OSCILLATORY FLOW OVER A POROUS FLEXIBLE SURFACE WITH SUCTION

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(Communicated by F. C. Auluck, F.N.A.)

(*Received 10 June 1974*)

A two-dimensional osciallatory flow over a flat flexible surface with variable suction is analysed. Low and high frequency solutions are developed separately. The response of skin friction to fluctuating free-stream and suction velocity is studied for variations in suction velocity and flexible wall parameters.

1. INTRODUCTION

Increasing attention is now being focussed on studying the effect of a flexible boundary on the characteristics of flow in the domain of boundary layer research. The basic stimulus comes from the possibilities of flow stabilization and consequent substantial drag reduction by the use of a flexible boundary. Kramer (1960) reported considerable drag reduction in water for under-water solid bodies of revolution covered with flexible surface coating. Since then considerable theoretical investigations have been conducted by Benjamin (1960), Landahl (1962), Kaplan (1964), Gyorgyfalvy (1967) and others to explore this new method of boundary layer control.

In the present paper a new aspect of laminar boundary layer flow over a flexible boundary is considered. Since foam or similar porous and rubber-like materials are being tried to serve as a flexible boundary, it appeared relevant to study flows over such surfaces. Oscillatory two-dimensional flow over a porous and flexible wall is considered. Both the free stream velocity and suction velocity at the wall are assumed to be oxcillating about a constant non-zero mean. The solution is obtained by perturbing the flow (Lighthill 1954) and solving the governing equations so obtained under appropriate boundary conditions. The flexible wall model considered is a membrane supported over an elastic base, which can be treated as a spring-mass-dashpot system undergoing forced oscillations from an external oscillating flow. Both low and high frequency solutions are developed. The response of skin friction to fluctuating free stream and suction velocity is studied for variations in suction velocity and flexible wall parameters.

2. Basic Equations

Consider a two-dimensional time-dependent laminar boundary layer over a plane porous flexible wall. Both the free stream velocity $\bar{U}$ and suction velocity $\bar{v}_s$ at the wall are oscillatory in nature and can be expressed as

$$\bar{U} = \bar{U}_0 (1 + \epsilon \cos \bar{\omega}\bar{t}), \qquad \ldots(2.1)$$

$$\bar{v}_s = \bar{v}_0 (1 + \epsilon \cos \bar{\omega}\bar{t}) \qquad \ldots(2.2)$$

where $\bar{\omega}$ is the frequency of oscillation and $\epsilon \ll 1$.

An overbar is used to represent the dimensional form of a quantity.

The basic governing equations for the flow are

$$\frac{\partial \bar{u}}{\partial \bar{t}} + \bar{u} \frac{\partial \bar{u}}{\partial \bar{x}} + \bar{v} \frac{\partial \bar{u}}{d \bar{y}} = \frac{\partial \bar{U}}{\partial \bar{t}} + \nu \frac{\partial^2 \bar{u}}{\partial \bar{y}^2} \quad , \qquad \ldots(2.3)$$

$$\frac{\partial \bar{u}}{\partial \bar{x}} + \frac{\partial \bar{v}}{\partial \bar{y}} = 0. \qquad \ldots(2.4)$$

Introducing dimensionless quantities

$$\left. \begin{aligned} x = \bar{x}\frac{U_0}{\nu}, \quad y = \bar{y}\frac{U_0}{\nu}, \quad t = \bar{t}\frac{U_0^2}{\nu}, \quad \omega = \bar{\omega}\frac{\nu}{U_0^2} \\ u = \frac{\bar{u}}{U_0}, \quad v = \frac{\bar{v}}{U_0}, \quad v_0 = \frac{\bar{v}_4}{U_0}, \quad v_s = \frac{\bar{v}_s}{U_0}, \quad U = \frac{\bar{U}}{U_0} \end{aligned} \right\} \qquad \ldots(2.5)$$

equations (2.1) through (2.4) take the following non-dimensional form

$$\left. \begin{aligned} U &= 1 + \epsilon \cos \omega t, \\ v_s &= v_0 (1 + \epsilon \cos \omega t), \end{aligned} \right\} \qquad \ldots(2.6)$$

$$\left. \begin{aligned} &\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\partial U}{\partial t} + \frac{\partial^2 u}{\partial y^2}, \\ &\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \end{aligned} \right\} \qquad \ldots(2.7)$$

The boundary conditions to be satisfied are

$$\left. \begin{aligned} &y = 0: \quad u = 0, \quad v = v_w + (-v_s) ; \\ &y \to o0: \quad u \to 1 + \epsilon \cos \omega t, \quad \epsilon \ll 1 \end{aligned} \right\} \qquad \ldots(2.8)$$

where v_w is the normal velocity of the flexible wall.

