SOME OPERATIONAL TECHNIQUES IN THE THEORY OF SPECIAL FUNCTIONS

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In this present paper, we have given a generalization of the double integral transform obtained by Srivastava and Panda [1973]. 3.6, p. 312] and show how this transformation would lead to interesting operational techniques for augmenting parameters of the $H^{m;n}_{p;q}$ [z] function.

1. INTRODUCTION

Fox (1961) introduced the H-function in the form of Mellin Barnes type contour integral which has been symbolically denoted as

$$H^{m,n}_{p,q}\left[x \left| \begin{matrix}\{(a_p, \alpha_p)\} \\ \{(b_q, \beta_q)\}\end{matrix}\right.\right]$$

$$=(2\pi i)^{-1}\int_L \frac{\prod_{j=1}^{m} \Gamma(b_j-\beta_j s) \prod_{j=1}^{n} \Gamma(1-a_j+\alpha_j s)}{\prod_{j=m+1}^{q} \Gamma(1-b_j+\beta_j s) \prod_{j=n+1}^{p} \Gamma(a_j-\alpha_j s)} x^s \, ds \tag{1.1}$$

where $\{(f_r, \gamma_r)\}$ stands for the set of the parameters (f_1, γ_1), (f_2, γ_2),..., (f_r, γ_r) ; x is not equal to zero and empty product is interpreted as unity ; p, q, m and n are integers satisfying $1 \leqslant m \leqslant q$; $0 \leqslant n \leqslant p$. α_j, $(j=1, ..., p)$; β_j $(j=1,..,q)$ are positive numbers and a_j $(j=1, ,p)$; b_j $(j=1,\cdots,q)$ are complex numbers such that no pole of $\Gamma(b_h-\beta_h s)$ $(h=1, .., m)$ coincides with any pole of $\Gamma(1-a_i+\alpha_i s)$ $(i=1....,n)$ and the integral in (1.1) converges if

$$|\arg x| < \tfrac{1}{2}\lambda\pi \tag{1.2}$$

with

$$\lambda \equiv \sum_{j=1}^{n} \alpha_j - \sum_{j=n+1}^{p} \alpha_j + \sum_{j=1}^{m} \beta_j - \sum_{j=m+1}^{q} \beta_j > 0 \tag{1.3}$$

$$A \equiv \sum_{j=1}^{q} \beta_j - \sum_{j=1}^{p} \alpha_j > 0 \tag{1.4}$$

and according to Braaksma (1963, 279),

$$H_{p,q}^{m,n}\left[x \,\middle|\, \begin{matrix}\{(a_p, \alpha_p)\}\\ \{(b_q, \beta_q)\}\end{matrix}\right] = 0\,(\,|x|^{\delta}\,) \text{ for small } x \text{ where}$$

$$\delta = \min_{1 \leqslant h \leqslant m} R\,(b_h/\beta_h) \tag{1.5}$$

$$= 0\,(\,|\,x\,|^{\beta}) \text{ for large } x, \text{ where } \beta = \max_{1 \leqslant i \leqslant n} R\left(\frac{a_i - 1}{\alpha_i}\right), \tag{1.6}$$

In subsequent discussions, we have used the result due to Edwards (1954, p. 177), i. e.

$$\int_0^{\infty}\int_0^{\infty} x^{\alpha-1} y^{\beta-1} \phi\,(x+y)\, dxdy = B\,(\alpha, \beta) \int_0^{\infty} \phi\,(z)\, z^{\alpha+\beta-1}\, dz, \tag{1.7}$$

$$\text{Re}\,(\alpha) > 0, \text{Re}\,(\beta) > 0,$$

a known result due to Mukherji and Prasad (1971), viz.

