SOME IDENTITIES AND HYPERGEOMETRIC FUNCTIONAL RELATIONS*

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In this paper, we have established five identities with the help of the difference operators defined by

$$E_\alpha f(\alpha) = f(\alpha + 1),$$

$$\triangle_\alpha f(\alpha) = f(\alpha + 1) - f(\alpha),$$

and the known results (Abdul-Halim and Al-Salam 1963). Also we have deduced many transformations of the hypergeometric series from these identities.

1. INTRODUCTION

Recently Agrawal (1973) gave some theorems involving the difference operators Δ and E defined as

$$E_\alpha f(\alpha) = f(\alpha + 1)$$

$$\Delta_\alpha f(\alpha) = f(\alpha + 1) - f(\alpha)$$

so that

$$\Delta_\alpha \equiv E_\alpha - 1$$

also

$$\Delta^n_\alpha f(\alpha) = \Delta^{n-1}_\alpha [\Delta_\alpha f(\alpha)]$$

and has obtained some transformations of hypergeometric series. In the present paper, we have established some identities involving these operators and have obtained some relations involving hypergeometric functions by using these identities. These relations are actually the direct consequences of our identities. As our identities are general in character, many other relations may be obtained by specializing the parameters involving in that identities.

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2. First Identity

$$(-1)^{-\lambda} \sum_{n=0}^{\infty} \frac{(\lambda)_n}{n!} t^n \Delta_\alpha^n \Delta_\beta^{-n-\lambda} f_1(\alpha)\, f_2(\beta)$$

$$= (1-t)^{-\lambda} \sum_{n,r=0}^{\infty} \frac{(\lambda)_{n+r}}{n!\,r!} \frac{(-t)^r}{(1-t)^{n+r}} f_1(\alpha+r)\, f_2(\beta+n), \qquad \ldots(2.1)$$

where $\{f(N)\}$ is a sequence of arbitrary complex numbers and it is assumed here that the series involved converge absolutely.

Proof : Left-hand side

$$= (-1)^{-\lambda}\ \Delta_\beta^{-\lambda} \left(1 - \frac{t\Delta_\alpha}{\Delta_\beta}\right)^{-\lambda} f_1(\alpha)\, f_2(\beta)$$

$$= (t \Delta_\alpha - \Delta_\beta)^{-\lambda} f_1(\alpha)\, f_2(\beta)$$

$$= \{t(E_\alpha - 1) - (E_\beta - 1)\}^{-\lambda} f_1(\alpha)\, f_2(\beta)$$

$$= (1 - t + t E_\alpha - E_\beta)^{-\lambda} f_1(\alpha)\, f_2(\beta)$$

$$= (1-t)^{-\lambda} \left\{1 - \frac{(E_\beta - tE_\alpha)}{(1-t)}\right\}^{-\lambda} f_1(\alpha)\, f_2(\beta) \qquad \ldots(2.2)$$

$$= (1-t)^{-\lambda} \sum_{n=0}^{\infty} \frac{(\lambda)_n}{(1-t)^n\, n!} \sum_{r=0}^{n} \binom{n}{r} E_\beta^{n-r} (-tE_\alpha)^{+r} f_1(\alpha)\, f_2(\beta)$$

$$= (1-t)^{-\lambda} \sum_{n=0}^{\infty} \sum_{r=0}^{\infty} \frac{(\lambda)_{n+r}}{n!\, r!} (-1)^r \frac{t^r}{(1-t)^{n+r}} f_1(\alpha+r)\, f_2(\beta+n)$$

This completes the proof of (2.1)

Example 1—Let

$$f_1(\alpha) = \frac{\Gamma[(a+\alpha)]}{\Gamma[(b+\alpha)]} x^\alpha$$

and

$$f_2(\beta) = \frac{\Gamma[(c+\beta)]}{\Gamma[(d+\beta)]} y^\beta$$

where (a) stands for a sequence $a_1, \ldots, a_A$ and similar interpretations for (b), (c) and (d).