3. Model of the Flexible Surface

The flexible surface model considered is a membrane supported over a foam-like elastic base and whose behaviour is characterised by its mass $\bar{m}$, tension $\bar{T}$

damping $\bar{d}_0$ and compression stiffness or spring constant $\bar{K}_w$, and thus can be treated as a spring-mass-dashpot system. In the absence of any oscillations in free stream, this flexible surface will behave more or less like a rigid one. But due to velocity and the corresponding pressure oscillations in the free stream, the flexible surface will undergo forced oscillations. Its equation of motion can be expressed as

$$\bar{T}\frac{\partial^2 \bar{y}}{\partial \bar{x}^2} - \bar{m}\frac{\partial^2 \bar{y}}{\partial \bar{t}^2} - \bar{d}_0\frac{\partial \bar{y}}{\partial \bar{t}} - \bar{K}_w \bar{y} = \bar{p}_w \qquad \text{...(3.1)}$$

where $\bar{p}_\omega$ is the pressure at the surface.

Introducing following non-dimensional quantities

$$\left.\begin{aligned} &T = \frac{\bar{T}}{\rho v U_0^2}, \quad m = \frac{\bar{m}}{\rho(v/U_0)^3}, \qquad d_0\frac{\bar{d}}{\rho\,(v/U_0)^2}, \quad K_w = \frac{\bar{K}}{\rho v\, U_0}, \\ &y = \frac{\bar{y}}{v/U_0}, \quad P_w = \frac{p_m}{\rho\, U_0^2}, \quad t = \frac{\bar{t}}{v/U_0^2}, \end{aligned}\right\} \qquad \text{...(3.2)}$$

eqn. (2.8) becomes

$$T\frac{\partial^2 y}{\partial x^3} - m\frac{\partial^2 y}{\partial t^3} - d_0\frac{\partial y}{\partial t} - K_\omega y = p_w, \qquad \text{...(3.3)}$$

where $p_w = \epsilon\, i\omega x\,.\, e^{i\omega t}$, is the dimensionless wall pressure.

The solution of eqn. (3.3) is

$$y = -\frac{i\hat{p}_w\, e^{i(\omega t - \phi)}}{z_m}, \qquad \text{...(3.4)}$$

where

$$\left.\begin{aligned} z_m &= [\omega^2 d_0^2 + (\omega^2 m - K_w)^2]^{1/2}, \\ \tan\phi &= \frac{\omega^2 m - K_w}{\omega d_0}, \end{aligned}\right\} \qquad \text{...(3.5)}$$

$$p_w = \epsilon\, i\omega x.$$

In the above result complex notation is used with the obvious implication that only real parts will have physical significance.

Setting $\eta = y/\delta$, δ being the boundary layer thickness, eqn. (3.4) can be recast in the form

$$\eta = \hat{\eta}\, e^{i\theta} \qquad \text{...(3.6)}$$

where

$$\eta = -\frac{i\hat{p}_w e^{-i\Phi}}{z_m \delta}.$$

The normal velocity of the flexible surface is then given by

$$v_w = \frac{\partial \eta_w}{\partial t} = i\omega \hat{\eta} e^{it}. \qquad ...(3.7)$$

4. Mathematical Analysis

The oscillatory flow field can be expressed as the sum of steady and oscillating components

$$\left.\begin{aligned} u &= u_1 + \epsilon u_2 e^{i\bar{\omega}t} \\ v &= v_1 + \epsilon v_2 e^{i\omega t}. \end{aligned}\right\} \qquad ...(4.1)$$

On substituting eqns. (4.1) into eqns. (2.7) it is found that the steady mean flow u_1, v_1 satisfies

$$\left.\begin{aligned} & u_1 \frac{\partial u_1}{\partial x} + v_1 \frac{\partial u_1}{\partial y} = \frac{\partial^2 u_1}{\partial y^2} \\ & \frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} = 0 \end{aligned}\right\} \qquad ...(4.2)$$

with the boundary conditions

$$\left.\begin{aligned} & y = 0: \quad u_1 = 0, \quad v_1 = -v_0; \\ & y \to \infty: \quad u_1 \to 1, \end{aligned}\right\} \qquad ...(4.3)$$

and that harmonic components u_2, v_2 satisfy the following equations

$$\left.\begin{aligned} & i\omega u_2 + u_1 \frac{\partial u_2}{\partial x} + u_2 \frac{\partial u_1}{\partial x} + v_2 \frac{\partial u_1}{\partial y} + v_1 \frac{\partial u_2}{\partial y} = i\omega + \frac{\partial^2 u_2}{\partial y^2} \\ & \frac{\partial u_2}{\partial x} + \frac{\partial v_2}{\partial y} = 0 \end{aligned}\right\} \qquad ...(4.4)$$

with the boundary conditions

$$\left.\begin{aligned} & y = 0: \quad u_2 = 0, \quad v_2 = \frac{i\omega\hat{\eta}}{\epsilon} - v_0, \quad \hat{\eta} \ll 1; \\ & y \to \infty: \quad u_2 = 1. \end{aligned}\right\} \qquad ...(4.5)$$