$$H_{p,q+1}^{m+1,n}\left[ax^{\sigma} \,\middle|\, \begin{matrix}\{(a_p, \alpha_p)\}\\ (b_0, \beta_0), \{(b_q, \beta_q)\}\end{matrix}\right]$$

$$= \frac{1}{\beta_0} \sum_{r=0}^{\infty} \frac{(-1)^r}{r\,!} \frac{\prod_{j=1}^{m} \Gamma(b_j - \beta_j \rho_r) \prod_{j=1}^{n} \Gamma(1 - a_j + \alpha_j \rho_r)}{\prod_{j=m+1}^{q} \Gamma(1 - b_j + \beta_j \rho_r) \prod_{j=n+1}^{p} \Gamma(a_j - \alpha_j \rho_r)} a^{\rho_r} x^{\sigma \rho_r} \tag{1.8}$$

where

$$\rho_r = \frac{b_0 + r}{\beta_0} \tag{1.9}$$

$\sigma > 0$, $\beta < R\,(b_0/\beta_0) < \delta$, $|\arg a| < \frac{1}{2} \lambda \pi$, $\lambda > 0$, $A > 0$. where λ, A, δ, β are defined in eqns. (1.3), (1.4), (1.5) and (1.6), and the double integral transform as

$$\int_0^{\infty}\int_0^{\infty} x^{\alpha-1} y^{\beta-1} (x+y)^{\sigma} H_{u,v}^{f,g}\left[\lambda\,(x+y) \,\middle|\, \begin{matrix}\{(A_u, \eta_u)\}\\ \{(B_v, \xi_v)\}\end{matrix}\right]$$

$$H_{p_1,q_1}^{m_1,n_1}\left[t x^{\rho_2} y^{\sigma_2} (x+y)^{r_2} \,\middle|\, \begin{matrix}\{(c_{p_1}, \gamma_{p_1})\}\\ \{(d_{q_1}, \delta_{q_1}\})\end{matrix}\right] dxdy$$

$$= \lambda^{-\alpha-\beta-\sigma} H_{p_1+v+2, q_1+u+1}^{g+m_1, 2+f+n_1}\left[\frac{t}{\lambda^{\theta}} \,\middle|\, \begin{matrix}(1-\alpha, \rho_2), (1-\beta, \sigma_2), \\ \{(\psi_g, \theta\,\eta_g)\}, \{(d_{q_1}, \delta_{q_1})\}.\end{matrix}\right.$$

$$\left.\begin{matrix}\{(\epsilon_f, \theta\,\xi_f)\}, \{(c_{p_1}, \gamma_{p_1})\}, (\epsilon_{f+1}, \theta\,\xi_{f+1}), \ldots, (\epsilon_v, \theta\,\xi_v)\\ (1-\alpha-\beta, \rho_2 + \sigma_2), (\psi_{g+1}\, \theta\eta_{g+1}), \cdots, (\psi_u, \theta\eta_u)\end{matrix}\right] \tag{1.10}$$

provided that $\rho_2, \sigma_2, r_2 > 0$, $\mathrm{Re}(\alpha) > 0$, $\mathrm{Re}(\beta) > 0, -\min\limits_{1 \leq j \leq f} R(B_j\, \xi_j)$

$$< \mathrm{Re}(\alpha+\beta+\sigma) < -\max_{1 \leq i \leq g} R\left(\frac{A_i - 1}{\eta_i}\right) \tag{1.11}$$

$$\theta = \rho_2 + \sigma_2 + r_2 \tag{1.12}$$

$$\epsilon_j = 1 - B_j - (\alpha+\beta+\sigma)\,\xi_j, \quad j=1,\cdots, v \tag{1.13}$$

and

$$\psi_j = 1 - A_j - (\alpha+\beta+\sigma)\,\eta_j, \quad j=1, \cdots. u. \tag{1.14}$$

