INFLUENCE OF POROSITY ON THE NON-STATIONARY FLOW BETWEEN TWO COAXIAL POROUS DISKS ROTATING WITH EQUAL TIME-VARIANT VELOCITIES

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The effect of a porous upper disk rotating coaxially with a time-variant angular velocity equal to that of the lower disk on the flow as well as heat transfer of an incompressible viscous fluid betewen them, has been examined. The influence of the suction, injection and absence of them at the upper disk, on the times of establishment of the non-stationery flow and heat transfer has also been discussed.

It is seen that if the suction velocity at the upper disk exceeds the injection velocity at the lower one, flow is establised earlier than the heat transfer. The utmost delay in the establishment of the flow as well as the heat transfer seems to occur in the case when the rate of injection through the lower disk is equal to the rate of suction through the upper-one.

1. INTRODUCTION

The study of unsteady motion between rotating disks has engaged the attention of several authors, due to their immense practical utility. A number of authors studied the unsteady flow of different fluids due to time-variant rotation of a single disk or two disks of infinite radii. The simple technique of expanding the flow functions in terms of certain parameters which involve the angular velocity of the disk and its higher derivatives, was employed after Sparrow and Gregg (1960) by Jawa (1966) and Rath and Iyengar (1968) during their study of similar problems. None of them however considered the influence of porosity on the flow, temperature functions and the time of establisment of the flow and heat transfer.

Here we have studied the flow, heat transfer and the times of their establishment, of a Newtonian-fluid due to the time-dependent rotations of two porous infinite disks about a common-axis. The equations set up are non-linear and have been solved numerically by Runge-Kutta method on an IBM-1130.

It is observed that the effect of suction and injection through the disks on the character of flow and heat-transfer is considerable. If the suction or injection velocity through the upper disk does not exceed the injection velocity through the lower disk, we see that heat transfer is established earlier than the flow. It is also interesting to note that the recovery of solid state rotation in the quasi-steady case takes place when the upper-disk sucks fluid at the same rate at which the lower disk injects, and that the establishment of flow and heat-transfer is considerably delayed in this case.

II. The Equations of Motion

We consider the motion of a Newtonian-viscous, incompressible fluid between two co-axial disks of large radii, separated by a relatively small gap. The disks are assumed (*i*) to rotate with time-dependent angular velocities $\omega(t)$ and $l\omega(t)$, (where l is real number) which might in general be continuously differentiable functions of time and (*ii*) to be porous, admitting suction and injection of fluid with time-variant velocities ; $W_0(t)$ and $mW_0(t)$, m being a real number. With u, v, ω, as the radial, azimuthal and axial components of the velocities, the equations of motion are as described in Sparrow and Gregg (1960).

The energy equation is

$$\rho_c (T_t + u T_r + w T_x) = k\nabla^2 T + \Phi_c, \tag{1}$$

where 'T' denotes the temperature, the suffixes denote the differentiation with respect to these variables, and Φ_c, dissipation function is given by

$$\Phi_c = \mu \left[8 \left\{ u_r^2 + \left(\frac{u}{r} \right)^2 + w_z^2 \right\} + v_z^2 + u_z^2 \right]. \tag{2}$$

The boundary conditions of the flow and energy equations are

$$\left.\begin{aligned}
&u(r, 0, t) = u(r, d, t) = 0\\
&v(r, 0, t) = r\,\omega(t),\ v(r. d, t) = l_r\,\omega(t)\\
&w(r. 0\,t) = W_0(t),\ \ W(r, d, t) = m\,W_0(t)\\
&T(r, 0, t) = T_{w_0} = \text{constant.}\\
&T_{w_1} = T(r, d, t) = n\,T(r, 0, t) = n\,T_{w_0}\\
&\text{'}n\text{' being a real number, with the initial conditions}\\
&u(r, z, 0) = v(r, z, 0) = w(r, z, 0) = 0.
\end{aligned}\right\} \tag{3}$$

III. Method of Solution

We nondimensionalise the equations by the transformations.