To solve eqn. (4.2), the following expression for u_1, consistent with boundary conditions (4.3), is assumed

$$u_1 = A_1 (\eta - 3\eta^2 + 3\eta^3 - \eta^4) + (6\eta^2 - 8\eta^3 + 3\eta^4) \qquad ...(4.6)$$

in which A_1 is an unknown function of x and is related to boundary layer thickness δ by the relation

$$A_1 = \frac{12}{(6 - v_0 \delta)} \, . \qquad \ldots(4.7)$$

The additional boundary conditions utilized are

$$\left.\begin{array}{l} \left(\dfrac{\partial^2 u_1}{\partial \eta^2}\right)_{\eta=0} = - v_0 \, \delta \left(\dfrac{\partial u_1}{\partial \eta}\right)_{\eta-0} \\ \left(\dfrac{\partial u_1}{v}\right)_{\eta\to 1} \to 0, \quad \left(\dfrac{\partial^2 u}{\partial \eta^2}\right)_{\eta\to 1} \to 0. \end{array}\right\} \qquad \ldots(4.8)$$

Integrating eqn. (4.2) from $y = 0$ to $y = \delta$ and using the boundary conditions (4.3), the following integral equation is obtained

$$\frac{\partial}{\partial x}\left(\delta \int_0^1 u_1^2 \, d\eta - \delta \int_0^1 u_1 \, d\eta\right) + \frac{1}{\delta}\left(\frac{\partial u_1}{\partial \eta}\right) = 0. \qquad \ldots(4.9)$$

After substituting for u_1 and δ and introducing suction parameter

$\xi = (- v_0)^2 \, x$, eqn. (4.9) takes the form

$$\frac{dA_1}{d\xi} = \frac{- 17 \cdot 5 \, A_1^4}{(5A_1^4 - 21A_1^3 + 22A_1^2 - 144 \, A_1 + 288)} \, . \qquad \ldots(4.10)$$

The initial condition for A_1 is

$$x = 0; \quad A_1 = 2. \qquad \ldots(4.11)$$

The above first order eqn. (4.10) is solved numerically by Runge-Kutta fourth order method on IBM 1130. Since there is a singularity at $\xi = 0$, the solution was started by inverting eqn. (4.10) and solving it as such for first two steps. With these current values of A_1 and ξ, the integration of eqn. (4.10) is then continued.

Also, the exact solution of eqn. (4.10) is

$$5A_1 - 21 \ln A_1 - \frac{22}{A_1} + \frac{72}{A_1^2} - \frac{96}{A_1^3} + 9 \cdot 55 = - 17 \cdot 5 \, \xi. \qquad \ldots(4.12)$$

Low Frequency Oscillations

In order to solve eqns. (4.4) for low frequency oscillations, u_2 and v_2 are expressed as the sum of in-phase and out-phase components

$$\left.\begin{array}{l} u_2 = u_{2r} + i \, u_{2i} \\ v_2 = v_{2r} + i \, v_{2i}. \end{array}\right\} \qquad \ldots(4.13)$$

Substituting in eqn. (4.4) and separating real and imaginary parts, the following sets of equations are obtained

$$\left.\begin{aligned} & -\omega u_{2i} + u_1 \frac{\partial u_{2r}}{\partial x} + u_{2r}\frac{\partial u_1}{\partial x} + \frac{\partial u_{2r}}{\partial y} + v_{2r}\frac{\partial u_1}{\delta y} = \frac{\partial^2 u_{2r}}{\partial y^2} \\ & \frac{\partial u_{2r}}{\partial x} + \frac{\partial v_{2r}}{\partial y} = 0. \end{aligned}\right\} \qquad \text{...(4.14)}$$

with the boundary conditions

$$\left.\begin{aligned} & y = 0; \quad u_{2r} = 0, \quad v_{2r} = -v_0; \\ & y \to \infty: \quad u_{2r} \to 1. \end{aligned}\right\} \qquad \text{...(4.15)}$$

and

$$\left.\begin{aligned} & u_{2r} + u_1 \frac{\partial u_{2i}}{\partial x} + u_{2i}\frac{\partial u_1}{\partial x} + v_1 \frac{\partial u_{2i}}{\partial y} + v_{2i}\frac{\partial u_1}{\partial y} = \omega + \frac{\partial^2 u_{2i}}{\partial y^2} \\ & \frac{\partial u_{2i}}{\partial x} + \frac{\partial v_{2i}}{\partial y} = 0 \end{aligned}\right\} \qquad \text{...(4.16)}$$

with the boundary conditions

$$\left.\begin{aligned} & y = 0: \quad u_{2i} = 0, \quad v_{2i} = \frac{\omega\hat{\eta}}{\epsilon}; \\ & y \to \infty: \quad u_{2i} \to 0. \end{aligned}\right\} \qquad \text{...(4.17)}$$