2. Generalized Double Transform

In this section we give our generalized double transform as follows :

$$\int_0^\infty \int_0^\infty x^{\alpha-1}\, y^{\beta-1}\, (x+y)^\sigma\, H^{f,g}_{u,v}\left[\lambda_1 (x+y) \,\middle|\, \begin{matrix} \{(A_u, \eta_u)\} \\ \{(B_u, \xi_u)\} \end{matrix}\right]$$

$$H^{m_1, n_1}_{p_1, q_1}\left[t x^{\rho_2} y^{\sigma_2} (x+y)^{r_2} \,\middle|\, \begin{matrix} \{(c_{p_1}, \gamma_{p_1})\} \\ \{(d_{q_1}, \delta_{q_1})\} \end{matrix}\right] .$$

$$H^{m+1, n}_{p, q+1}\left[\lambda_2\, x^{\rho_1} y^{\sigma_1} (x+y)^{r_1} \,\middle|\, \begin{matrix} \{(a_p, \alpha_p)\} \\ (b_0, \beta_0), \{(b_q, \beta_q)\} \end{matrix}\right] dx\, dy$$

$$= \frac{1}{\beta_0} \sum_{r=0}^{\infty} \frac{(-1)^r}{r!} \frac{\prod\limits_{j=1}^{m} \Gamma(b_j - \beta_j\, \rho_r) \prod\limits_{j=1}^{n} \Gamma(1 - a_j + \alpha_j\, \rho_r)}{\prod\limits_{j=m+1}^{q} \Gamma(1 - b_j + \beta_j\, \rho_r) \prod\limits_{j=m+1}^{p} \Gamma(a_j - \alpha_j\, \rho_r)} \lambda_2^{\rho_r}.$$

$$\lambda_1^{-\alpha-\beta-\sigma-\rho_r(\rho_1+\sigma_1+r_1)}\, H^{g+m_1, 2+f+n_1}_{p_1+v+2, q_1+u+1}\left[\frac{t}{\lambda_1^\theta} \,\middle|\, \begin{matrix} (1-\alpha-\rho_1\, \rho_r, \rho_2), \\ \{(\psi_g, \theta\, \eta_g)\}, \end{matrix}\right.$$

$$\left.\begin{matrix} (1-\beta-\sigma_1\, \rho_r, \sigma_2)\, \{(\epsilon_f, \theta\, \xi_f)\}, \{(c_{p_1}, \gamma_{p_1})\}, (\epsilon_{f+1}, \theta\, \xi_{f+1}), \ldots, (\epsilon_v, \theta\, \xi_v) \\ \{(d_{q_1}, \delta_{q_1})\}, \{1-\alpha-\beta-\rho_r\, (\rho_1+\sigma_1), \rho_2+\sigma_2\}, (\psi_{g+1}, \theta\eta_{g+1}), \ldots, (\psi_u, \theta\eta_u) \end{matrix}\right] \tag{2.1}$$

provided that $\rho_1, \sigma_1, r_1, \rho_2, \sigma_2, r_2 > 0$, $\beta < R(b_0/\beta_0) < \delta$, $|\arg \lambda_2| < \frac{1}{2} \lambda\pi$, $\lambda > 0$, $A > 0$, $R\left(\alpha + \rho_1 \frac{b_0}{\beta_0}\right) > 0$, $R\left(\beta + \sigma_1 \frac{b_0}{\beta_0}\right) > 0$, $\delta^1 < R(\alpha+\beta+\sigma+b_0/\beta_0)$ $(\rho_1+\sigma_1+r_1) < \beta'$, where δ' and β' are the first and last members of the inequality (1.11), $\delta, \beta, \lambda, A$ are given by equations (1.5), (1.6), (1.3), (1.4), θ, ϵ_j, ψ_j and ρ_r are given by eqns. (1.12) (1.13), (1.14.) and (1.9).

Proof: Expanding the last H-function of (2.1) with the help of (1.8) and interchanging the integration and summation, the left hand side of (2.1) reduces to

$$\frac{1}{\beta_0}\sum_{r=0}^{\infty}\frac{(-1)^r}{r!}\frac{\prod_{j=1}^{m}\Gamma(b_j-\beta_j\rho_r)\prod_{j=1}^{n}\Gamma(1-a_j+\alpha_j\rho_r)}{\prod_{j=m+1}^{q}\Gamma(1-b_j+\beta_j\rho_r)\prod_{j=n+1}^{p}\Gamma(a_j-\alpha_j\rho_r)}\lambda_2^{\rho_r}.$$