$$\left.\begin{aligned}
&\frac{u}{r\omega} = f(\eta, \beta_n) = F(\eta) + \sum_{n=1}^{\infty} \beta_n F_n(\eta) \\
&v/r\omega = g(\eta, \beta_n) = G(\eta) + \sum_{n=1}^{\infty} \beta_n G_n(\eta) \\
&w/\sqrt{\nu\omega} = h(\eta, \beta_n) = H(\eta) + \sum_{n=1}^{\infty} \beta_n H_n(\eta) \\
&\frac{T(r, z, t) - T_{w_0}}{T_{w_0}} = \tau(\eta) + \sum_{n=1}^{\infty} \beta_n \tau_n(\eta) \\
&= R^2 s(\eta, \beta_n) + q(\eta, \beta_n) \\
&\frac{P}{\mu\omega} = \left[P^*(\eta, \beta_n) + \frac{1}{2}\frac{r^2}{d^2} \right. . \lambda^*(\beta_n), \text{ where} \\
&P^*(\eta, \beta_n) = P(\eta) + \sum_{n=1}^{\infty} \beta_n . P_n(\eta), \text{ and} \\
&\lambda^*(\beta_n) = \lambda + \sum_{n=1}^{\infty} \beta_n \lambda_n, \ \eta = z/d, \ R = r/d
\end{aligned}\right\} \quad (4)$$

where

$\beta_1 = \dot{\omega}/\omega^2$, $\beta_2 = \ddot{\omega}/\omega^3$ etc. ; r = radius of the disk and d = distance between the two-disks. The first terms in each of the series are associated with the quasi-steady state and the succeeding terms denote the deviations from the quasi-steady state. The equations of continuity, motion and energy in terms of the new functions are,

Continuity equation

$$h' = -2/\alpha . f(b) \text{ where } \alpha^2 = \frac{\omega d^2}{\nu} = \text{Taylor's number.}$$

Equations of motion

$$f'' = \alpha^2 (f^2 - g^2 + \beta_1 . f + \beta_2 . \frac{\partial f}{\partial \beta_1} + \lambda^*) + \alpha h f' \quad (6)$$

To eliminate the constant λ^*, we differentiate with respect to η and get,

$$f''' = \alpha^2 (2ff' - 2gg' + \beta_1 f' + \beta_2 \frac{\partial f'}{\partial \beta_1}) + \alpha(hf'' + h'f') \quad (6')$$

$$g'' = \alpha^2 (2fg + \beta_1 . g + \beta_2 . \frac{\partial g}{\partial \beta_1}) + \alpha h g' \quad (7)$$

$$h'' = \alpha^2 \left(\frac{1}{2} \beta_1 . h + \beta_1 \frac{\partial h}{\partial \beta_1} \right) + \alpha(\overset{*}{P'} + h\, h') \quad (8)$$

Equation of energy

$$s'' = Pr\left[\alpha^2\left(2fs+\beta_1 s+\beta_2 \frac{\partial s}{\partial \beta_1}\right)+ahs'\right]-Br\left(f'^2+g'^2\right) \tag{9}$$

$$q'' = Pr\left(\alpha^2 \beta_2 \frac{\partial q}{\partial \beta_1} \quad \alpha hq'\right)-4\alpha^2 s - \frac{16B_r}{\alpha^2}\left(\alpha^4+2\right) f^2 \tag{10}$$

where $Pr = \mu c/k$, μ = coefficient of viscosity,

c = specific heat, k = coefficient of thermal conductivity.

$$Br = Pr\, x E \quad \frac{\omega\nu}{c\,(T_{w_0}-T_{w_1})}, \tag{11}$$

E = Eckert number

Boundary conditions are

$$\begin{aligned} &\eta = 0; \quad f = 0. \quad g = 1, \quad h = Rs, \quad s = q = 0 \\ &\eta = 1, \quad f = s = 0, \quad g = l, \quad h = m\,Rs, \quad q = n-1. \end{aligned} \tag{12}$$

where

$$Rs = \frac{W_0\,(t)}{\sqrt{\omega\nu}} \tag{13}$$

We have inserted the above series into the equations of continuity, motion and energy and equated the free terms and coefficient functions of β_1, β_2, separately to zero. We have got three sets of equations with the appropriate boundary conditions which have been numerically integrated by the Runge-Kutta method as indicated in Iyengar and Rath (1968).

The pressure functions are found out when the H-functions are known. They are

$$P = -\frac{1}{2\alpha^2}\left(4F+\alpha^2 H^2\right)+\frac{1}{2} Rs^2 \tag{14}$$

$$P_1 = -\frac{1}{\alpha^2}\left(2F_1 + \alpha^2\, HH_1\right) - \frac{1}{2}\alpha \int_0^\eta H\, d\eta \tag{15}$$

$$P_2 = -\frac{1}{\alpha^2}\left(2F_2 + \alpha^2 HH_2\right) - \frac{1}{2}\alpha \int_0^\eta H_1\, d\eta \tag{16}$$

IV. Discussion of the Results

It is clear that the upper disk is non-porous if $m=0$, and admits suction or injection according as 'm' is positive or negative.