For the case of low frequency oscillations, the quasisteady solution of eqn. (4.14) is obtained by assuming the following expression for u_{2r}

$$u_{2r} = (4\eta^3 - 3\eta^4) + A_{2r}(\eta - 3\eta^3 + 2\eta^4) + \frac{(-v_0)\,\delta}{2} \times (A_{2r} + A_1)(\eta^2 - 2\eta^3 + \eta^4) \qquad \text{...(4.18)}$$

which in addition to boundary conditions (4.15) also satisfies

$$\left.\begin{aligned} & -v_0\left(\frac{\partial u_{2r}}{\partial y}\right)_{y=0} - v_0\left(\frac{\partial u_1}{\partial y}\right)_{y=0} = \left(\frac{\partial u^2{}_{2r}}{\partial y^2}\right)_{y=0} \\ & \left(\frac{\partial u_{2r}}{\partial y}\right)_{y\to\infty} \to 0. \end{aligned}\right\} \qquad \text{...(4.19)}$$

Integrating eqn. (4.14) from $y = 0$ to $y = \delta$ and using boundary conditions (4.15), the following integral equation is obtained

$$\frac{\partial}{\partial x}\left(2\delta \int_0^1 u_1 u_{2r}\, d\eta\right) - \frac{\partial}{\partial x}\left(\delta \int_0^1 (u_1 + u_{2r})\, d\eta\right) + \frac{1}{\delta}\left(\frac{\partial u_{2r}}{\partial \eta}\right)_{\eta=0} = 0. \qquad \text{...(4.20)}$$

Substituting for u_1, u_{2r} and instroducing ξ, the eqn. (4.20) becomes.

$$T_1 \frac{dA_{2r}}{d\xi} + \frac{1}{v_0^2 \delta^2} A_{2r} + T_2 \frac{dA_1}{d\xi} + T_3 \frac{1}{\delta} \frac{d\delta}{d\xi} = 0, \qquad \text{...(4.21)}$$

where

$$\left.\begin{aligned} T_1 &= \frac{10}{504} A_1 - \frac{1}{504} v_0 \delta A_1 - \frac{1}{210} v_0 \delta + \frac{2}{105} \\ T_2 &= \left(\frac{10}{504} - \frac{1}{504} v_0 \delta\right) A_{2r} - \frac{2}{504} v_0 \delta A_1 \\ &\quad - \frac{4}{105} v_0 \delta - \frac{1}{30} \\ T_3 &= \frac{2}{105} A_{2r} + \frac{10}{504} A_{2r} A_1 - \frac{v_0 \delta}{504} (A_2 + A_{2r}) A_1 \\ &\quad - \frac{1}{210} v_0 \delta (A_{2r} + A_1) - \frac{A_1}{30} - \frac{2}{7}. \end{aligned}\right\} \qquad \text{...(4.22)}$$

Equation (4.21) can be put in the following convenient form

$$\frac{dA_{2r}}{d\xi} = \frac{-\left(T_2 \frac{dA_1}{d\xi} + \frac{12}{v_0 A_1^2} \frac{dA_1}{d\xi} \frac{T_3}{\delta} + \frac{A_{2r}}{v_0^2 \delta^2}\right)}{T_1}. \qquad \text{...(4.23)}$$

As it is not possible to solve eqn. (4.23) analytically, this first order equation is solved numerically on IBM 1130 by using Runge-Kutta fourth order method. Integration was started by using the relation $A_{2r} = A_1^2/2$ [obtained from boundary condition $(\partial^3 u_{2r}/\partial \eta^2)_{\eta=1} = 0$] for the initial step. It is found that the function A_{2r} is not sensitive to slight changs in initial value.

Next eqns. (4.16) are considered. Consistent with the boundary condition (4.17) the following expression for u_{2i} is assumed

$$u_{2i} = A_{2i} (\eta - 3\eta^3 + 2\eta^4) - \left[\frac{v_0 \delta}{2} A_{2i} + \frac{\omega \delta^2}{2}\left(1 + \frac{W_K A_1}{2}\right)\right]$$
$$\times (\eta^2 - 2\eta^3 + \eta^4). \qquad \text{...(4.24)}$$

Additional boundary conditions imposed are

$$\left.\begin{aligned} &- v_0 \left(\frac{\partial u_{2i}}{\partial y}\right)_{y=0} + \left(v_{2i} \frac{\partial u_1}{\partial y}\right)_{y=0} = \omega + \left(\frac{\partial^2 u_{2i}}{\partial y^2}\right)_{y=0} \\ &\left(\frac{\partial u_{2i}}{\delta y}\right)_{y \to \infty} \to 0. \end{aligned}\right\} \qquad \text{...(4.25)}$$

