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EFFECT OF STATIC MAGNETIC FIELD ON BLOOD FLOW IN A BRANCH

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Mathematical simulation of blood flow through a bifurcation in the presence of transverse static magnetic field is presented here. A simple bifurcation model has been analysed to study the effect of magnetic field on the nature of local disturbances and high shear forces which cause certain cardiovascular lesions like atherosclerotic plaques, aneurysms and intimal cushions etc. near bifurcation. The extent of the effect of magnetic field on the flow is determined by the magnitude of the Hartmann number *M*. It has been observed that within certain limits, the applied magnetic field reduces the strength of blockage at the apex of bifurcation, shear stress parameters and changes the speed of blood.

1. INTRODUCTION

The application of magnetohydrodynamic principles in medicine and biology is of growing interest in the literature of bio-mathematics (Vardanyan 1973, Sud *et al.* 1974, 1978). By Lenz's law, the Lorentz's force will oppose the motion of conducting fluid. Since blood is an electrically conducting fluid (Katz and Kolin 1938), the MHD principles may be used to deaccelerate the flow of blood in a human arterial system and thereby it is useful in the treatment of certain cardiovascular disorders (Korchevskii and Marochunik 1965) and in the diseases with accelerated blood circulation like haemorrhages and hypertension etc.

The idea of electromagnetic fields in medical research was firstly given by Kolin (1936) and later Korchevskii *et al.* (1965) discussed the possibility of regulating the movement of blood in human system by applying magnetic field. Recently Vardanyan (1973) studied the effect of magnetic field on blood flow theoretically and his work was later corroborated by Sud *et al.* (1974, 1978) by considering different models. It was observed by these authors that the effect of constant magnetic field slows down the speed of blood. However, the published work yet lacks any mathematical analysis of the magnetic effect on blood flow in a branch, which is of great interest in the treatment of some cardiovascular lesions like atherosclerotic plaques, intimal cushions etc. and aneurysms in brain circulation system at and near the apex of bifurcation. Thus to start with, a simple mathematical model

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for blood flow through an equally branched channel is analysed to study various hemodynamic factors in the presence of static magnetic field.

2. Mathematical Model and Basic Equations of Motion

The real blood circulation system consist of three-dimensional elastic tubes of varying cross-section and angle of bifurcation. For the sake of mathematical convenience, we discuss the magnetic effect on unsteady flow of blood through a two-dimensional, non-conducting, parallel plate and equally branched channel such that one stream of blood from trunk is branched into two different streams. The thickness of the bifurcating wall is assumed to be of negligibly small magnitude. The geometry of the bifurcation is same as taken by Zamir and Roach (1973) and is shown in Fig. 1. For this analysis, blood has been considered to be Newtonian, incompressible, homogeneous and viscous fluid. The Fahreus Lindquist effect is significant only when the vessel diameter is less than 1 mm (Burton 1965) unlike the case here. As such the Reynolds number does not vary much in the region of any one bifurcation. Thus the viscosity of the blood is treated as constant.

The static magnetic field B_0 is applied in a direction perpendicular to the flow of blood. We make the following assumptions for electromagnetic interactions:

(i) The induced magnetic field and the electric field produced by the motion of blood are negligible [since blood has low magnetic Reynolds number (Sud *et al.* 1974)].

(ii) No external electric field is applied.

With the above assumptions, the unsteady flow of blood in the presence of static magnetic field is governed by the two-dimensional boundary layer equations

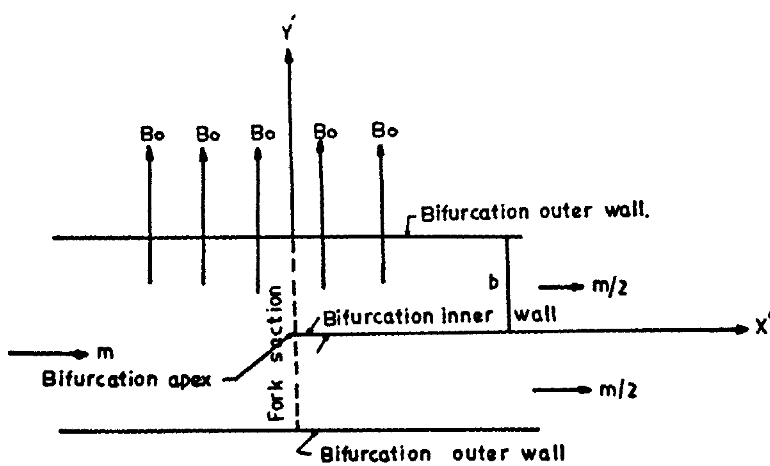

FIG. 1. Unsteady flow of blood in a parallel plate channel in the presence of external magnetic field B_0.