Radial flow—The radial velocity profile is parabolic for all values of m. The fluid velocity increases with injection and decreases with suction through the upper disk. For $\beta_1>0$, the profile deviates appreciably from the quasi-steady profile, the extent of this deviation being utmost midway between the disks. There is almost non-radial flow in the planes $\eta=0.3$ and $\eta=0.8$ (see Fig 1.)

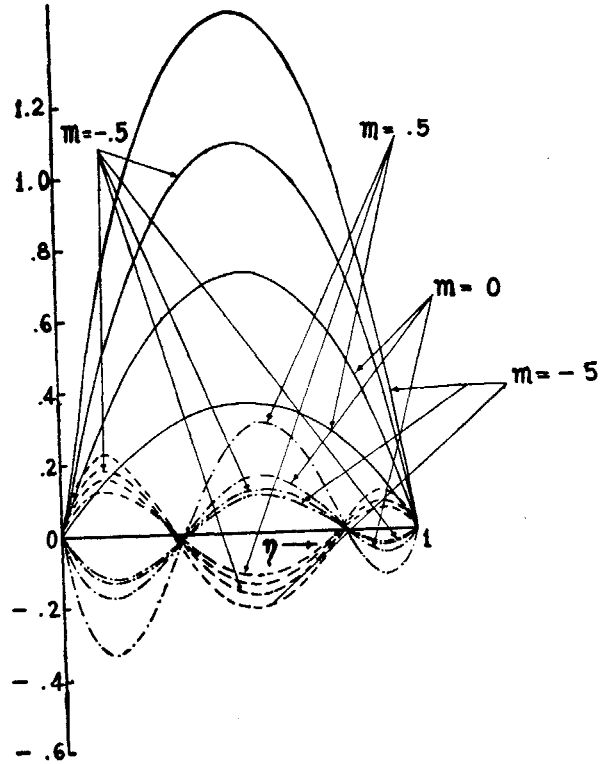

FIG. 1. Radial flow functions for different values of m
$F=$ —— , $10F_1=$ – – – –, $100F_2=$ — —. —. —.

Tangential flow—With increasing injection through the upper-disk, the quasi-steady fluid rotation becomes slower and slower as compared to the disk-rotation and for a certain negative value of m, the fluid in $\eta=0.5$ shall not rotate. On the other hand, suction through the upper-disk seems to have the effect of recovering the solid-state rotation which was evidently destroyed due to injection through the lower disk. The perfect recovery of the solid state rotation, as suggested by intuition, occurs for $m=1$, which sounds all right from physical considerations (See Fig 2). For $\beta_1 > 0$, the tangential velocity is slowed down every where, and the degree of slowing down increases considerably with increasing suction. The layer most affected is that midway between the disks.

Axial Flow—When $m=1$, the quasisteady axial-flow rate is constant and equal to the injection velocity at the lower disk (or suction velocity at the upper). As the suction velocity through the upper-disk is decreased the

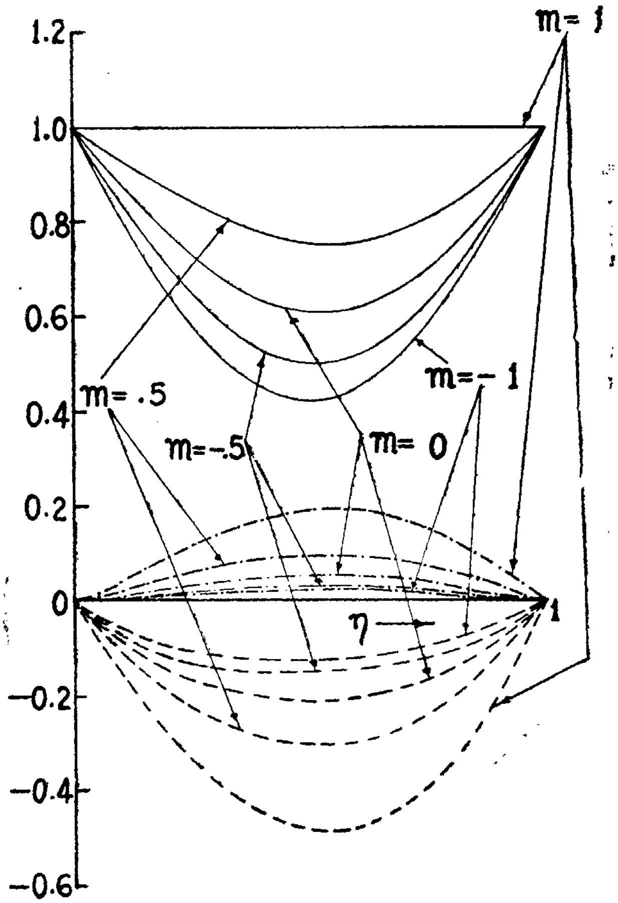

FIG. 2. Tangential flow functions for different values of m
G= ——, G_1= — — — —, G_2= —. —. —. —.