Integration of eqn. (4.16) over the entire boundary layer thickness leads to

$$\omega \int_0^1 (u_{2r} - 1)\, d\eta + \frac{2}{\delta} \frac{\partial}{\partial x}\left(\delta \int_0^1 u_{2i}\, u_1\, d\eta\right) + \frac{1}{\delta^2}\left(\frac{\partial u_{2i}}{\partial \eta}\right)_{\eta=0}$$

$$- \frac{1}{\delta}\frac{\partial}{\partial x}\left(\delta \int_0^1 u_{2i}\, d\eta\right) = 0. \qquad \ldots(4.26)$$

Substituting for v_1, u_{2r}, u_{2i} and introducing ξ in eqn. (4.26) yields

$$\frac{dA_{2i}}{d\xi} T_1 + A_{2i} T_4 + \frac{A_{2i}}{v_0^2 \delta^2} + \frac{1}{\delta}\frac{d\delta}{d\xi}\left\{A_{2i}\left(\frac{5A_1}{252} + \frac{2}{105}\right)\right.$$

$$\left. - \left[\frac{v_0 \delta}{2} A_{2i} + \frac{\delta^2}{2}\left(1 + \frac{W_K A_1}{2}\right)\right]\left(\frac{A_1}{252} + \frac{1}{105}\right)\right\} - T_5 = 0 \ldots(4.27)$$

where

$$T_4 = \frac{1}{504}\left[5 \frac{dA_1}{d\xi} - \frac{v_0 \delta}{2}\frac{dA_1}{d\xi} - \frac{v_0}{2}\frac{d\delta}{d\xi} A_1\right] + \frac{5}{840} v_0 \frac{d\delta}{d\xi}$$

$$T_5 = \frac{\omega}{v_0^2}\left\{\left[\left(24 \frac{v_0 \delta}{A_1} + v_0^2 \delta^2\right)\left(\frac{1}{504} + \frac{W_K}{420}\right) + \frac{12}{105}\frac{v_0 \delta}{A_1}\right.\right.$$

$$\left. + \frac{W_K}{504}(12\, v_0 \delta + v_0^2 \partial^3 A_1)\right]\frac{dA_1}{d\xi} + \frac{v_4 \delta}{60} \qquad \ldots(4.28)$$

$$\left. - (A_{2r} + A_1) - \frac{3}{20} A_{2r} + \frac{3}{5}\right\}.$$

The first order eqn. (4.27) is integrated numerically on IBM 1130 using Runge-Kutta fourth order method. Integration was started by using the relation

$$A_{2i} = \frac{\delta^2 A_1}{12}\left(1 + \frac{W_K A_1}{2}\right),$$

$\left[\text{obtained from the boundary condition } \left(\frac{\partial^2 u_{2i}}{\partial \eta^2}\right)_{\eta-1} = 0\right]$ for the first step.

High Frequency Oscillations

For high frequency solution of eqn. (4.4), u_2, v_2 are expanded in following series

$$\left.\begin{aligned} u_2 &= \sum_{n=0}^{\infty} (i\omega)^{-\frac{n}{2}} u_1^{(n)}(x, y), \\ v_2 &= \sum_{n=0}^{\infty} (i\omega)^{-\frac{n+1}{2}} v_1^{(n)}(x, Y), \end{aligned}\right\} \qquad \ldots(4.29)$$

where

$$Y = y\,(i\omega)^{1/2},$$

Inserting eqn. (4.29) into eqn. (4.4) and collecting coefficients of like powers of $(i\omega)^{-1/2}$ yields the following equations

$$\left.\begin{aligned}
&\frac{\partial^2 u_2^{(0)}}{\partial Y^2} - u_2^{(0)} + 1 = 0 \\
&\frac{\partial^2 u_2^{(1)}}{\partial Y^2} - u_2^{(1)} + v_0 \frac{\partial u_2^{(0)}}{\partial Y} = 0 \\
&\frac{\partial^2 u_2^{(2)}}{\partial Y^2} - u_2^{(2)} + v_0 \frac{\partial u_2^{(1)}}{\partial Y} = 0 \\
&\frac{\partial^2 u_2^{(3)}}{\partial Y^2} - u_2^{(3)} - b_4' Y u_2^{(0)} \\
&\qquad + b_4' \frac{Y^2}{2} \frac{\partial u_2^{(0)}}{\partial Y} \\
&\qquad - v_0 \frac{\partial u_2^{(2)}}{\partial Y} + b_4 v_2^{(0)} = 0
\end{aligned}\right\} \qquad \ldots(4.30)$$

where

$$b_A = \frac{A_1}{\delta} \quad \text{and} \quad b_4' = \frac{1}{\delta}\left[\frac{dA_1}{dx} - \frac{A_1}{\delta}\frac{d\delta}{dx}\right].$$