Then for $\alpha = \beta = 0$, we obtain a bilinear generating function,

$$\sum_{n=0}^{\infty} \frac{[\lambda]_n}{n!} {}_{A+1}F_B\left[\begin{matrix}-n, & (a); \\ & (b);\end{matrix} x\right] {}_{C+1}F_D\left[\begin{matrix}\lambda+n, & (c); \\ & (d);\end{matrix} y\right] t^n$$

$$= (1-t)^{-\lambda} F\left[\begin{matrix}\lambda: (a); (c); \\ —: (b); (d);\end{matrix} \frac{xt}{t-1}, \frac{y}{1-t}\right], \qquad \ldots(2.3)$$

which is formula (4.6) of Srivastava (1972, p. 82).

In (2.3), when we put $x = 0$ we obtain,

$$\sum_{n=0}^{\infty} \frac{[\lambda]_n}{n!} {}_{C-1}F_D\left[\begin{matrix}\lambda+n, & (c); \\ & (d);\end{matrix} y\right] t^n$$

$$= (1-t)^{-\lambda} {}_{C+1}F_D\left[\begin{matrix}\lambda, & (c); \\ & (d);\end{matrix} \frac{y}{1-t}\right] \qquad \ldots(2.4)$$

when t is replaced by $-t$ and $C = D = 1$; it reduces to

$$(1+t)^{-\lambda} {}_2F_1\left[\lambda, c; d; \frac{y}{1+t}\right]$$

$$= \sum_{n=0}^{\infty} (-)^n \frac{[\lambda]_n}{n!} {}_2F_1 [\lambda+n, c; d; y] t^n \qquad \ldots(2.5)$$

which is formula (5.4) of Abdul-Halim and Al-salam (1963).

Also when $y = 0$ and t replaced by z, it reduces to Chaundy's (1943) formula

$$\sum_{n=0}^{\infty} \frac{(\lambda)_n}{n!} {}_{A+1}F_B\left[\begin{matrix}-n, & (a); \\ & (b)\end{matrix} x\right] z^n$$

$$= (1-z)^{-\lambda} {}_{A+1}F_B\left[\begin{matrix}\lambda, & (a); \\ & (b)\end{matrix} \frac{-xz}{1-z}\right]$$

this formula has also been discussed directly by Singhal (1975).

By appealing to the familiar result (Abdul-Halim and Al-Salam 1963)

$$(1-x-y)^{-b} = \sum_{r=0}^{\infty} \frac{(b)_r}{r!} (xy)^r \sum_{m,n=0}^{\infty} x^m y^n \frac{(b+r)_m (b+r)_n}{m!\, n!} \qquad \ldots(2.6)$$

it is easily seen that the identity (2.1) with (2.2) comes in the form:

$$(-1)^{-\lambda} \sum_{n=0}^{\infty} \frac{[\lambda]_n}{n!} t^n \Delta_\alpha^n \Delta_\beta^{-n-\lambda} f_1(\alpha)\, f_2(\beta)$$

$$= (1-t)^{-\lambda} \sum_{r=0}^{\infty} \frac{[\lambda]_r}{r!} \frac{(-t)^r}{(1-t)^{2r}} \sum_{m,\,n=0}^{\infty} \frac{(\lambda+r)_m (\lambda+r)_n}{m!\, n!}$$

$$\times \left(\frac{1}{1-t}\right)^n \left(\frac{-t}{1-t}\right)^m f_1(\alpha+m+r)\, f_2(\beta+n+r) \qquad ...(2.7)$$

Taking $f_1(\alpha)$ and $f_2(\beta)$ as in Example 1, we get the following transformation

$$\sum_{n=0}^{\infty} \frac{[\lambda]_n}{n!}\, {}_{A+1}F_B\left[\begin{matrix}-n, (a); \\ (b);\end{matrix}\ x\right] {}_{C+1}F_D\left[\begin{matrix}\lambda+n, (c); \\ (d);\end{matrix}\ y\right] t^n$$

$$= (1-t)^{-\lambda} \sum_{r=0}^{\infty} \frac{[\lambda]_r [(a)]_r [(c)]_r}{r!\, [(b)]_r\, [(d)]_r}\, {}_{A+1}F_B\left[\begin{matrix}\lambda+r, (a+r); \\ (b+r)\end{matrix}\ \frac{-xt}{1-t}\right]$$

$$\times\, {}_{C+1}F_D\left[\begin{matrix}\lambda+r,\ (c+r); \\ (d+r)\end{matrix}\ \frac{yt}{1-t}\right] \left\{\frac{-txy}{(1-t)^2}\right\}^r. \qquad ...(2.8)$$