xial velocity is everywhere towards the upper disk when the upper disk is non-porous, the axial velocity, which is everywhere positive, steadily decreases from the injection velocity at the lower disk to the zero value at the upper-one. In the presence of injection through both the disks the quasi-steady axial velocity is away from the lower-disk in the range $0 < \eta < 0.6$ for $m=-0.5$ and in the range $0 < \eta < 0.5$ for $m=-1$. In the rest of the fluid-space, in both the above cases, the axial flow is towards the lower disk. or $\beta_1 > 0$, the axial velocity is diminished in the region, $0 < \eta < 0.6$ and increased in the region nearer the upper-disk in all cases where $m>0$. (see Fig. 3).

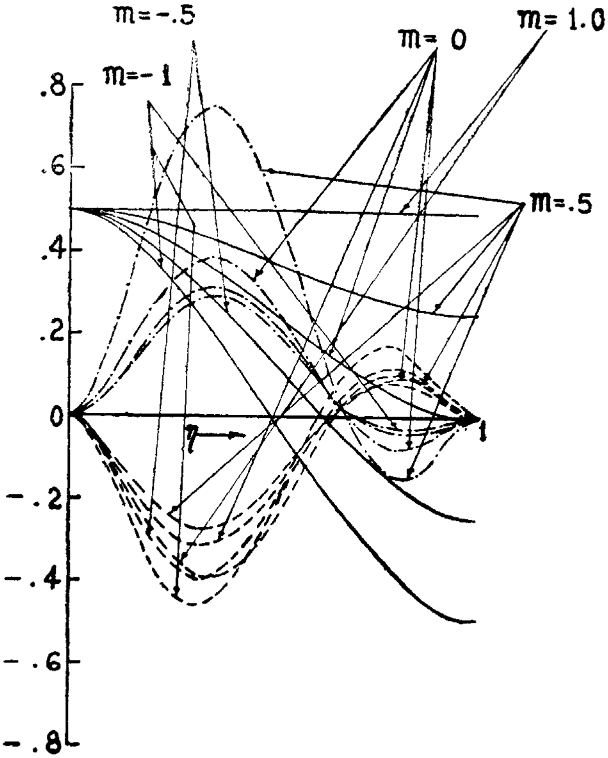

FIG. 3. Axial flow functions for different values of m
$H=$ ——, $100H_1=$ — — — —, $1000H_2=$ —. —. —. —

Pressure Function—The pressure function consists of two parts of which one is a function of the perpendicular distance from any disk and the other varies as the square of the radial-distance. The first part in all cases of quasi-steady rotation has negative values almost throughout the fluid, except very near the upper-disk. One peculiarity observed is that $P(\eta)$ vanishes when the upper-disk has the same velocity as the lower one. With lesser injection-velocity, it vanishes again (besides its zero value at the lower disk) a little away from the upper-disk. The point where it vanishes recedes further from the upper-disk as the injeetion-velocity decreases. From the graph (Fig. 4) it appears that the upper-disk experiences an upward-thurst in all cases of suction through it or injection with velocity less than the injection velocity at the lower one. Thus quasi-steady pressure nagatively increases as the injection velocity through it increases. $P_1(\eta)$ takes negative values in

$0 < \eta < 0.3$ and again in $0.8 < \eta < 1$ and is positive in $0.3 < \eta < 0.8$. It attains its maximum on the negative side near the disks at $\eta = 0.1$ and $\eta = 0.9$ and on the positive side at $\eta = 0.5$.