$$\int_0^\infty\int_0^\infty x^{\alpha+\rho_1\rho_r-1}\,y^{\beta+\sigma_1\rho_r-1}\,(x+y)^{\sigma+r_1\rho_r}.$$

$$H_{v,v}^{f,g}\left[\lambda,(x+y)\;\Big|\;\begin{matrix}\{(A_u,\eta_u)\}\\ \{(B_v,\xi_v)\}\end{matrix}\right]H_{p_1,q_1}^{m_1,n_1}$$

$$\left[tx^{\rho_2}\,y^{\sigma_2}\,(x+y)^{r_2}\;\Big|\;\begin{matrix}\{(c_{p_1},\gamma_{p_1})\}\\ \{(d_{q_1},\delta_{q_1})\}\end{matrix}\right]dxdy \tag{2.2}$$

provided that $\rho_1, \sigma_1. r_1>0$, $\beta<R(b_0/\beta_0)<\delta$, $|\arg\lambda_2|<\frac{1}{2}\lambda\pi$, $\lambda>0$, $A>0$. Now making use of the relationship (1.10), we arrive at the right hand side of (2.1). The change of integration and summation is permissible due to the convergence of the series involved in (2.1.) under the given conditions.

3. Particular Cases

(*i*) Putting $m=n=p=q=b_0=0$, $\beta_0=1$ in (2.1) and taking $\lambda_2=0$ in the expanded form the result (2.1), we arrive at the result given by Srivastava and Panda (1973, 3. 6).

(*ii*) On taking $\eta_1=\eta_2=\ldots=\eta_4=1=\xi_1=\xi_2=\ldots=\xi_v$, $r_1=r_2=\ldots=r_{p_1}=1=\delta_1=\delta_2=\ldots=\delta_{q_1}$ in (2.1) we arrive at the double transform given by Pandey and Pandey (1971).

(*iii*) On taking $f=4=v$, $g=0$, $u=2$, $A_1=\frac{1}{2}\sigma$,

$A_2=\frac{1}{2}(\sigma+1)$, $\eta_1=\eta_2=1$, $B_1=\frac{1}{2}(\sigma+\mu+\nu)$, $B_2=\frac{1}{2}(\sigma-\mu+\nu)$

$B_3=\frac{1}{2}(\sigma-\mu-\nu)$, $B_4=\frac{1}{2}(\sigma+\mu-\nu)$, $\xi_1=\xi_2=\xi_3=\xi_4=1$, $m_1=1$,

$n_1=p_1$, $q_1=q_1+1$, $\gamma_1=\gamma_2=\ldots=\gamma_{p_1}=1=\delta_1=\delta_\lambda=\ldots=\delta_{q_1}$,

replacing c_{p_1} and d_{q_j} by $1-C_{p_i}$ and $1-d_{q_j}$ where $i=1, 2, \ldots, p_1$, $j=0, 1, \ldots, q_1$ and $d_0=1$, we arrive at the double transform given by Srivastava and Panda (1973) 1.7, p. 309).

(*iv*) On taking $f=2=v$, $g=0$, $u=1$, $A_1=\frac{1}{2}$, $B_1=\nu$

$B_2=-\nu$, $\eta_1=1=\xi_1=\xi_2$, $m_1=1$, $n_1=p_1$, $q_1=q_1+1$,

$\gamma_1 = \gamma_2 = \ldots = \gamma_{p_1} = 1 = \delta_1 = \delta_2 = \ldots = \delta_{q_1}$, replacing C_{p_i} and d_{q_j} by $1-C_{p_i}$ and $1-d_{q_j}$, $i = 1, 2, \ldots, p_1$, $j = 0, 1, 2, \ldots, q_1$ and $d_0 = 1$, we arrive at the double transform studied by Srivastava and Singhal (1968, 1.5, p. 309).

