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INTERACTION OF WALL POROSITY AND HALL EFFECTS IN THE HYDROMAGNETIC FREE AND FORCED CONVECTION FLOW THROUGH A CHANNEL

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The interactions of wall porosity, free convection and Hall effects in the flow between two infinite parallel porous walls have been considered. The presence of a transverse magnetic field, a pressure gradient and linearly varying wall temperature with the distance along the pressure gradient have been assumed. Closed form solutions for the velocity, temperature function and induced magnetic field have been obtained. Critical values of the Grashof number for reversal of the main and cross-flow velocities are found. It is seen that flow functions are odd functions of the porosity and free convection parameters, on which flow reversal depends.

1. INTRODUCTION

Free convective flow past a semi-infinite hot vertical plate in the presence of a magnetic field was considered among others by Gupta (1960), Cramer (1962) and Riley (1964). While Gupta (1969) investigated the free and forced convective flow of an electrically conducting fluid in the presence of a transverse magnetic field assuming axial temperature variation along a wall, Majumder *et al.* (1976) extended Gupta's study by considering the effects of Hall current on the flow and heat transfer. Since the effects of such a study finds application in cooling of nuclear reactors, we assume the walls to be porous and investigate the possible effects of liquid suction or injection through the walls on the flow and heat transfer characteristics.

2. FORMULATION OF THE PROBLEM

We consider an electrically conducting liquid flowing between two horizontal porous walls—distance $2L$ apart—such that x and y axes are along and transverse to the walls, the origin being midway between them. It is assumed that a strong uniform magnetic field H_0 acts transverse to the walls and the flow is taking place under a uniform axial pressure gradient so that $\partial p/\partial x$ is constant. The fully developed steady state flow at a large distance from the entrance region will obviously have all its

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physical variables except pressure, dependent on y alone, so that $\nabla \cdot \vec{H} = 0$, $\nabla \cdot \vec{q} = 0$ where $\vec{H} = (H_x, H_y, H_z)$, $\vec{q} = (u, v, w)$, lead to $H_y = \text{const.} = H_0$ and $v = \text{const} = v_0$ respectively.

The equations governing the motion are

$$\rho_0 v_0 \frac{du}{dy} = -\frac{\partial p}{\partial x} + \mu \frac{d^2 u}{dy^2} + \mu_e H_0 \frac{dH_x}{dy} \qquad \ldots(1)$$

$$0 = -\frac{\partial p}{\partial y} - \rho g - \tfrac{1}{2}\mu_e \frac{d}{dy}(H_x^2 + H_z^2) \qquad \ldots(2)$$

$$\rho_0 v_0 \frac{dw}{dy} = \mu \frac{d^2 w}{dy^2} + \mu_e H_0 \frac{dH_z}{dy} \qquad \ldots(3)$$

where ρ_0 is the value of density ρ at a reference temperature T_0, μ the coefficient of viscosity and μ_e the magnetic permeability. $\partial p/\partial z$ is assumed zero, the motion being due to eddy current velocity. Maxwell's equations are $\nabla \times \vec{E} = 0, \nabla \times \vec{H} = \vec{J}$. Neglecting slip effects due to imperfect coupling between ions and neutrons and neglecting also the electron pressure gradient, Ohm's law can be written with the inclusion of Hall effects as (Cowling 1957) $\vec{J} + \frac{\omega\tau}{H_0} \vec{J} \times \vec{H} = \sigma(\vec{E} + \mu_e \vec{q} \times \vec{H})$, where $\vec{J}$, ω, τ and σ represent current density, electron Larmor frequency, electron collision and electrical conductivity respectively. From Maxwell's equations and Ohm's Law, the x and z components on elimination of E (since the y-components of the equations are identically satisfied) yield

$$\frac{d^2 H_z}{dy^2} + \omega\tau \frac{d^2 H_x}{dy^2} = \sigma\mu_e \left(v_0 \frac{dH_z}{dy} - H_0 \frac{dw}{dy} \right) \qquad \ldots(4)$$

and

$$-\frac{d^2 H_x}{dy^2} + \omega\tau \frac{d^2 H_z}{dy^2} = \sigma\mu_e \left(H_0 \frac{du}{dy} - v_0 \frac{dH_x}{dy} \right). \qquad \ldots(5)$$