$$\frac{\partial u^*}{\partial t^*} + \frac{1}{\rho}\frac{dp^*}{dx^*} = \frac{\eta}{\rho}\frac{\partial^2 u^*}{\partial y^{*2}} - \frac{\sigma B_0^2 u^*}{\rho} \qquad \ldots(1)$$

$$\frac{\partial u^*}{\partial x^*} + \frac{\partial v^*}{\partial y^*} = 0 \qquad \ldots(2)$$

where u^* and v^* are velocity components in the direction of x^* and y^* respectively at time t in the flow field. σ, ρ and η denotes electrical conductivity, density and viscosity of blood while p^* and B_0 stands for pressure and applied magnetic field.

The Reynolds number $R = \rho \bar{V} b/\eta$ is related to the mass flow m since $m = 2b\rho\bar{V}$ and hence $R = m/2\eta$, where $\bar{V}$ is the mean velocity of flow and b the branch diameter.

We express the following non-dimensional quantities by defining

$$\left.\begin{aligned} &x = \frac{x^*}{b},\ u = \frac{u^*}{(m/2b\rho)},\ h(x,t) = \frac{(dp^*/dx^*)}{(\eta m/2b^3\rho)},\\ &y = \frac{y^*}{b},\ v = \frac{v^*}{(m/2b\rho)},\qquad R = \frac{m}{2\eta},\\ &t = \frac{t^*}{(b^2\rho/\eta)} \qquad \text{and} \qquad M = bB_0\sqrt{\sigma/\eta}. \end{aligned}\right\} \qquad \ldots(3)$$

Substituting (3), eqns. (1) and (2) become dimensionless as

$$\frac{\partial u}{\partial t} + h = \frac{\partial^2 u}{\partial y^2} - M^2 u \qquad \ldots(4)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \qquad \ldots(5)$$

3. Initial and Boundary Conditions

(i) Initially at

$$t = 0;\ u = u_0 \text{ for all } y. \qquad \ldots(6)$$

(ii) The boundary conditions in the trunk and branches are 'no-slip' condition, i.e., for

$$x < 0;\ u = v = 0 \quad \text{for } y = \pm 1 \qquad \ldots(7)$$

and

$$x \geqslant 0;\ u = v = 0 \quad \text{for } y = 0, \pm 1. \qquad \ldots(8)$$

(iii) The law of conservation of mass is preserved by the equation

$$\int_{-b}^{b} \rho u^*\, dy^* = m \qquad \ldots(9)$$

and because of symmetry about $y = 0$, it can be written in dimensionless form as

$$\int_0^1 u\, dy = 1. \qquad \ldots(10)$$

4. Momentum and Moment of Momentum Integral Equations

As the pressure gradient parameter (h) is unknown and the conventional boundary layer solutions of eqns. (4) and (5) does not satisfy the boundary condition on $y = \pm 1$ in the branches, so it will be appropriate to analyse the problem based on integral form of the boundary layer equations. Further, since the flow configuration is symmetrical about $y = 0$, it is sufficient to restrict our analysis to one of the branches i.e., the region bounded by

$$0 \leqslant x \leqslant +\infty, \qquad 0 \leqslant y \leqslant +1. \qquad \ldots(11)$$

By taking Laplace transform on both sides of momentum eqn. (4), we get

$$s\bar{u} - u_0 + \bar{h} = \frac{\partial^2 \bar{u}}{\partial y^2} - M^2 \bar{u} \qquad \ldots(12)$$

where $\bar{u}(s) = \int_0^\infty e^{-st}\, u(t)\, dt.$

The moment of momentum in the direction of flow is thus governed by the equation

$$ys\bar{u} - yu_0 + y\bar{h} = y\frac{\partial^2 \bar{u}}{\partial y^2} - yM^2 \bar{u}. \qquad \ldots(13)$$