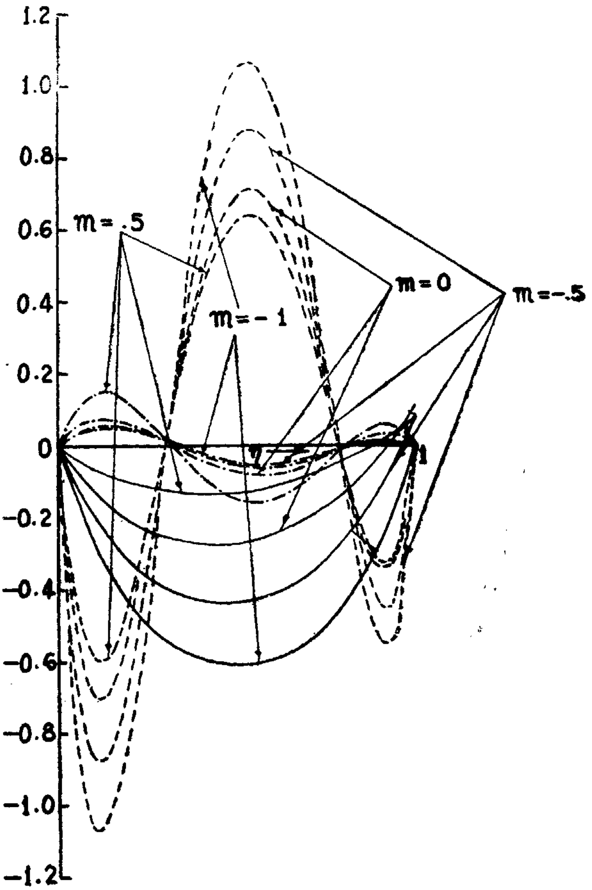

FIG. 4. Pressure functions for different values of m
P= —— , $100P_1$= — — — — , $100P_2$= —. —. —. — .

The Temperature Functions—Each of the temperature functions T consists of two parts : the functions Q which show the variation of temperature along the 'axis' and the functions S which give the variations along the radial-directions on any particular plane parallel to the disks. The quasisteady temperature in the axial-direction increases everywhere in the fluid with increasing injection through the upper disk. Its profile is nearly parabolic. With suction through the upper disk there is negligible variation of temperature along the axis.

The differences of axial temperature variation in the instantaneous-state from that in the established state is sufficiently marked and the tendency is one of lowering down the axial variation of temperature (Fig. 5). Dependence of this deviation on the injection and suction velocity at the upper disk

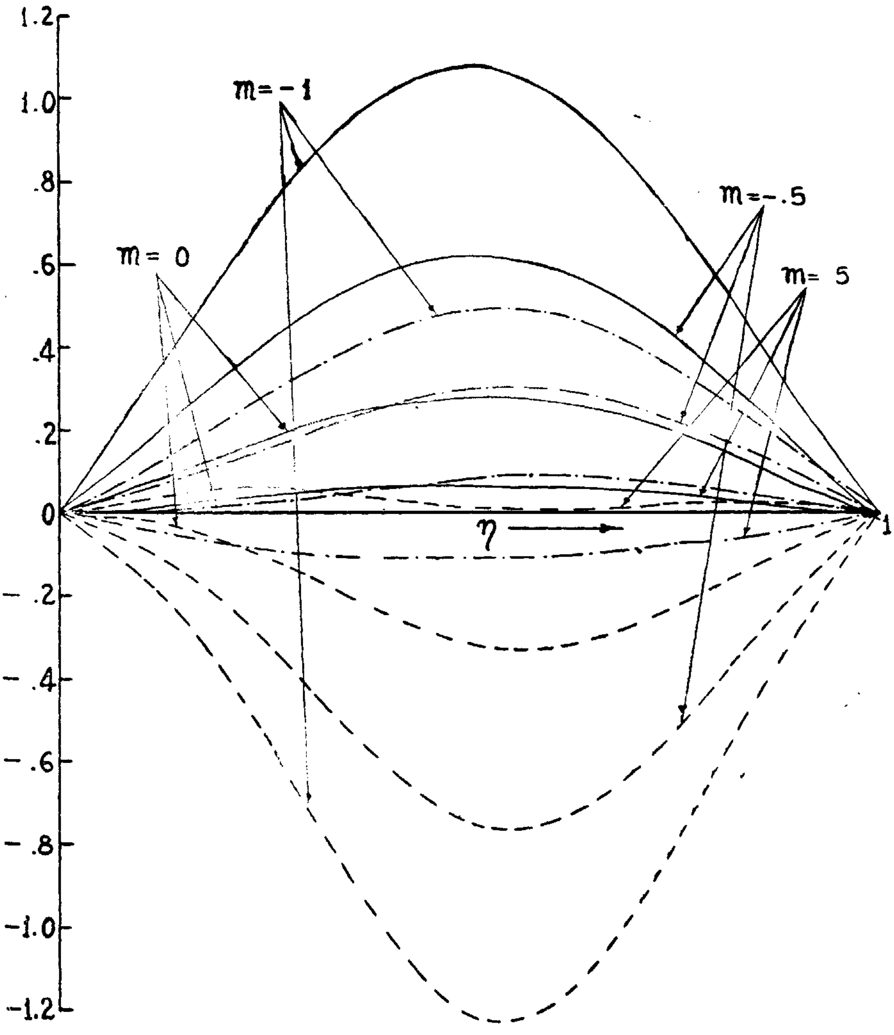

FIG. 5. Temperature functions along the axial directions for different values of m
$Q/50$= —— , Q_1= — — — — , Q_2= —. —. —. —.