(*v*) On taking $f = 2 = v$, $g = 0$, $u = 1$, $A_1 = \sigma-\mu+1$, $B_1 = \sigma+\nu+\frac{1}{2}$, $B_2 = \sigma-\nu+\frac{1}{2}$, $\eta_1 = 1 = \xi_1 = \xi_2$, together with the remaining substitutions given in (*iv*), we obtain the double Whittaker transform studied by Srivastava and Joshi (1967).

4. Applications

In terms of the linear operator, we define our operator as,

$$\Omega_{\alpha,\beta,\sigma}^{\lambda_1,[f,g,u,v],[m_1,n_1,p_1,q_1]}\{\ \}$$

$$= \int_0^\infty \int_0^\infty x^{\alpha-1} y^{\beta-1} (x+y)^\sigma H_{u,v}^{f,g}\left[\lambda_1(x+y)\,\middle|\,\begin{matrix}\{(A_u,\eta_u)\}\\ \{(B_v,\xi_v)\}\end{matrix}\right]$$

$$H_{p_1,q_1}^{m_1,n_1}\left[t\,x^{\rho_2} y^{\sigma_2} (x+y)^{r_2}\,\middle|\,\begin{matrix}\{(C_{p_1},\gamma_{p_1})\}\\ \{(d_{q_1},\delta_{q_1})\}\end{matrix}\right]\{\ \}\,dx\,dy. \tag{4.1}$$

The operator defined in (4.1) is more general than defined by Srivastava and Panda (1973) we mention few applications of the above operator as given below, the proofs of which can be verified by specializing the parameters of the third H-function involved in the left-hand side of result (2.1)

(*i*) $\Omega_{\alpha,\beta,\sigma}^{\lambda_1,[f,g,u,v],[m_1,n_1,p_1,q_1]}\{1\}$

$$= \lambda_1^{-\alpha-\beta-\sigma} H_{p_1+v+2,\,q_1+u+1}^{g+m_1,\,2+f+n_1}\left[\frac{t}{\lambda_1^{\theta}}\,\middle|\,\begin{matrix}(1-\alpha,\rho_2),\\ \{(\psi_g,\theta\eta_g)\},\end{matrix}\right.$$

$$\left.\begin{matrix}(1-\beta,\sigma_2),\{(\epsilon_f,\theta\xi_f)\},\{(c_{p_1},\gamma_{p_1})\},(\epsilon_{f+1},\theta\xi_{f+1}),\ldots,(\epsilon_v,\theta\xi_v)\\ \{(d_{q_1},\delta_{q_1})\},(1-\alpha-\beta,\rho_2+\sigma_2),(\psi_{g+1},\theta\eta_{g+1}),\ldots,(\psi_u,\theta\eta_u)\end{matrix}\right] \tag{4.2}$$

where the conditions given in (1.10) are satisfied.

(*ii*) $\Omega_{\alpha,\beta,\sigma}^{\lambda_2,[f,g,u,v],[m_1,n_1,p_1,q_1]}\{K_\nu[x^{\rho_1} y^{\sigma_1}(x+y)]^{r_1}, K_\mu[x^{\rho_1} y^{\sigma_1}(x+y)^{r_1}]\}$

$$= \lambda_1^{-\alpha-\beta-\sigma-(\mu+\nu)(\rho_1+\sigma_1+r_1)}\,\frac{\sqrt{\pi}}{2}\,.$$

$$\sum_{r=0}^{\infty} \frac{(-1)^r}{r!}\,\frac{\Gamma(-\mu-r)\,\Gamma(-\nu-r)\,\Gamma(-\mu-\nu-r)}{\Gamma(-\frac{1}{2}\nu-\frac{1}{2}\mu-r)\,\Gamma(\frac{1}{2}-\frac{1}{2}\nu-\frac{1}{2}\mu-r} \lambda_1^{-2(\rho_1+\sigma_1+r_1)r}\,.$$

$$H^{g+m_1,\,2+f+n_1}_{p_1+v+2,\,q_1+u+1}\left[\frac{t}{\lambda_1^{\theta}}\left|\begin{matrix}(1-\alpha-\rho_1(\nu+\mu+2r),\rho_2),(1-\beta-\sigma_1(\nu+\mu+2r),\sigma_2)\\ \{\psi_g,\theta\eta_g)\},\{(d_{q_1},\delta_{q_1})\},(1-\alpha-\beta-(\rho_1+\sigma_1)\end{matrix}\right.\right.$$