Each term of eqns. (12) and (13) can be integrated from $y = 0$ to $y = 1$ to get integral form of the boundary layer equations respectively,

$$\int_0^1 (s + M^2)\,\bar{u}\, dy - \int_0^1 u_0 dy + \int_0^1 \bar{h}\, dy = \int_0^1 \frac{\partial^2 \bar{u}}{\partial y^2}\, dy \qquad \ldots(14)$$

$$\int_0^1 y(s + M^2)\,\bar{u} dy - \int_0^1 yu_0 dy + \int_0^1 y\bar{h}\, dy = \int_0^1 y\,\frac{\partial^2 \bar{u}}{\partial y^2}\, dy. \qquad \ldots(15)$$

Integrating by parts and noting that

$$\int_0^1 \frac{\partial^2 \bar{u}}{\partial y^2}\, dy = \left(\frac{\partial \bar{u}}{\partial y}\right)_{y=1} - \left(\frac{\partial \bar{u}}{\partial y}\right)_{y=0} \qquad \ldots(16)$$

$$\int_0^1 y\,\frac{\partial^2 \bar{u}}{\partial y^2}\, dy = \left(\frac{\partial \bar{u}}{\partial y}\right)_{y=1} \qquad \ldots(17)$$

eqns. (14) and 15 become, respectively,

$$(s + M^2)\int_0^1 \bar{u}\,dy - \int_0^1 u_0\,dy + \bar{h} = \left(\frac{\partial \bar{u}}{\partial y}\right)_{y=1} - \left(\frac{\partial \bar{u}}{\partial y}\right)_{y=0} \qquad ...(18)$$

$$(s + M^2)\int_0^1 y\bar{u}\,dy - \int_0^1 yu_0\,dy + \frac{\bar{h}}{2} = \left(\frac{\partial \bar{u}}{\partial y}\right)_{y=1}. \qquad ...(19)$$

Equations (18) and (19) can be solved by separating independent variables in u, i.e. by assuming u as a polynomial in y with coefficients which are functions of x and t only as under:

$$u(x, y, t) = f_0(x, t) + yf_1(x, t) + y^2 f_2(x, t) + y^3 g(x, t) \qquad ...(20)$$

where f_0, f_1, f_2 and g are arbitrary functions of x and t.

Taking Laplace transform on both sides of eqn. (20), we get

$$\bar{u}(x, y, s) = \bar{f}_0(x, s) + y\bar{f}_1(x, s) + y^2 f_1(x, s) + y^3 g(x, s). \qquad ...(21)$$

Applying Laplace transform on boundary conditions (8), (10) and using (21), we find the coefficients $\bar{f}_0$, $\bar{f}_1$, $\bar{f}_2$ in terms of $\bar{g}$ as

$$\bar{f}_0 = 0,\ \bar{f}_1 = \left(\frac{\bar{g}}{2} + \frac{6}{s}\right) \text{ and } \bar{f}_2 = -\left(\frac{3}{2}\bar{g} + \frac{6}{s}\right). \qquad ...(22)$$

Hence eqn. (21) becomes,

$$\bar{u} = \frac{1}{s}\left(6y - 6y^2\right) + \bar{g}\left(\frac{y}{2} - \frac{3}{2}y^2 + y^3\right). \qquad ...(23)$$

Substituting (23) in eqns. (18) and (19) to get

$$\bar{h} = \frac{-(M^2 + 12)}{s} \quad \text{and} \quad \bar{g} = \frac{g_0}{[s + (M^2 + 60)]} \qquad ...(24)$$

where g_0 is the value of g at $x = 0$.

Now by taking Laplace inverse, (23) and (24) can be simplified to give

$$u = (6y - 6y^2) + (\tfrac{1}{2}y - \tfrac{3}{2}y^2 + y^3)\,g_0 \exp\left(-(M^2 + 60)t\right) \qquad ...(25)$$

and

$$h = -(M^2 + 12). \qquad ...(26)$$

Using continuity eqn. (5), we get

$$v = -\int_0^y \frac{\partial u}{\partial x}\,dy = 0. \qquad ...(27)$$

In addition, the shear stress parameters τ_0 and τ_1 at the inner and outer walls of bifurcation can be obtained from (25) in non-dimensional form respectively, as

$$\tau_0 = \left(\frac{\partial u}{\partial y}\right)_{y=0} = \tfrac{1}{2} g_0 \exp\left(-(M^2+60)t\right) + 6 \qquad ...(28)$$

$$\tau_1 = \left(\frac{\partial u}{\partial y}\right)_{y=1} = \tfrac{1}{2} g_0 \exp\left(-(M^2+60)t\right) - 6. \qquad ...(29)$$

For $M = 0$, in the non-magnetic case, the mathematical results obtained from (25) to (29) are consistent with the results obtained by Suri and Suri (1981).