Assuming a linear uniform axial temperature variation along the lower wall such as $T = T_0 + Nx$, where N is a constant, the temperature of the fluid can be written $T - T_0 = Nx + \phi(y)$. This, along with the equation of state $\rho = \rho_0 [1 - \beta(T - T_0)]$, β being the coefficient of volume expansion, can be inserted into (2) so that the latter on integration gives

$$p = -\rho_0 g y + \rho_0 g \beta N x y + \rho g \beta \int \phi(y)\, dy - \tfrac{1}{2}\mu_e (H_x^2 + H_z^2) + F(x). \qquad \ldots(6)$$

We now non-dimensionalise the quantities through the following:

$$\left.\begin{aligned} &\eta = \frac{y}{L}, \quad \bar{u} = \frac{uL}{\nu P_x}, \quad \frac{\partial p}{\partial x} = P_x = \frac{-L^3}{\rho_0 \nu^2}\frac{dF}{dx}, \quad \bar{w} = \frac{wL}{\nu P_x} \\ &\bar{H}_x = \frac{H_x}{\sigma \mu_e H_0 \nu P_x}, \quad \bar{H}_z = \frac{H_z}{\sigma \mu_e H_0 \nu P_x}, \quad M^2 = \mu_e^2 H_0^2 L^2 \sigma/\rho_0 \nu \\ &G = \beta g N L^4/\nu P_x, \quad R = \frac{v_0 L}{\nu}. \end{aligned}\right\} \qquad \ldots(7)$$

Introducing (6) and (7) in eqns. (1) and (2) and omitting the bars for convenience, we obtain

$$\frac{d^2u}{d\eta^2} - R\frac{du}{d\eta} + M^2\frac{dH_x}{d\eta} - G\eta = -1 \qquad \ldots(8)$$

$$\frac{d^2w}{d\eta^2} - R\frac{dw}{d\eta} + M^2\frac{dH_z}{d\eta} = 0. \qquad \ldots(9)$$

Equations (4), (5) and (8), (9) together yield

$$\frac{d^2U}{d\eta^2} - R\frac{dU}{d\eta} + M^2\frac{dh}{d\eta} - G\eta = -1 \qquad \ldots(10)$$

$$\frac{d^2h}{d\eta^2} = \frac{RP_m}{1+i\omega\tau}\frac{dh}{d\eta} - \frac{1}{1+i\omega\tau}\frac{dU}{d\eta} \qquad \ldots(11)$$

where $U = u + iw$, $h = H_x + iH_z$ and $P_m = \sigma\mu_e\nu$.

Elimination of h between (10) and (11) gives

$$\frac{d^3U}{d\eta^3} - R\left[\frac{P_m}{1+i\omega\tau} + 1\right]\frac{d^2U}{d\eta^2} + \left[R^2\frac{P_m}{1+i\omega\tau} - \frac{M^2}{1+i\omega\tau}\right]\frac{dU}{d\eta}$$

$$+ \frac{GRP_m}{1+i\omega\tau}\eta - \frac{RP_m}{1+i\omega\tau} - G = 0. \qquad \ldots(12)$$

Integrating (12), we have

$$\frac{d^2U}{d\eta^2} - R(1 + M_2^2)\frac{dU}{d\eta} + (R^2M_2^2 - M_1^2)\,U$$

$$= -\tfrac{1}{2}GRM_2^2\eta^2 + (RM_2^2 + G)\,\eta + C_1 \qquad \ldots(13)$$

where $M_1^2 = \dfrac{M^2}{1+i\omega\tau}$, $M_2^2 = \dfrac{P_m}{1+i\omega\tau}$ and $C_1 = \text{const.}$