3. Second Identity

$$\sum_{m,\,n=0}^{\infty} \frac{(\lambda)_m\, (\lambda)_n}{m!\ n!} f_1(\alpha+m)\, f_2(\beta+n)$$

$$= \sum_{r=0}^{\infty} \frac{(-1)^r [\lambda]_r}{r!} \sum_{m,\,n=0}^{\infty} \frac{(\lambda+r)_{m+n}}{m!\ n!} f_1(\alpha+m+r)\, f_2(\beta+n+r), \qquad ...(3.1)$$

provided the series involved converge absolutely.

Proof: Left-hand side

$$= \sum_{m,\,n=0}^{\infty} (\lambda)_m (\lambda)_n \frac{E_\alpha^m}{m!} \frac{E_\beta^n}{n!} f_1(\alpha)\, f_2(\beta)$$

$$= (1-E_\alpha)^{-\lambda}\, (1-E_\beta)^{-\lambda} \cdot f_1(\alpha)\, f_2(\beta).$$

Now using the result Abdul-Halim and Al-Salam (1963. p. 61).

$$(1-x)^{-a}(1-y)^{-a}=\sum_{r=0}^{\infty}(-)^r\frac{(a)_r}{r!}(xy)^r\sum_{m,n=0}^{\infty}\frac{(a+r)_{m+n}}{m!\,n!}x^m y^n$$

we get the right-hand side of (3.1).

Example 2—Choosing $f_1(\alpha)$ and $f_2(\beta)$ similar to Example 1, we get (for $\alpha=\beta=0$)

$${}_{A+1}F_B\left[\begin{matrix}\lambda,\ (a);\\ (b);\end{matrix}\ x\right]{}_{C+1}F_D\left[\begin{matrix}\lambda,\ (c);\\ (d);\end{matrix}\ y\right]$$

$$=\sum_{r=0}^{\infty}(-1)^r\frac{[\lambda]_r\,[(a)]_r\,[(c)]_r}{r!\,[(b)]_r\,[(d)]_r}x^r y^r F\left[\begin{matrix}\lambda+r:(a+r);(c+r);\\ -:(b+r);(d+r);\end{matrix}\ x,y\right] \qquad \ldots(3.2)$$

which evidently provides a generalization to the result (Jain 1972, p. 5)

$${}_4F_3\,[a,\alpha,\beta,\gamma;\ \Delta(3,\delta);\ x]\ {}_4F_3\,[a,\alpha',\beta',\gamma';\ \Delta(3,\delta');\ y]$$

$$=\sum_{r=0}^{\infty}\frac{(-1)^r(a)_r(\alpha)_r(\beta)_r(\gamma)_r(\alpha')_r(\beta')_r(\gamma')_r}{r!\left(\frac{\delta}{3}\right)_r\left(\frac{\delta+1}{3}\right)_r\left(\frac{\delta+2}{3}\right)_r\left(\frac{\delta'}{3}\right)_r\left(\frac{\delta'+1}{3}\right)_r\left(\frac{\delta'+2}{3}\right)_r}(xy)^r$$

$$\times F\left(\begin{matrix}a+r;\ \alpha+r,\ \alpha'+r;\ \beta+r,\ \beta'+r;\\ \gamma+r,\ \gamma'+r;\\ -:\ \frac{\delta}{3}+r,\ \frac{\delta'}{3}+r;\ \frac{\delta+1}{3}+r,\ \frac{\delta'+1}{3}+r;\\ \frac{\delta+2}{3}+r,\ \frac{\delta'+2}{3}+r;\end{matrix}\quad x,\ y\right)$$