5. Discussion

It is obvious from eqns. (25) through (29) that the hemodynamic factors are dependent on the value of g_0 for interpreting and discussing the behaviour of these

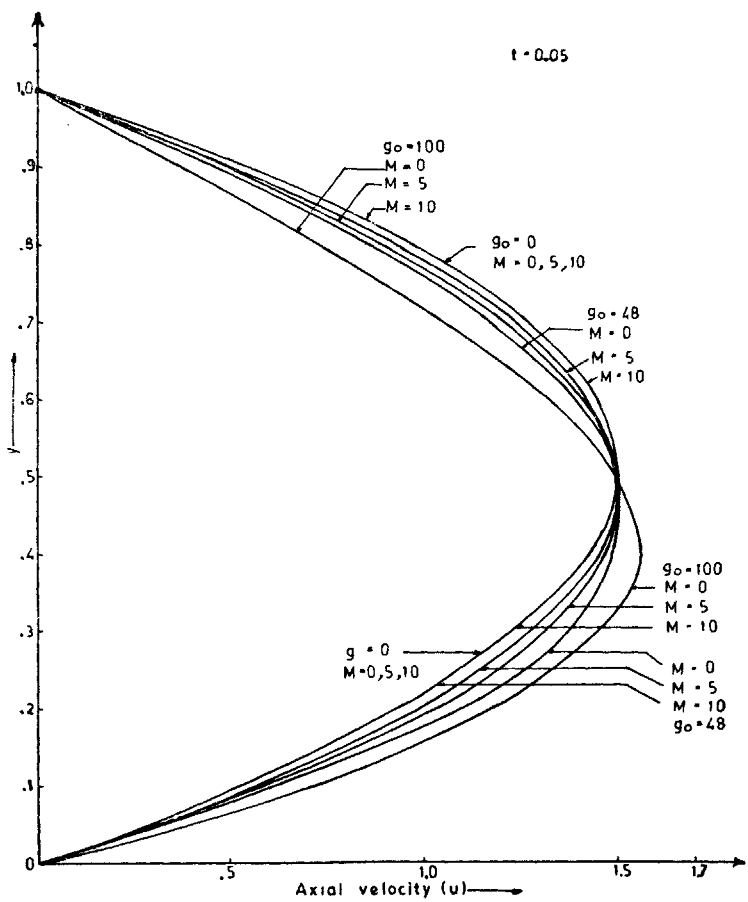

FIG. 2. Main velocity profiles for different values of g_0 and M.

parameters. The value of g_0 will depend upon the flow conditions at the apex of bifurcation i.e. the thickness and shape of the bifurcation apex. In human blood circulation system, arteries or veins cannot have leading edge of zero thickness and solid boundary as is considered in the present model. By keeping all these assumptions in mind, we may expect that in real flow, g_0 will assume a high value, the exact magnitude of which will depend upon the thickness of the bifurcating wall and apex angle. Thus in such preliminary unsteady flow models, it is beneficial to study hemodynamic factors with different values of g_0 rather than one specific value.

Figure 2 illustrates the behaviour of axial velocity profiles at $t = 0.05$ for $g_0 = 0$, 48, 100 and for different values of Hartmann number ($M = 0$, 5 and 10) which measures the extent of applied magnetic field. It is observed that the velocity component (u) increases with the increase in the value of g_0 and decreases with the increase in magnetic field for $0 < y \leqslant 0.5$ while the reverse effect is observed for