Since there is no slip at the walls and the walls electrically nonconducting, we have

$$U(\pm 1) = 0, \; h(\pm 1) = 0. \qquad \ldots(14)$$

Further we can write

$$\frac{du}{d\eta} = F_1 G + F_2 \qquad \ldots(14a)$$

where $F_1 = F_1(\eta, M, P_m, R, \omega\tau)$ and $F_2 = F_2(\eta, M, P_m, R, \omega\tau)$.

Putting $G = 0$ in (14*a*) we have

$$F_2 = \left(\frac{du}{d\eta}\right)_{G=0}.$$

Putting $G = 1$ in (14*a*) we have

$$F_1 = \left(\frac{du}{d\eta}\right)_{G=1} - F_2.$$

So that

$$F_1 = \left(\frac{du}{d\eta}\right)_{G=1} - \left(\frac{du}{d\eta}\right)_{G=0}.$$

From (14*a*) when $du/d\eta = 0$, we can define a critical value of G for the reversal of the primary flow

$$G_{\text{crit}} = -\frac{F_2}{F_1}.$$

Since flow reversal is of consequences at the walls only, we obtain the two values of G_{crit} at $\eta = \pm 1$ from

$$(G_{\text{crit}})_{\eta=\pm 1} = -\left(\frac{F_2}{F_1}\right)_{\eta=\pm 1} = \left[\frac{\left(\frac{du}{d\eta}\right)_{G=0}}{\left(\frac{du}{d\eta}\right)_{G=0} - \left(\frac{du}{d\eta}\right)_{G=1}}\right]_{\eta=\pm 1}$$

Further the cross flow at both the plates has incipient flow reversal when

$$G = \pm G'_{\text{crit}} \qquad \ldots(14b)$$

for injection and suction respectively.

3. Energy Equation

The equation of energy including viscous and Ohmic dissipation is

$$u\frac{\partial T}{\partial x} + v_0 \frac{\partial T}{\partial y} = k\frac{\partial^2 T}{\partial y^2} + \frac{\mu}{\rho c}\left[\left(\frac{du}{dy}\right)^2 + \left(\frac{dw}{dy}\right)^2\right] + \frac{1}{\sigma\rho c}\left[\left(\frac{dH_x}{dy}\right)^2 + \left(\frac{dH_z}{dy}\right)^2\right] \qquad \ldots(15)$$

where k is the thermal diffusivity, c the specific heat and T temperature of the fluid. Using the expression for T as stated earlier and (13) we can write (15) in terms of dimensionless quantities as

$$\frac{d^2\theta}{d\eta^2} - RP\frac{d\theta}{d\eta} = Pu - k_1\left[\frac{dU}{d\eta}\frac{d\bar{U}}{d\eta} + S_1^2 \frac{dh}{d\eta}\frac{d\bar{h}}{d\eta}\right] \qquad \ldots(16)$$

where P is the Prandtl number, $k_1 = \nu^3 P_x/cKNL^3$, $\theta = \phi/NLP_x$, $S_1 = M(\rho/\rho_0)^{-1/2}$ while $\bar{U}$ and $\bar{h}$ are complex conjugates of U and h respectively.

Since the temperature at the lower wall is assumed to be $T_0 + Nx$, it is easy to see that

$$\left.\begin{aligned} &\theta(-1) = 0, \\ \text{and} \quad &\theta(1) = \phi(L)/NLP_x = N_1 = \text{wall temperature parameter.} \end{aligned}\right\} \qquad \ldots(17)$$

4. Solution of the Equations

Equation (13) is first integrated using the relevant part of the conditions (14) to give, U, which is used in eqn. (10) for its integration under the conditions (14) relating to h. The solutions for U and h are split into their respective real and imaginary parts for finding out the solutions for u, w, H_x and H_z. Further the values of U and h are used in (16) which is integrated under the boundary conditions (17). Want of space forbids us here either to describe calculation or to state the forms of the solutions for u, w, H_x, H_z and θ. Profiles of the functions (except θ) have been drawn while skin friction and Nusselt numbers have been tabulated for various values of G, R and $\omega\tau$.