where $\Delta(s,\alpha)$ stands for a set of 's' parameters;

$$\frac{\alpha}{s},\ \frac{\alpha+1}{s},\ldots,\frac{\alpha+s-1}{s}.$$

On specializing the parameters, (3.2) also reduces to;

$${}_2F_1\,[a,b;\ c;\ x]\ {}_2F_1\,[a,b';\ c';\ y]$$

$$=\sum_{r=0}^{\infty}\frac{(-)^r(a)_r(b)_r(b')_r}{r!\,(c)_r(c')_r}(xy)^r F_2\,(a+r,b+r,b'+r;c+r,c'+r;x,y)$$

which is formula (27) of Burchnall and Chaundy (1940) and where F_2 is Appell's double hypergeometric function defined as (Appell and Kampe de Fériet 1926)

$$F_2[a, b, b'; c, c'; x, y] = \sum_{m,n=0}^{\infty} \frac{(a)_{m+n}(b)_m(b')_n}{m!\,n!\,(c)_m(c')_n} x^m y^n.$$

It can easily be seen that the identity (3.1) reduces in particular to a known result [Burchnall and Chaundy 1940, p. 121 (51)]

$$\begin{aligned} &{}_1F_1(a; c; x)\, {}_1F_1(a; c'; y) \\ &= \sum_{r=0}^{\infty} (-)^r \frac{(a)_r}{r!\,(c)_r\,(c')_r} x^r y^r\, \psi_2(a+r; c+r; c'+r; x, y) \end{aligned}$$

where ψ_2 is Humbert's confluent double hypergeometric function defined as (Erdelyi *et al.* 1953)

$$\psi_2(a; c, c'; x, y) = \sum_{m,n=0}^{\infty} \frac{(a)_{m+n}}{m!\,n!\,(c)_m(c')_n} x^m y^n$$

4. Third Identity

$$\begin{aligned} &\sum_{m,n=0}^{\infty} \frac{(\lambda)_{m+n}}{m!\,n!} f_1(\alpha+m+n)\, f_2(\beta+m)\, f_3(\gamma+n) \\ &= \sum_{r=0}^{\infty} \frac{(\lambda)_r}{r!} \sum_{m,n=0}^{\infty} \frac{(\lambda+r)_m(\lambda+r)_n}{m!\,n!} f_1(\alpha+m+n+2r) \\ &\quad \times f_2(\beta+m+r)\, f_3(\gamma+n+r) \qquad \ldots(4.1) \end{aligned}$$

provided that the series involved converge absolutely.

Proof : Left-hand side

$$\begin{aligned} &\sum_{m,n=0}^{\infty} \frac{(\lambda)_{m+n}}{m!\,n!} E_\alpha^{m+n} E_\beta^m E_\gamma^n \cdot f_1(\alpha)\, f_2(\beta)\, f_3(\gamma) \\ &= (1 - E_\alpha E_\beta - E_\alpha E_\gamma)^{-\lambda} \cdot f_1(\alpha)\, f_2(\beta)\, f_3(\gamma) \end{aligned}$$

using the relation (2.6) we get right-hand side.

Example 3—Let

$$f_1(\alpha) = 1^\alpha, f_2(\beta) = \frac{\Gamma[(a+\beta)]}{\Gamma[(b+\beta)]} x^\beta$$

and

$$f_3(\gamma) = \frac{\Gamma[(c+\gamma)]}{\Gamma[(d+\gamma)]} y^{\gamma}$$

we get (for $\alpha = \beta = \gamma = 0$)

$$F\left[\begin{matrix}\lambda: (a); (c); \\ -: (b); (d);\end{matrix} x, y\right] = \sum_{r=0}^{\infty} \frac{[\lambda]_r [(a)]_r [(c)]_r}{r! \, [(b)]_r \, [(d)]_r} x^r y^r$$

$$\times {}_{A+1}F_B\left[\begin{matrix}\lambda + r, \ (a+r); \\ (b+r);\end{matrix} x\right] {}_{C+1}F_D\left[\begin{matrix}\lambda + r, \ (c+r); \\ (d+r)\end{matrix} y\right].$$

which is formula (92) of Burchnall and Chaundy (1940).