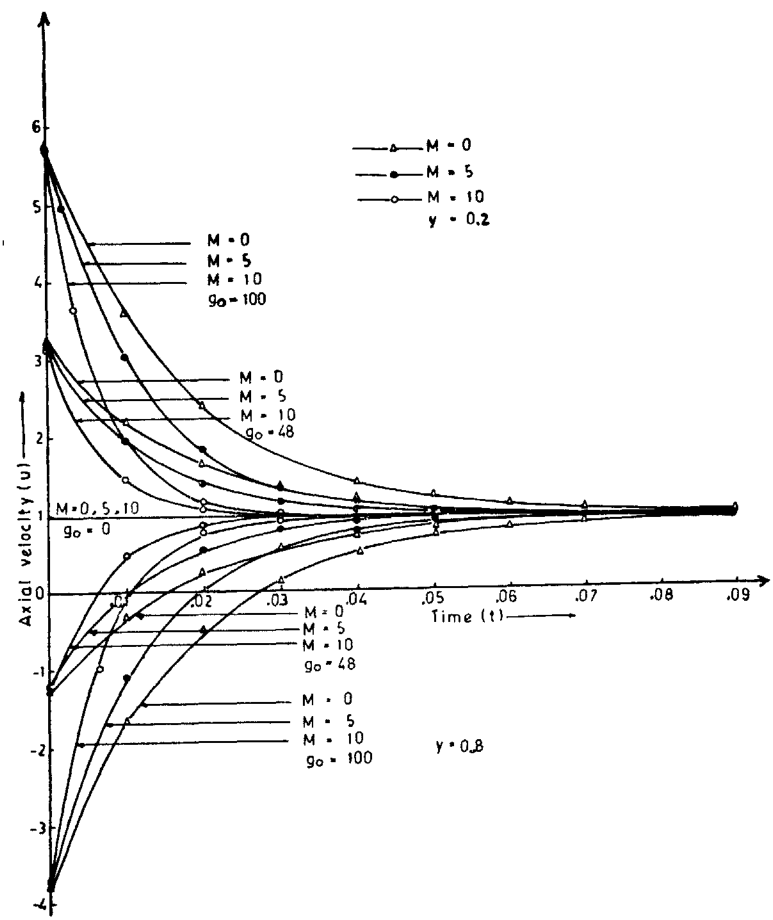

FIG. 3. Velocity profiles with respect to time at $Y = 0.2$ and $Y = 0.8$ for different values of g_0 and M.

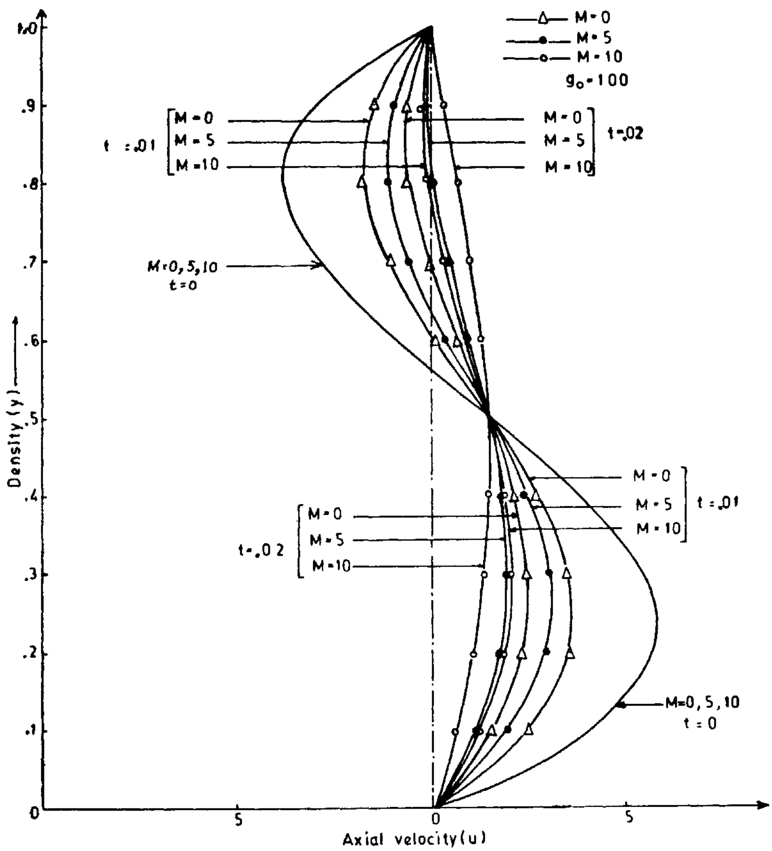

FIG. 4. Velocity profile at $t = 0, 0.01, 0.02$, $M = 0, 5, 10$ and for fixed value of $g_0 = 100$.