5. Discussion of the Results

From Figs. 1 and 2 it is obvious that negative G and injection induce flow reversal, while positive G and suction prevent the same. From these figures it is also apparent

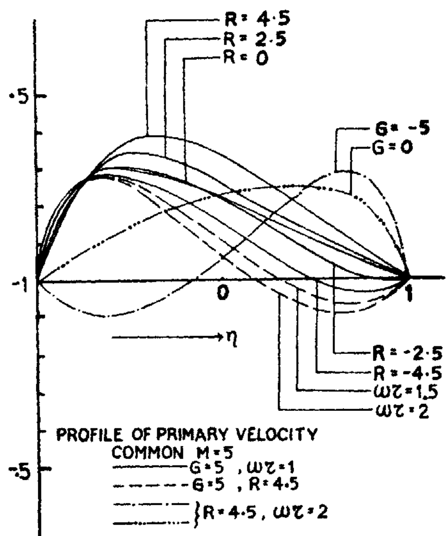

Fig. 1.

that $u(-R, \;-G) = -u(R, G)$ and $w(-R, \;-G) = -w(R, G)$ [observe the cases $G = 5$, $R = -4.5$ and $G = -5$, $R = 4.5$ for $\omega\tau = 2$ in Fig. 1 and for $\omega\tau = 1$ in Fig. 2] so that both of u and w are odd functions of R and G. Suction has a significant role on the velocity and magnetic field. It prevents the reversal of flow and seems to pull the magnetic profiles of the induced magnetic field H_z towards the upper wall, a role which is also played by positive G (Fig. 3). As far as the lines of field H_x are concerned both G and suction at the upper wall have similar effects, of pulling magnetic profiles (Fig. 4).

From Table I we see that heat transfer rate at the lower wall increases for positive G and decreases with a negative G. For a negative G, with an increase in the Hall current, heat transfer rate increases at the lower wall and decreases at the upper one, the effect being opposite when $G \geqslant 0$. Suction depresses the wall heat transfer rate and injection inflates it for fixed values of other parameters.

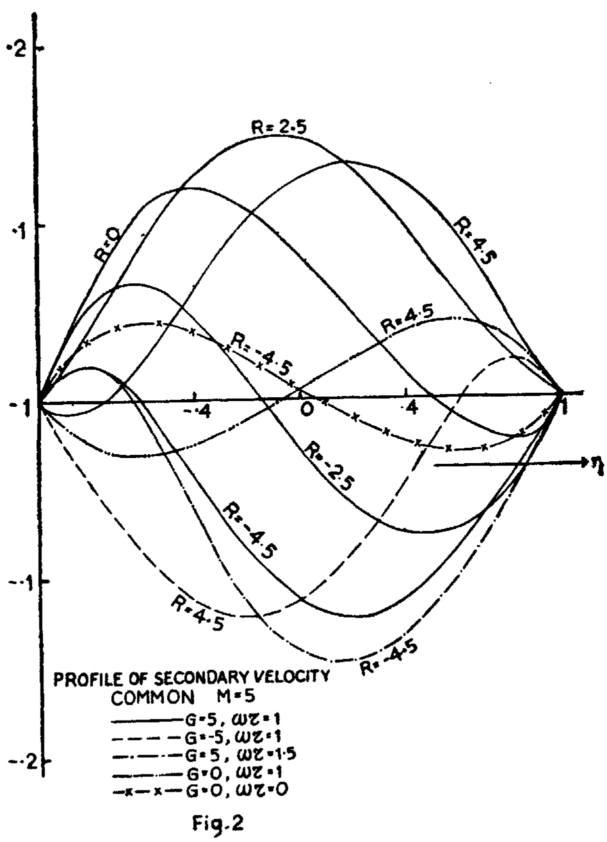

FIG. 2.