$$\left.\begin{matrix}\{(\epsilon_f,\theta\xi_f)\},\{(c_{p_1},\gamma_{p_1})\},(\epsilon_{f+1},\theta\xi_{f+1}),\ldots,(\epsilon_v,\theta\xi_v)\\ (\nu+\mu+2r),\rho_2+\sigma_2),(\psi_{g+1},\theta\eta_{g+1}),\ldots\ldots,(\psi_u,\theta\eta_u)\end{matrix}\right] \tag{4.2}$$

provided that $\text{Re}(\mu)<0$, $\text{Re}(\nu)<0$, $\text{Re}(\alpha+\rho_1(\mu+\nu))>0$, $\text{Re}(\beta+\sigma_1(\mu+\nu))>0$ $\rho_2,\sigma_2,r_2,\rho_1,\sigma_1,r_1>0$, $\delta'<\text{Re}(\alpha+\beta+\sigma+(\nu+\mu)(\rho_1+\sigma_1+r_1)<\rho'$, where δ',β', $\rho_r,\theta,\epsilon,\psi$, being the same as in (2.1).

(*iii*) $$\Omega^{\lambda_1,[f,g,u,v],[m_1,n_1,p_1,q_1]}\{W_{k',m'}[x^{\rho_1}y^{\sigma_1}(x+y)]^{r_1}W_{-k',m'}[x^{\rho_1}y^{\sigma_1}(x+y)^{r_1}]\}=\tfrac{1}{2}\lambda_1^{-\alpha-\beta-\sigma-(\frac{1}{2}+m')(\rho_1+\sigma_1+r_1)}\cdot\frac{1}{\sqrt{\pi}}\times$$

$$\times\sum_{r=0}^{\infty}\frac{(-\frac{1}{4})^r}{r!}\,\frac{\Gamma(\frac{1}{2}-m'-r)\,\Gamma(-2m'-r)\,\Gamma(-m'-r)}{\Gamma(\frac{1}{2}+k'-m'-r)\,\Gamma(\frac{1}{2}-k'-m'-r)}\,\lambda_1^{-2(\rho_1+\sigma_1+r_1)r}\times$$

$$\times H^{g+m_1,\,2+f+n_1}_{p_1+v+2,\,q_1+u+1}\left[\frac{t}{\lambda_1^{\theta}}\left|\begin{matrix}(1-\alpha-\rho_1(\frac{1}{2}+m'+r),\rho_2)\,(1-\beta-\sigma_1(\frac{1}{2}+m'+r),\sigma_2),\\ \{(\psi_g,\theta\eta_g)\},\{(d_{q_1},\delta_{q_1})\},\{1-\alpha-\beta-(\rho_1+\sigma_2)\end{matrix}\right.\right.$$

$$\left.\begin{matrix}\{(\epsilon_f,\theta\xi_f)\},\{(c_{p_1},\gamma_{p_1})\},(\epsilon_{f+1},\theta\xi_{f+1}),\ldots,(\epsilon_v,\theta\xi_v)\\ (\frac{1}{2}+m'+r),\rho_2+\sigma_2\},(\psi_{g+1},\theta\eta_{g+1}),\cdots,(\psi_u,\theta\eta_v)\end{matrix}\right] \tag{4.3}$$

provided that $\rho_2,\sigma_2,r_2,\rho_1,\sigma_1,r_1>0$, $\text{Re}(m')<0$, $\text{Re}(\alpha+(\frac{1}{2}\pm m')\rho_1)>0$, $\text{Re}(\beta+(\frac{1}{2}\pm m')\sigma_1)>0$, $\delta'<\text{Re}(\alpha+\beta+\sigma+\frac{1}{2}\pm m')(\rho_1+\sigma_1+r_1)<\beta'$, where, $\delta',\beta',\theta,\epsilon,\psi$ are given in (2.1).