follows the same pattern as in the quasi-steady case. The radial temperature variation in the quassi-steady state increases with injection at the upper disk aud with suction through the upper-disk variation is sufficiently small. The deviation in the instantaneous steady state from the established-state is relatively less compared to the case of axial variation.

From Fig. 6 it is obvious that the suction through the upper disk has a great controlling effect on the temperature of the fluid which for any particular value of the radius in the case of a non-porous upper disk is considerably diminished. The temperature of the fluid mid-way between the disks in particular diminishes to $\frac{1}{10}$ th of its value with suction (m=0.5) and increases to twice its value with injection (m = − 0.5) through the upper disk.

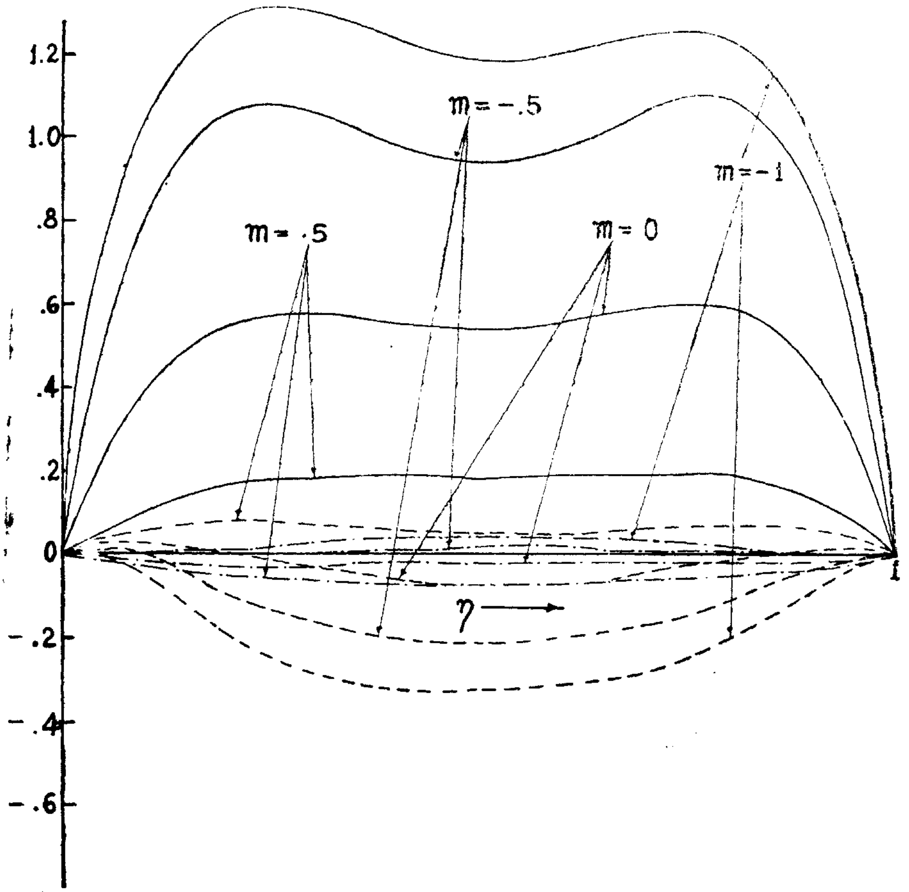

FIG. 6. Temperature functions along the radial directions for different values of m

S= ——, S_1= – – – – , S_2= —.—.—.— .

As the radial distance from the disk increases, the quasi-steady temperature increases retaining the parabolic character of the profile irrespective of whether there is suction, injection or none of them through the upper-disk. The instantaneous temperature everywhere is lower than that in established state for $m=0, -0.5$, but higher for $m=0.5$, i. e. in presence of suction through the upper disk. The Figs. 7, 8 and 9 confirm many of our earlier conclusions drawn from other graphs.