$y > 0.5$. The same effect is observed in Fig. 3 which shows the trend of main velocity component with respect to dimensionless time (t) for different values of g_0, M and for fixed values of $y = 0.2$ and $y = 0.8$. In addition, it is also noticed that the flow stabilizes at $t = 0.08$ and with the increase in magnetic field, u starts converging for lower values of t. Velocity profiles for $t = 0, 0.01, 0.02, g_0 = 100$ and $M = 0, 5, 10$ for various values of y bounded by the walls of the branch under consideration are shown in Fig. 4 illustrating the interesting feature of the back flow. It is evident that the negative axial velocity is developed near the outer wall of bifurcation for $t \lesssim 0.02$. Zamir and Roach (1973) have observed a similar feature with respect to g_0 in the steady state model for non-magnetic case. In the present case where time-dependent flow has been considered by neglecting inertia terms in the presence of externally applied magnetic field. Thus reverse flow with respect to time is in accordance with the expectations and thereby confirming our analysis. Further, it is concluded from Fig. 4 that the reverse flow decreases with the increase in magnetic

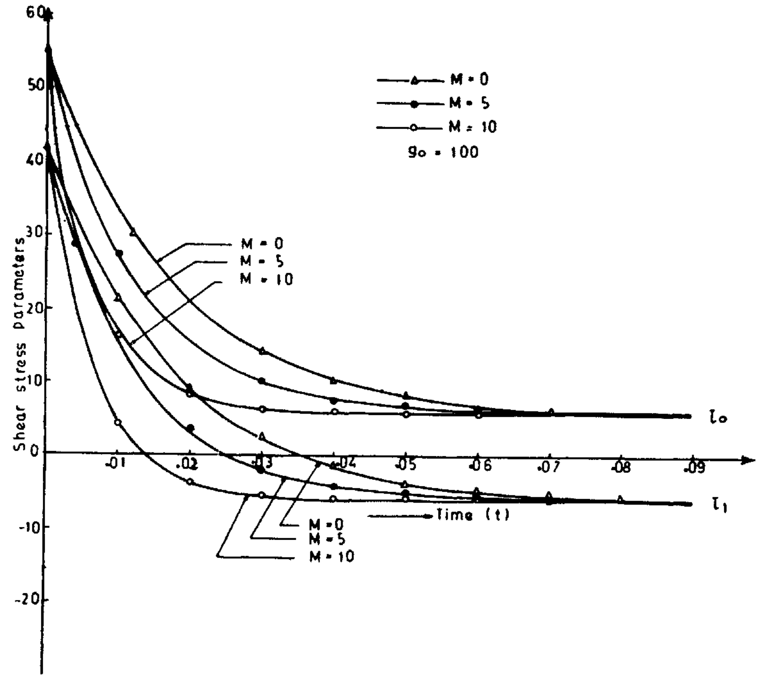

FIG. 5. Non-dimensional shear stress parameters τ_0 and τ_1 versus dimensionless time for different values of M.

field (i.e., the Hartmann number M) which may have interesting medical applications. Hassler (1962a, b) suggested from pathological studies that intimal cushions always develop at the lateral angles of intracranial bifurcations. He felt that this could be due to some sort of flow disturbances occurring in this area. Zamir and Roach (1973) suggested that these intimal cushions may occur due to reverse flow in this region. Thus our analysis may be useful in the treatment of such lesions as the effect of external magnetic field is to decrease back flow near the outer wall of bifurcation.

The non-dimensional shear stress parameters τ_0 and τ_1 at the inner and outer walls of bifurcation with respect to time are plotted in Fig. 5 for different values of Hartmann number $M = 0$, 5 and 10 and for fixed value of $g_0 = 100$. It is observed that the stresses become constant earlier in magnetic case. Fry (1968, 1969a, b) showed in his experiments that high shear rates near bifurcation might be a predisposing factor for atherosclerosis. It is evident from Fig. 5, the effect of externally applied magnetic field is to reduce the shear stresses. Thus our analysis can also be effectively used in curing diseases like atherosclerosis.