Integration of eqn. (4.30) leads to

$$\left.\begin{aligned}
&u_2^{(0)} = 1 - e^{-Y} \\
&u_2^{(1)} = -v_0\, e^{-Y} Y \\
&u_2^{(2)} = v_0^2\, e^{-Y} Y (1 - Y), \\
&u_2^{(3)} = b_5 (1 - e^{-Y}) + b_4'\left[- Y + e^{-Y}\left(\frac{3}{8} Y + \frac{3}{8} Y^2\right.\right. \\
&\qquad \left.\left. + \frac{1}{12} Y^3\right)\right] + v_0^3\, e^{-Y} Y^2 \left(1 - \frac{Y}{3}\right),
\end{aligned}\right\} \qquad \ldots(4.31)$$

where b_4 is a constant which shall be determined from the following boundary condition at the flexible wall

$$\left(v_1 \frac{\partial u_2}{\partial y}\right)_{y=0} + \left(v_2 \frac{\partial u_1}{\partial y}\right)_{y=0} = i\omega + \left(\frac{\partial^2 u_2}{\partial y^2}\right)_{y=0}. \qquad \ldots(4.32)$$

Utilising eqns. (4.32) the solution finally takes the form

$$\left.\begin{aligned} u_2 = {} & (1 - e^{-Y})\left(1 + \frac{A_1 W_K}{2}\right) + (i\omega)^{-1/2}\left[3v_0 + \frac{v_0 A_1 W_k}{2}\right. \\ & \left. - e^{-Y}\left(3v_0 + \frac{v_0 A_1 W_K}{2}\right) - v_0 e^{-Y} Y\right] \\ & + (i\omega)^{-1}\left[(1 - e^{-Y})\left(b_4 v_0 - 2v_0^2 + \frac{v_0^2 A_1 W_K}{2}\right.\right. \\ & \left.\left. + v_0^2 e^{-Y} Y (1 - Y)\right)\right] + (i\omega)^{-3/2}\left\{\left(- b_4 v_0^2 + 11 v_0^3\right.\right. \\ & \left. - \frac{v_0^3 A_1 W_K}{2}\right)(1 - e^{-Y}) + v_0^3 e^{-Y} Y^2 (1 - Y/3) \\ & \left. + b_4'\left[- Y + e^{-Y}\left(\frac{3}{8} Y + \frac{3}{8} Y^2 + \frac{1}{12} Y^3\right)\right]\right\}. \end{aligned}\right\} \quad ...(4.33)$$

5. Discussion of Results

For low frequency oscillations the longitudinal component of velocity is given by

$$u = u_1 + \epsilon R_4 \cos(\omega t + \alpha_4), \quad ...(5.1)$$

where

$$\left.\begin{aligned} R_4 &= (u_{2r}^2 + u_{2i}^2)^{1/2} \\ \alpha_4 &= \tan^{-1}\frac{u_{2i}}{u_{2r}}. \end{aligned}\right\} \quad ...(5.2)$$

The oscillating component of skin friction at the flexible wall, in case of low frequency oscillations, is given by

$$\tau_{2L} = \epsilon R_5 \cos(\omega t + \alpha_5) \quad ...(5.3)$$

where

$$\left.\begin{aligned} R_5 &= \frac{1}{\delta}(A_{2r}^2 + A_{2i}^2)^{1/2} \\ \alpha_5 &= \tan^{-1}\frac{A_{2i}}{A_{2r}}. \end{aligned}\right\} \quad ...(5.4)$$

For high frequency oscillations, the oscillating component of skin friction at the flexible wall is given by

$$\tau_{2H} = \epsilon R_6 \cos(\omega t + \alpha_6). \quad ...(5.5)$$

where

$$\left.\begin{aligned}
R_6 &= (b_6^2 + b_7^2)^{1/2} \\
\alpha_6 &= \tan^{-1}\frac{b_7}{b_6} \\
b_6 &= \left(1 + \frac{A_1 W_K}{2}\right)\frac{\omega^{1/2}}{\sqrt{2}} + \left(2v_0 + \frac{v_0 A_1 M_K}{2}\right. \\
&\quad + \left(b_4 v_0 - v_0^2 + \frac{v_0^2 A_1 W_K}{2}\right)\frac{\omega^{-1/2}}{\sqrt{2}} \\
b_7 &= \left(1 + \frac{A_1 W_K}{2}\right)\frac{\omega^{1/2}}{\sqrt{2}} - \left(b_4 v_0 - v_0^2 + \frac{v_0^2 A_1 W_K}{2}\right. \\
&\quad \times \frac{\omega^{1/2}}{\sqrt{2}} - \left(b_4 v_0^2 + v_0^3 + \frac{v_0^3 A_1 W_K}{2} - \frac{5}{8} b_4'\right)\omega^{-1}.
\end{aligned}\right\} \quad \ldots(5.6)$$

The results discussed below are based on the output of a computer programme which makes use of the values of A_1, A_{2r} and A_{2i} obtained from another programme.

The steady velocity component u_1, and unsteady in phase velocity component u_{2r} depend upon the suction velocity, characterised by suction parameter $\xi = (-v_0)^2 x$, but these are independent of the flexible wall properties. The functions u_1 and u_{2r} are shown in Figs. 1 and 2 respectively for various values of suction parameter.

