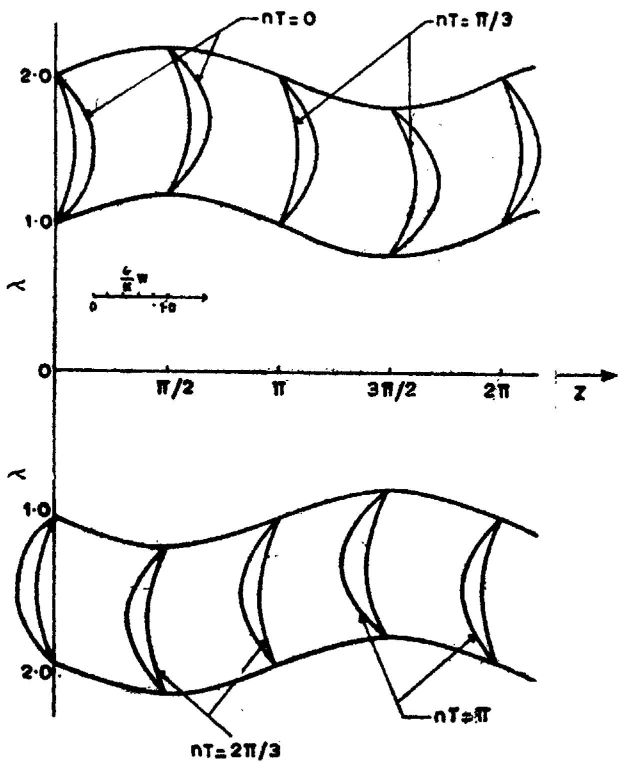

Fig. 2. The axial velocity profiles at different sections of a roughness wave for $\sigma = 2$, $\epsilon = 0.2$ and $nT = 0, \pi/3, 2\pi/4$ and π.

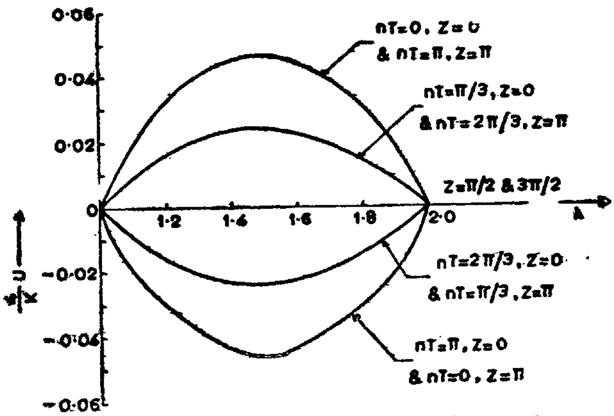

FIG. 3. The radial velocity profiles of a roughness wave for $\sigma = 2$, $\varepsilon = 0.1$ and $nT = 0, \pi/3, 2\pi/3$ and π, at different sections $Z = 0, \pi/2, \pi$ and $3\pi/2$.

for various values of nT. In Fig. 1, axial velocity profiles for $nT = 0$ and $\pi/3$ have been shown in the upper half of the section of annulus which would be the same in the lower half also. For $nT = 2\pi/3$ and $nT = \pi$, these have been shown in the lower half, which would be the same in the upper half also. Figure 2 shows the axial velocity profiles in the same fashion for $\epsilon = 0.2$. The increase in the roughness parameter results in the increase in the axial velocity at various cross-sections.

The radial velocity profiles for different values of nT and Z and for $\sigma = 2$, $l = 1$ and $\epsilon = 0.1$ have been depicted in Fig. 3. For all values of nT the radial velocity remains zero at the sections $Z = \pi/2$ and $Z = 3\pi/2$. From the analysis it is clear that the radial velocity is directly proportional to the roughness parameter.

Combining the effects of the two velocities it can be noted that for forward pressure gradient, the fluid presses the outer wall when $0 < Z < \pi/2$, inner wall when $\pi/2 < Z < 3\pi/2$ and again the outerwall for $3\pi/2 < Z < 2\pi$. For an annulus without wall roughness, there is only one component of velocity (axial). Roughness of the walls, therefore, gives rise to the radial velocity because of which the direction of the fluid flow becomes towards the outer/inner wall.

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