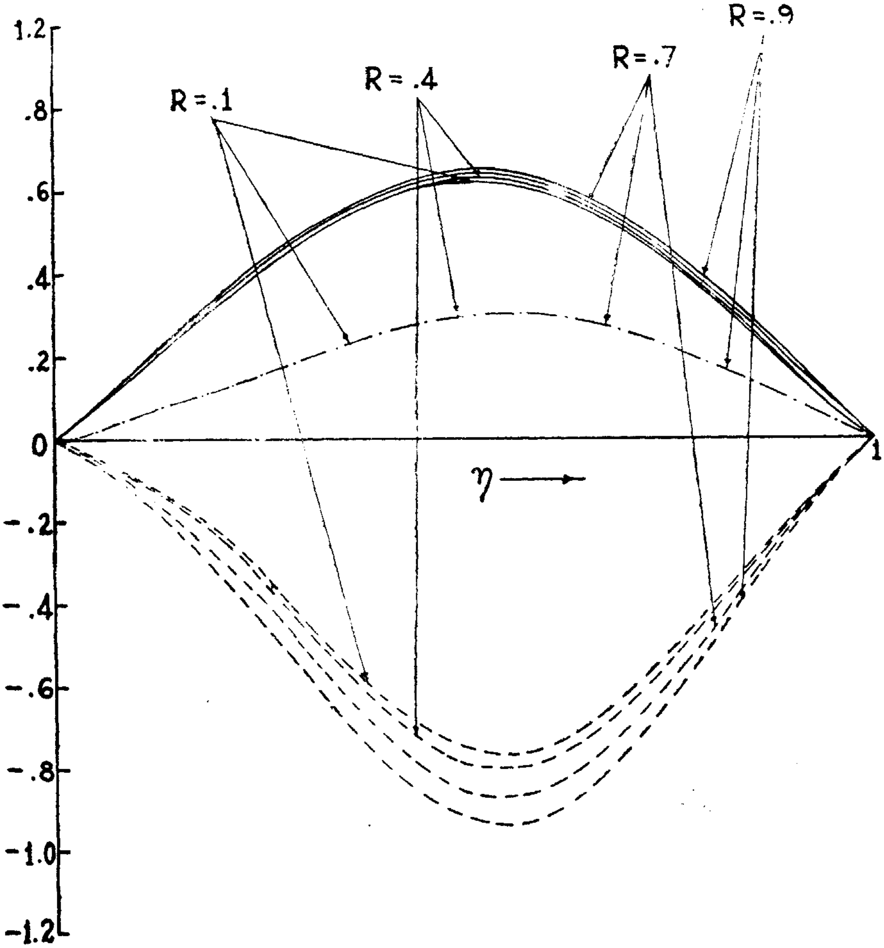

FIG. 7. Effects of injection on the temperature function $T/50$ = ——, T_1 = — — — —, T_2 = —. —.—. — .