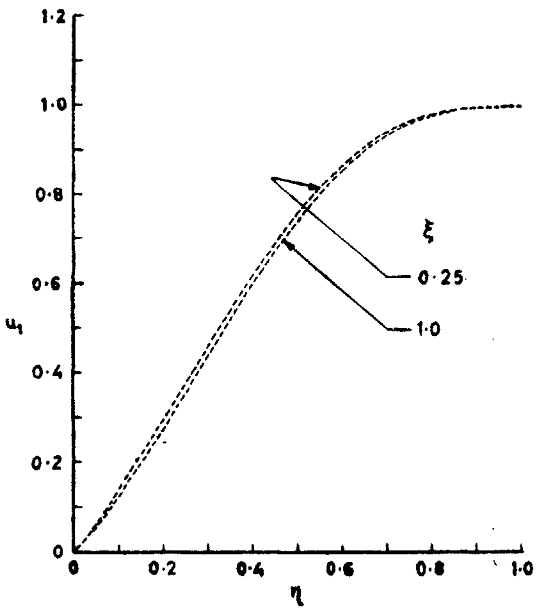


FIG. 1. Steady velocity component u_1 versus η for different values of suction parameter ξ

The unsteady out-phase velocity component u_{2i} is a function of wall properties ω_0, d_0 and m, suction parameter ϵ, and the frequency parameter $\zeta = \omega/(-v_0)^2$.

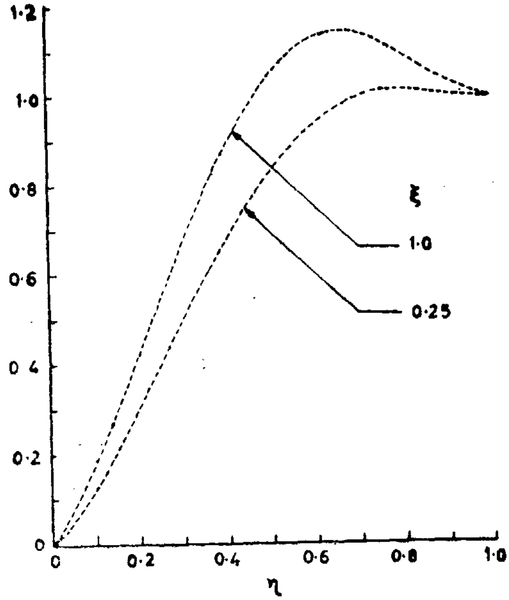


FIG. 2. Unsteady in-phase velocity component u_{2_r} versus η for different values of suction parameter ξ.

It is desirable to study the effect of ϵ and on u_{2i}. Keeping the wall properties and ξ unchanged, the variation of u_{2i} versus η for different values of ζ is exhibited in Fig. 3. Increase in ζ results in an increase in u_{2i} at all points

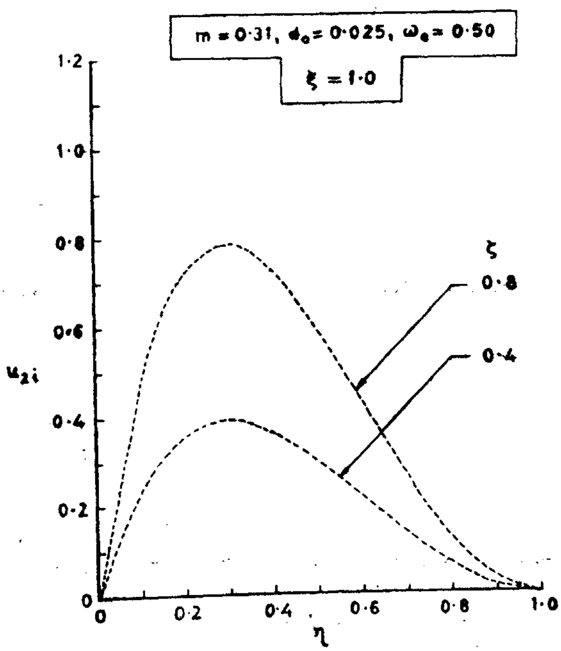


FIG. 3. Unsteady out-phase velocity component u_{2i} versus η for different values of frequency parameter ξ.

inside the boundary layer. One new feature to be noted is that u_{2i} is positive throughout the boundary layer, that is, near the flexible wall as well as near the edge of the boundary layer. Thus the velocity oscillations in this case are always leading the free stream oscillations. Obviously this change in the nature of function u_{2i} is due to the introduction of suction at the wall. Also it is relevant to point out here that the u_{2i} curves for rigid surface for the sake of comparison are not shown, since these themselves are functions of ξ and ζ, and have the same nature.