Also in particular, for the Humbert's confluent double hypergeometric function ϕ_1, ψ_1, ψ_2 and E_1 (cf., e.g., Erdelyi *et al.*, 1953, p. 225); we are led to the following known special cases [Burchnall and Chaundy 1941, pp. 119 (28), 120 (34), 121 (50)], of (2.1);

$$\psi_1(\alpha: \beta; \ \gamma, \ \gamma'; \ x, \ y)$$

$$= \sum_{r=0}^{\infty} \frac{(\alpha)_r (\beta)_r}{r! (\gamma)_r (\gamma')_r} x^r y^r \, {}_2F_1\left[\begin{matrix}\alpha + r, \beta + r; \\ \gamma + r\end{matrix} x\right] {}_1F_1\left[\begin{matrix}\alpha + r; \\ \gamma' + r\end{matrix} y\right] \quad \ldots(4.2)$$

$$\phi_1 \ (\alpha; \ \beta; \ \gamma; \ x, \ y)$$

$$= \sum_{r=0}^{\infty} \frac{(\alpha)_r (\beta)_r}{(\gamma)_{2r} \ r!} x^r y^r \, E_1(\alpha + r, \ \alpha + r, \beta + r, \gamma + 2r; x, y) \quad \ldots(4.3)$$

and

$$\psi_2 \ (\alpha; \ \gamma, \ \gamma'; \ x, \ y)$$

$$= \sum_{r=0}^{\infty} \frac{(\alpha)_r}{r! (\gamma)_r (\gamma')_r} \, {}_1F_1\left[\begin{matrix}\alpha + r; \\ \gamma + r\end{matrix} x\right] {}_1F_1\left[\begin{matrix}\alpha + r; \\ \gamma' + r\end{matrix} y\right] \quad \ldots(4.4)$$

5. Fourth Identity

$$\sum_{r=0}^{\infty} \frac{(\lambda)_n}{n!} f(\alpha + n) \ (x+y)^n$$

$$= \sum_{r=0}^{\infty} \frac{(\lambda)_r}{r!} x^r y^r \sum_{m,n=0}^{\infty} \frac{(\lambda + r)_m (\lambda + r)_n}{m! \ n!} f(\alpha + m + n + 2r) x^m y^n$$

$$\ldots(5.1)$$

provided that the series involved converge absolutely.

PROOF : Using the identity (Singhal 1974)

$$\sum_{N=0}^{\infty} f(N) \frac{(x_1 + \ldots + x_n)^N}{N!}$$

$$= \sum_{m_1, \ldots, m_n=0}^{\infty} f(m_1 + \ldots + m_n) \frac{x_1^{m_1}}{m_1!} \ldots \frac{x_n^{m_n}}{m_n!} \quad \ldots(5.2)$$

where $\{f(N)\}$ is a sequence of arbitrary complex numbers and it is assumed that the series involved converge absolutely, we get

Left-hand side of (5.1)

$$= \sum_{m, n=0}^{\infty} \frac{(\lambda)_{m+n}}{m! \; n!} \; f(\alpha + m + n) \; x^m y^n$$

$$= \sum_{m, n=0}^{\infty} (\lambda)_{m+n} \frac{(E_\alpha x)^m}{m!} \frac{(E_\alpha y)^n}{n!} \cdot f(\alpha)$$

$$= (1 - E_\alpha x - E_\alpha y)^{-\lambda}, f(\alpha)$$

using (2.6), we get the right-hand side.

This completes the proof of (5.1).