5. Generating Functions

In Bateman's generating function for the classical Jacobi polynomials, viz. Rainville (1965, p. 256)

$$\sum_{n,k=0}^{\infty}\frac{(\frac{1}{2}x'-\frac{1}{2})^n\,(\frac{1}{2}x'+\frac{1}{2})^k\,t^{n+k}}{n!\,k!\,(1+\alpha)_n\,(1+\beta)\,k}=\sum_{n=0}^{\infty}\frac{t^n}{(1+\alpha)_n\,(1+\beta)_n}\,P_n^{(\alpha,\beta)}(x') \tag{5.1}$$

if we replace t by txy and operate upon both sides by the operator

$$\Omega_{a,b,c}^{\lambda_1,[f,g,u,v],[m_1,n_1,p_1,q^1]} \tag{5.2}$$

then (5.1) reduces to,

$$\sum_{n,k=0}^{\infty} \frac{(\frac{1}{2}x'-\frac{1}{2})^n (\frac{1}{2}x'+\frac{1}{2})^k t^{n+k}}{n!\,k!\,(1+\alpha)_n (1+\beta)_k} \left\{ H^{g+m_1, 2+f+n_1}_{p_1+v+2, q_1+u+1} \left[\frac{t}{\lambda_1^{\theta}} \times \right.\right.$$

$$\times \left| \begin{matrix} (1-a-n-k, \rho_1), (1-b-n-k, \sigma_1), \{(\epsilon_f, \theta\xi_f)\}, \{(c_{p_1}, \gamma_{p_1})\}, \\ \{(\psi_g, \theta\eta_g)\}, \{(d_{q_1}, \delta_{q_1})\}, (1-a-b-2n-2k, \rho_2+\sigma_2), \end{matrix} \right.$$

$$\left.\left. \begin{matrix} (\epsilon_{f+1}, \theta\xi_{f+1}), \ldots, (\epsilon_v, \theta\xi_v) \\ (\psi_{g+1}, \theta\eta_{g+1}), \ldots, (\psi_u, \theta\eta_u) \end{matrix} \right]\right\}$$

$$= \sum_{n=0}^{\infty} \frac{P_n^{(\alpha,\beta)}(x')\, t^n}{(1+\alpha)_n (1+\beta)_n} \left\{ H^{g+m_1, 2+f+n_1}_{p_1+v+2, q_1+u+1} \left[\frac{t}{\lambda_1^{\theta}} \right.\right.$$

$$\left| \begin{matrix} (1-a-n, \rho_1), (1-b-n, \sigma_1), \{(\epsilon_f, \theta\xi_f)\}, \{(c_{p_1}, \gamma_{p_1})\}, \\ \{(\psi_g, \theta\eta_g)\}, \{(d_{q_1}, \delta_{q_1})\}, (1-a-b-2n, \rho_2+\sigma_2), \end{matrix} \right.$$

$$\left.\left. \begin{matrix} (\epsilon_{f+1}, \theta\xi_{f+1}), \ldots, (\epsilon_v, \theta\xi_v) \\ (\psi_{g+1}, \theta\eta_{g+1}), \ldots, (\psi_u, \theta\eta_u) \end{matrix} \right]\right\} \tag{5.3}$$

provided that $\mathrm{Re}(a) > 0$, $\mathrm{Re}(b) > 0$, $\mathrm{Re}(\alpha) > -1$, $\mathrm{Re}(\beta) > -1$, $\delta' < \mathrm{Re}(a+b+c) < \beta'$ and the notations, θ, ϵ, ψ are given in (2.1).