Next keeping the wall properties unchanged, the effect of suction parameter ξ on u_{2i} is studied. This is illustrated in Fig. 4. An increase in the value of suction parameter causes an increase in u_{2i} at all points inside the boundary layer.

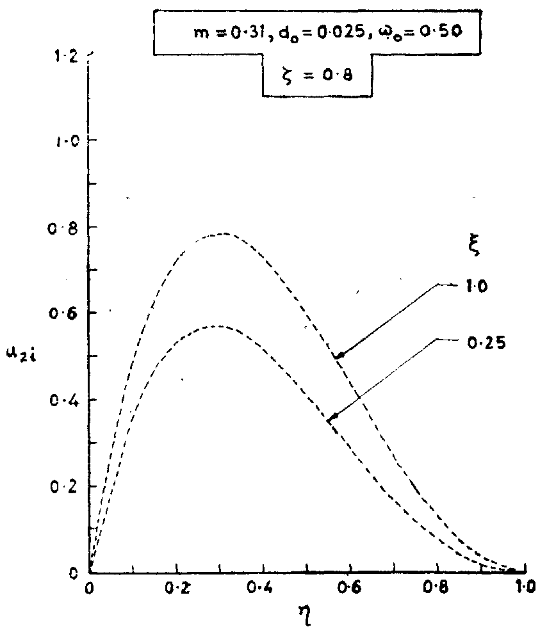


FIG. 4. Unsteady out-phase velocity component u_{2i} versus η for different values of suction parameter ζ.

By keeping ξ and ζ constant, the effect of each wall properties on u_{2i} can be studied. It is found that increase in ω_0 below a critical value causes increase in u_{2i}, and thereafter any increase in ω_0 causes decrease in u_{2i}, increase in d_0 results in a decrease in u_{2i}, and lastly and increase in m causes a decrease in u_{2i}.

The variation of oscillating skin friction amplitude $R_5\delta$ versus frequency parameter ζ for various values of suction parameter ξ is shown in Fig. 5. It is found that with increase in suction parameter ξ, the amplitude $R_5\delta$ also increases. Also it may be noted that the value of $R_5\delta$ for any ξ is always higher than the corresponding value for the rigid wall.

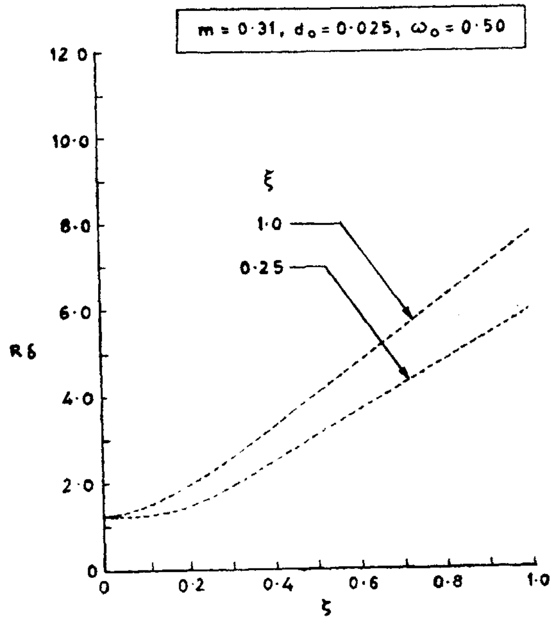
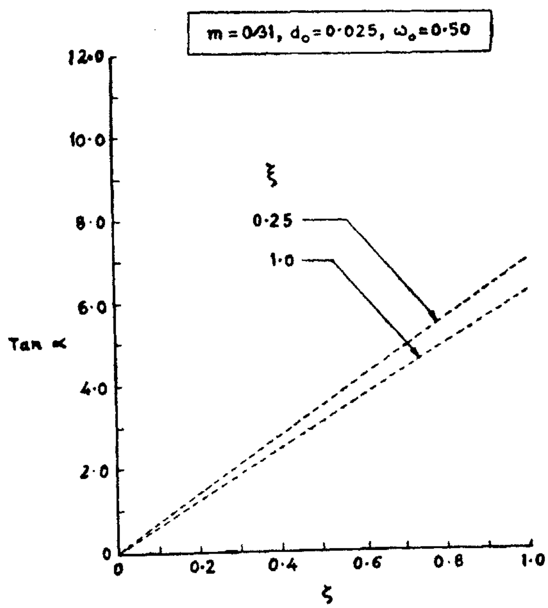


Fig. 5. Skin friction amplitude Rδ versus frequency parameter ζ for different values of suction parameter ξ.

In Fig. 6 is shown the variation of skin friction phase tan α_5 versus frequency parameter ζ for various values of suction parameter ξ. The value of phase tan α_5 decreases with increase in the value of suction parameter.



Fig, 6. Skin friction phase tan α versus frequency parameter ζ for different values of suction parameter ξ.

ACKNOWLEDGEMENT

The financial assistance by C.S.I.R. is gratefully acknowledged.

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