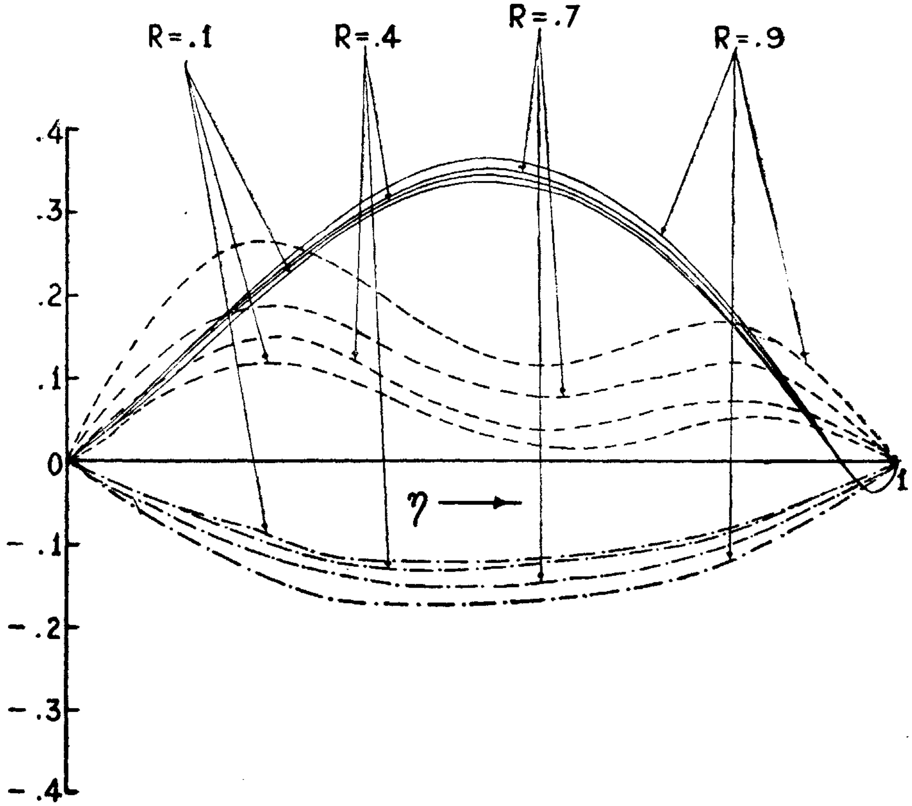

FIG. 8. Effects of suction on the temperature function
T= ——— , $5T_1$= — — — — . T_2 = —. —. —. —.

From Table 1 we conclude that in the presence of a upper disk, where the suction or injection-velocity is not greater than the injection velocity at

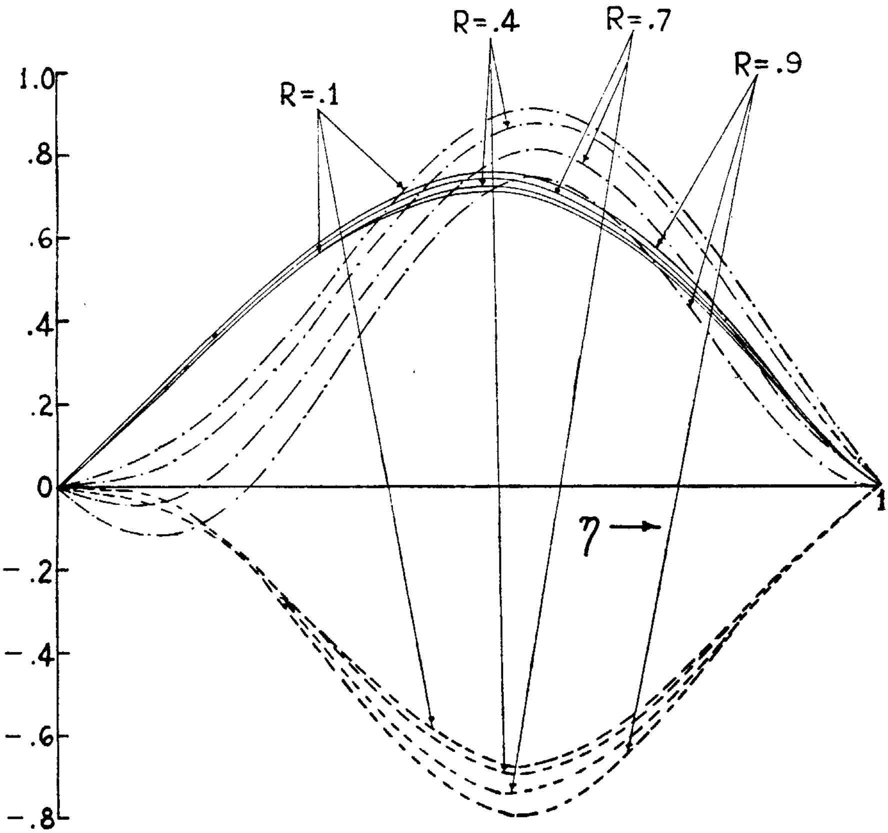

FIG. 9. Effect of non porocity on the temperature function $T\,20=$ ——, $2T_1=$ – – – –, $10T_2=$ —. —. —. —.

TABLE 1

Acccleration and time for the establishment of flow and temperature

Cases	M	Acc. in radians per sec² A_F	A_H	T_F in secs.	T_H in secs.
1.	0.5	6525	19050	1.95	1.15
2.	—0.5	20500	183275	1.29	0.37
3.	0	13300	70050	1.37	0.61
4.	1	3750	1875	2.58	3.97
5.	−1	27750	342.000	0.95	0.27
6.	2	8125	4625	1.57	2.26

A_F is acceleration in radians/sec² of flow, A_H acceleration in radians per sec² of heat transfer, T_F time for the establishment of flow and T_H time for the establishment of heat transfer.

the lower disk, heat transfer is established earlier than the flow. The time of establishment of flow as well as heat transfer decreases or increases according as there is injection or suction through the upper-disk with velocity less than or equal to the injection velocity at the lower-one. If the suction velocity at the upper-disk exceeds the injection velocity at the lower one, flow seems to be established earlier than the heat-transfer. The utmost delay in the establishment of the flow as well as the heat transfer seems to occur in the case when the of injection through the lower-disk is the same as the rate of suction through the upper-one.

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