(*ii*) A similar application of the operator (5.2) to the generating relations [10, p. 74] of Srivastava, viz.

$$\sum_{n=0}^{\infty} \frac{(\lambda')_n}{(-\alpha-\beta)_n} H^{(\alpha_{-n}, \beta_{-n})} [\rho, \sigma, x'] t^n$$

$$= F_2 [\lambda', -\alpha, \rho; -\alpha-\beta, \sigma; -t, x' t] \tag{5.4}$$

involving the generalized Rice polynomials given as

$$H_n^{(\alpha, \beta)} [\zeta, p', v'] = \binom{\alpha+n}{n} {}_3F_2 \left[\begin{matrix} -n, \alpha+\beta+n+1, \zeta \\ \alpha+1, p'; v' \end{matrix} \right] \tag{5.5}$$

and (5.4) gives the result in the form

$$\sum_{n=0}^{\infty} \frac{(\lambda')_n}{(-\alpha-\beta)_n} H_n^{(a_{-n}, \beta_{-n})} [\rho, \sigma, x'] t^n \left\{ H^{g+m_1, 2+f+n_1}_{p_1+v+2, q_1+u+1} \right.$$

$$\left[\frac{t}{\lambda_1^{\theta}} \left| \begin{matrix} (1-a-n, \rho_1), (1-b-n, \sigma_1), \{(\epsilon_f, \theta\xi_f)\}, \{c_{p_1}, \gamma_{p_1}\}\} \\ \{(\psi_g, \theta\eta_g)\}, \{(d_{q_1}, \delta_{q_1})\}, (1-a-b-2n, \rho_2+\sigma_2), \end{matrix} \right.\right.$$

$$\left.\left. \begin{matrix} (\epsilon_{f+1}, \theta\xi_{f+1}), \ldots, (\epsilon_v, \theta\xi_v) \\ (\psi_{g+1}, \theta\eta_{g+1}), \ldots, (\psi_v, \theta\eta_u) \end{matrix} \right]\right\}$$

$$= \sum_{n,k=0}^{\infty} \frac{(\lambda')_{n+k}\,(-\alpha)_k\,(\rho)_n\,(x')^n\,(-t)^{n+k}}{(-\alpha-\beta)_k\,(\sigma)_n\,n!\,k!} \left\{ H^{g+m_1,\,2+f+n_1}_{p_1+v+2,\,q_1+u+1} \left[\frac{t}{\lambda_1^{\theta}} \middle| \begin{matrix} (1-a-n-k,\rho_1),(1-b-n-k,\sigma_1),\{(\epsilon_f,\theta\xi_f)\}, \\ \{(\psi_g,\theta\eta_g)\},\{(d_{q_1},\delta_{q_1})\},(1-a-b-2n-2k,\rho_2+\sigma_1), \end{matrix} \right. \right.$$

$$\left. \left. \begin{matrix} \{(c_{p_1},\gamma_{p_1})\},(\epsilon_{f+1},\theta\xi_{f+1}),\ldots(\epsilon_v,\theta\xi_v) \\ (\psi_{g+1},\theta\eta_{g+1}),\ldots,(\psi_u,\theta\eta_u) \end{matrix} \right] \right\} \tag{5.6}$$

provided that $\mathrm{Re}\,(a) > 0$, $\mathrm{Re}\,(b) > 0$, $\mathrm{Re}\,(\alpha) > -1$, $\mathrm{Re}\,(\beta) > -1$, $\delta' < R(a+b+c) < \beta'$, and the notation θ, ϵ, ψ are given in (2.1).

Sub Case

Putting $\rho_1 = \sigma_2 = 0$ in (5.3), we get as

$$\sum_{m=0}^{\infty} \frac{(a)_n\,(b)_n}{2^{2n}(1+\alpha)_n\,(1+\beta)_n\,(\frac{1}{2}(a+b)\,)_n\,(\frac{1}{2}(a+b+1)} P_n^{(\alpha,\beta)}(x')t^n$$

$$= F\left[\begin{matrix} a, b, -\,; -\,; \\ \qquad\qquad \frac{1}{8}(x'-1)t,\ \frac{1}{8}(x'+1)t \\ \frac{a+b}{2},\ \frac{a+b+1}{2}\ 1+\alpha\,;\ 1+\beta\,; \end{matrix} \right]$$

where $F(x, y)$ denotes a generalized Appell functions in the contracted notation of Burchnall and Chaundy (1941 p. 112).

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