Example 4—Let

$$f(\alpha) = \frac{\Gamma[(a + \alpha)]}{\Gamma[(b + \alpha)]},$$

we get (for $\alpha = 0$)

$${}_{A+1}F_B \left[\begin{matrix} \lambda, & (a); \\ & (b) \end{matrix} \; x + y \right]$$

$$= \sum_{r=0}^{\infty} \frac{[\lambda]_r \, [(a)]_{2r}}{r! \; [(b)]_{2r}} x^r y^r \; F \left[\begin{matrix} (a + 2r): & \lambda + r; & \lambda + r; \\ (b + 2r): & -; & -; \end{matrix} \; x, y \right] \quad \ldots(5.3)$$

or particularly we obtain

$${}_2F_1 \; [a, \; b; \; c; \; x + y]$$

$$= \sum_{r=0}^{\infty} \frac{(a)_{2r} \, (b)_r}{r! \; (c)_{2r}} x^r y^r F_1 \; [a + 2r, b + r, b + r; c + 2r; x, y] \quad \ldots(5.4)$$

which is formula (38) of Burchnall and Chaundy (1940).

The function F_1 is Appell's double hypergeometric function defined as (Appell and Kampé de Fériet 1926):

$$F_1\,[a,\ b,\ b';\ c;\ x,\ y] = \sum_{m,\,n=0}^{\infty} \frac{(a)_{m+n}\,(b)_m\,(b')_n}{(c)_{m+n}\,m!\ n!}\,x^m\,y^n$$

Also in particular (5.1) reduces to a known result [Burchnall and Chaundy 1941, p. 121 (44)]

$$_1F_1\ [a;\ c;\ x+y]$$

$$= \sum_{r=0}^{\infty} \frac{(a)_r}{r!\ (c)_{2r}}\,x^r\,y^r\ \ \phi_2\,(a+r,\ a+r;\ c+2r;\ x,\ y), \qquad \ldots(5.5)$$

It may be noted here that the formulas (44) and (45) of Burchnall and Chaundy (1940), i.e.,

$$_2F_1\ (a,\ b;\ c;\ x+y-xy)$$

$$= \sum_{r=0}^{\infty} \frac{(-1)^r\,(a)_r\,(b)_r}{r!\,(c)_r}\,x^r\,y^r\ {_2F_1}\,(a+r,\ b+r;\ c+r;\ x+y) \quad \ldots(5.6)$$

and

$$_2F_1\ (a,\ b;\ c;\ x+y)$$

$$= \sum_{r=0}^{\infty} \frac{(a)_r\,(b)_r}{r!\ (c)_r}\,x^r\ y^r\ {_2F_1}\ (a+r,\ \ b+r;\ \ c+r;\ \ x+y-xy) \qquad \ldots(5.7)$$

are direct consequences of the identity (5.2).

6. Fifth Identity

$$\sum_{n=0}^{\infty} \frac{t^n}{n!}\,\Delta_\alpha^n\ \Delta_\beta^n\ \ f_1(\alpha)\ \ f_2(\beta)$$

$$= e^t \sum_{m,\,n,\,p=0}^{\infty} f_1(\alpha+m+p)\,f_2\ (\beta+m+n)\,\frac{(-1)^{n+p}}{m!\,n!\,p!}\,t^{m+n+p} \qquad \ldots(6.1)$$

provided that the series involved converge absolutely. As the proof of this identity is very easy, we are not giving here.

Example 5—Taking $f_1(\alpha)$ and $f_2(\beta)$ as in Example 1, we get the following result

$$e^t \sum_{n=0}^{\infty} \frac{[(a)]_n\,[(c)]_n}{[(b)]_n\,[(d)]_n} \frac{(xyt)^n}{n!}\ {}_AF_B\left[\begin{matrix}(a+n); \\ (b+n);\end{matrix} -xt\right]\ {}_CF_D\left[\begin{matrix}(c+n); \\ (d+n)\end{matrix} -yt\right]$$

$$= \sum_{n=0}^{\infty} {}_{A+1}F_B\left[\begin{matrix}(a), -n; \\ (b),\end{matrix}\ x\right]{}_{C+1}F_D\left[\begin{matrix}(c); -n; \\ (d);\end{matrix}\ y\right]\frac{t^n}{n!} \qquad \ldots(6.2)$$

In a forthcoming communication, we shall discuss some other identities.

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