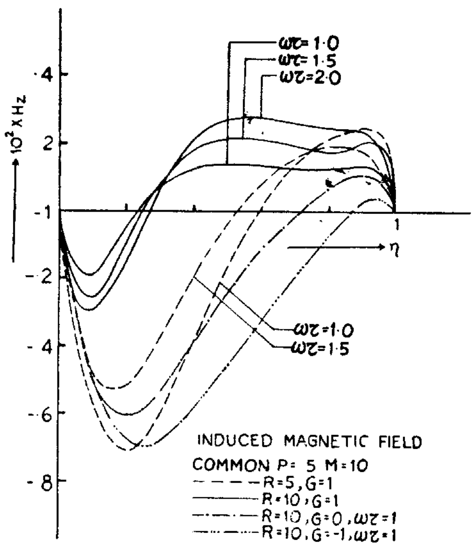

FIG. 3.

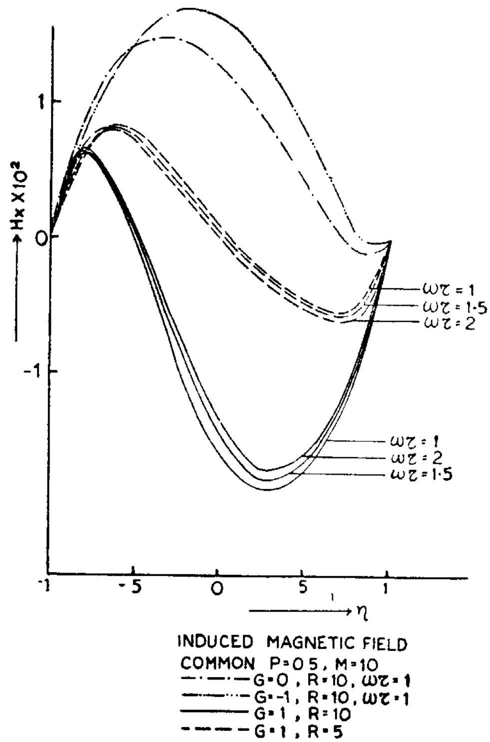

FIG. 4.

TABLE I

Nusselt number

$M = 5.0, \quad P = 0.5, \quad K_1 = 0.5$

		$\eta = -1$			$\eta = 1$		
G	$R \setminus \omega\tau$	1.00	1.50	2.00	1.00	1.50	2.00
1.0	−5.0	0.45714	0.46860	0.47909	0.46042	0.46335	0.46530
	−4.5	0.47784	0.50588	0.51533	0.44892	0.45425	0.45870
	0.0	0.61536	0.62673	0.63790	0.40417	0.39381	0.39583
	5.0	0.76359	0.68354	0.65339	0.32438	0.30614	0.29381
−5.0	5.0	0.90644	1.04946	1.13928	1.62802	1.42209	1.11789
−1.0	5.0	0.53962	0.54659	0.55390	0.45256	0.42210	0.38651
0.0	5.0	0.61518	0.58291	0.57096	0.35016	0.33220	0.21742
5.0	5.0	1.78512	1.72922	1.63678	0.98709	0.84329	0.65407