In Fig. 6, the axial velocity component is plotted against Hartmann number for different values of g_0 and for fixed values of vertical distance $y = 0.2$, 0.4, 0.7

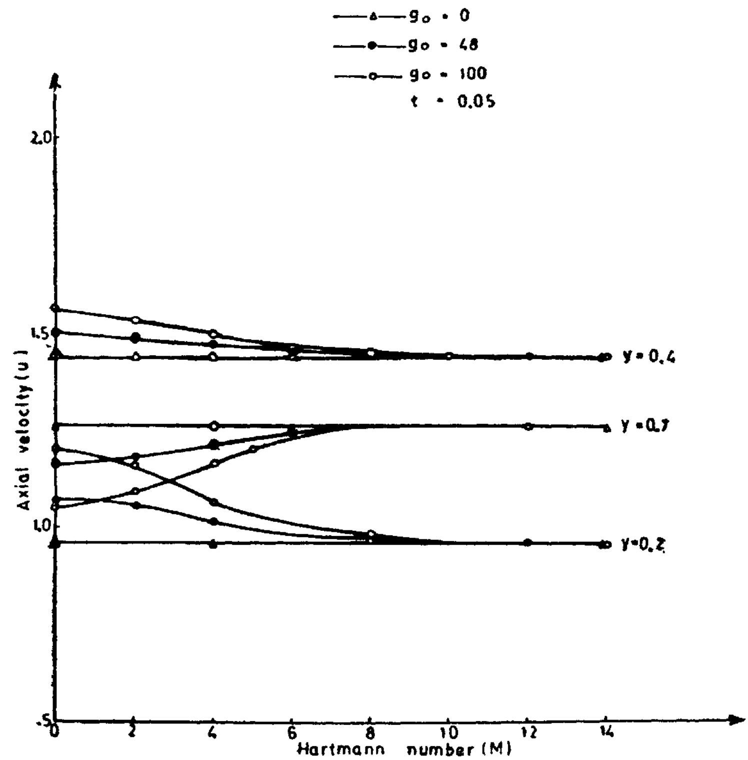

FIG. 6. Axial velocity component versus Hartmann number for different values of g_0.

and $t = 0.05$. It can be noticed that u decreases with the increase in magnetic field for $y \leqslant 0.5$ (e.g. in case of $y = 0.4$) and it increases with the increase in magnetic field for $y > 0.5$ (e.g. for $y = 0.7$). We also observe that the effect of external magnetic field on the velocity component (u) for $M \geqslant 10$ is negligibly small—that is of the order of $\leqslant 10^{-3}$. Further from eqn. (26), it is seen that the dimensionless pressure gradient parameter decreases with the increase in magnetic field and eqn. (27) depicts that the normal velocity component (v) is zero in this particular flow model and is not affected by externally applied magnetic field. Finally, above discussion concludes that the flow becomes steady as $t \to \infty$, which confirms our analysis for unsteady blood flow model.

Blockage effect at the fork section is an important factor in the study of flow through arterial bifurcation, as it may lead to certain vascular lesions like atherosclerotic plaques and intracranial aneurysms etc. Blockage near the apex of bifurcation is caused by the development of boundary layers on the inner walls of bifurcation and a region of increasing pressure in the neighbourhood of apex. When the bifurcation angle is large, the main blood stream is almost fully stagnated at the fork section and hence the boundary layer formation is low. While on the other hand, when the angle of bifurcation is small, the formation of boundary layer is rapid as the main blood stream retains most of its momentum as it crosses the fork section and hence a region of high shear is developed.

The magnitude of the strength of 'blockage effect' is the value of pressure gradient at the fork section and is obtained from eqns. (3) and (26) as

$$\left(\frac{dp^*}{dx^*}\right)_{x^*=0} = -(M^2 + 12)\frac{\eta m}{2b^3\rho} \quad \text{...(30)}$$

which shows that the 'blockage effect' decreases rapidly as square of the strength of applied magnetic field determined by the Hartmann number M. Equation (30) also shows that the 'blockage effect' decreases linearly with the rate of mass flow (m) and increases rapidly with third power of breadth (b) of the channel.

6. Conclusion

In this paper, we discusses a preliminary model of unsteady flow of blood through a bifurcation in the presence of transverse static magnetic field. This analysis will encourage medical researchers/bio-medical engineers to control the flow of blood in human cardiovascular and neural circulation systems artificially by applying static magnetic field in the direction perpendicular to the direction of flow. Hence subsequently, it will be of great importance in the treatment of cardiovascular lesions such as atherosclerotic plaques, intimal cushions and aneurysms etc. which tend to occur near the apex of bifurcation and the diseases related with accelerated circulation like hypertension and brain haemorrhages etc.

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