Figure 5 indicates that as R increases through positive values, since at the upper wall it implies increasing suction which counters flow reversal, value, of G_{crit} decreases and remains independent of $\omega\tau$. At the lower wall the increasing fluid influx requires increasing value of G to counter the onset of flow reversal. From Fig. 6 we observe that the flow reversal in the presence of moderately small injection velocity, say up to $R = 5$ can be checked by a positive G of very nearly same magnitude for all positive values of $\omega\tau$. From this we conclude that cross-flow reversal in the absence of either porosity or Hall current cannot happen in the presence of free convection. Figure 6 also indicates that with an increase of injection at the wall beyond a certain value say $R = 10$ the cross-flow reversal gets stronger but only for small values of $\omega\tau$ say up to 1. But injection effect passes through a peculiar transition for some value of $\omega\tau$ between 1.0 and 1.5 beyond which injection opposes flow reversal instead of helping it. Thus for $R = 10$, $G'_{crit} = 0$ is attained twice once for

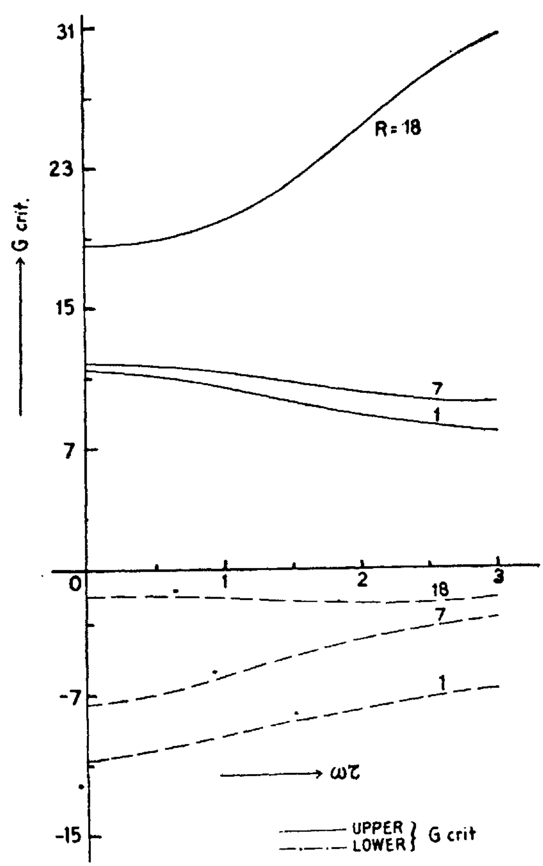

FIG. 5.

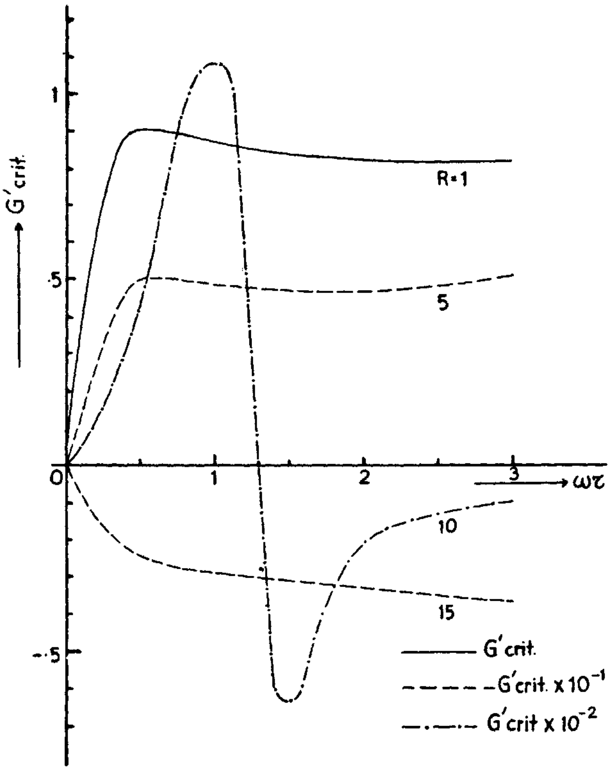

FIG. 6.

$\omega\tau = 0$ and again for $1.0 < \omega\tau < 1.5$. For further increase in R, say $R = 15$, G'_{crit} remains negative increasing very slowly with an increase in Hall effect.

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