OSCILLATORY HYDROMAGNETIC FREE CONVECTION FLOW PAST AN INFINITE VERTICAL FLAT PLATE WITH SUCTION

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Oscillatory hydromagnetic laminar free convection flow of viscous incompresible and electrically conducting fluid past a vertical infinite porous flat plate is studied in the presence of a transverse magnetic field when the temperature or heat flux at the surface oscillates in magnitude but not in direction. Expressions for temperature, velocity and skin-friction are obtained in closed form when the suction velocity is uniform. Effect of magnetic field on velocity and skin friction is studied for Prandtl numbers 0.1 and 1.

1. INTRODUCTION

Menold and Yang (1962) have obtained solutions for unsteady free convection flow of an incompressible fluid past an infinite plate with time-dependent arbitary surface temperature or heat flux variations. Gupta (1960) studied the effect of horizontal magnetic field on two-dimensional unsteady laminar free convection flow past a vertical flat plate undergoing a stepwise change in temperature. Singh (1964*a*) discussed the unsteady laminar free convection flow of an electrically conducting fiuid past a vertical infinite flat plate in the presence of a constant horizontal magnetic field when the temperature or heat flux at the surface varies as some power of time. Recently, Ballabh and Singh (1967) have considered the unsteady free convection flow of an incompressible fluid past a vertical infinite porous flat plate with time dependent arbitrary surface temperature or heat flux variations when suction velocity is uniform.

The object of this paper is to study the effect of a constant horizontal magnetic field on oscillatory laminar free convection flow of viscous incompressible and electrically conducting fluid past a vertical infinite porous flat plate for which the wall temperature or surface heat flux oscillates in magnitude but not in direction when the suction velocity is uniform. Graphs showing variations of velocity, temperature, skin friction, rate of heat transfer and plate temperature difference are plotted for different values of magnetic field and Prandtl number. It is seen that the effect of magnetic field is to decrease the velocity and skin friction. Further if the Prandtl number of the fluid is increased the effectiveness of magnetic field in reducing the velocity and skin friction decreases.

Such problems have a wide range of application in the large scale convective motions of the atmospheric air. Their study may also, be of great interest and use in the global general circulation with convection in equatorial zone, subsidence in the horse latitudes, ascent in the polar frontal zones and subsidence near the poles.

Similar free and forced convection problems have been considered by Gupta (1962), Singh (1963, 1964 *b*) and Suryaprakasarao (1962).

2. Analysis

With x-axis along the plate and y-axis normal to it, the time dependent equations which describe the unsteady hydromagnetic laminar free convection flow past a vertical infinite flat plate are

$$\frac{\partial v}{\partial y} = 0, \tag{2.1}$$

$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + g\beta\theta - \frac{\sigma B_0^2}{\rho}u, \tag{2.2}$$

$$\frac{\partial \theta}{\partial t} + v\frac{\partial \theta}{\partial y} = \frac{\nu}{P_r}\frac{\partial^2 \theta}{\partial y^2}. \tag{2.3}$$

Here g is the gravitational acceleration, u the velocity along the plate, σ the electrical conductivity of the fluid, ρ the density, ν the kinematic viscosity, β the expansivity and θ the excess of the local temperature $\bar{T}$ over the temperature $\bar{T}_\infty$ at a large distance from the plate. B_0 is the magnetic induction, t the time and P_r the Prandtl number.

Fluid property variations have been considered only to the extent of a density variation which provides a buoyancy force. Viscous dissipation has been neglected in the energy equation (2.3) because of small velocities usually encountered in free convection flows. Further since the Joule heating is usually of the same order as the viscous dissipation, it has been also neglected. Throughout this paper rationalised MKS units have been used.

In (2.2), as in Singh (1964*a*), it is assumed that the magnetic Reynolds number is small, so that the induced magnetic field is negligible in comparison to the imposed magnetic field. Further, since no external electric field is applied and the effect of polarisation of ionized fluid is negligible it can be assumed that the electric field is zero.

Equation (2.1), on integration, gives $v = -V_0$ (constant) where V_0 (>0) represents the suction velocity.

Equations (2.2) and (2.3) can now be written as

$$\frac{\partial u}{\partial t} - V_0 \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} + g\beta\theta - \frac{\sigma B_0^2}{\rho} u \tag{2.4}$$

$$\frac{\partial \theta}{\partial t} - V_0 \frac{\partial \theta}{\partial y} = \frac{\nu}{P_r} \frac{\partial^2 \theta}{\partial y^2} . \tag{2.5}$$

The boundary conditions are

$$\left.\begin{aligned} & u(0, t) = u(\infty, t) = \theta(\infty, t)\ 0 \\ & \theta(0, t) = \theta_0(t) = ae^{i\omega t} \\ \text{or} \quad & \frac{\partial \theta(0, t)}{\partial y} = -\frac{q(t)}{K} = -be^{i\omega t} \end{aligned}\right\} \tag{2.6}$$

where, $\theta_0 = \bar{T}_w(t) - \bar{T}_\infty$, and $\bar{T}_w(t)$ is the plate temperature, $q(t)$ is the prescribed surface heat flux, K the thermal conductivity and a and b are constants.

3. Solution of the Equation for Oscillatory Surface Temperature Variation

It is clearly seen from the energy equation (2.5) that the temperature is not affected by the magnetic field.

Putting $\theta = ae^{i\omega t} f(\eta)$ in (2.5), we get

$$f'' + P_r f' - \frac{i\Omega P_r}{4} f = 0 \tag{3.1}$$

with boundary conditions

$$f(0) = 1, \ f(\infty) = 0 \tag{3.2}$$

where

$$\eta = \frac{V_0 y}{\nu}, \quad \Omega = \frac{4\nu\omega}{V_0^2}$$

and primes denote the differentiation with respect to η.

Solution of (3,1) satisfying the boundary conditions (3.2) is given by

$$f = \exp\left\{-\frac{\eta}{2}\left(P_r + \sqrt{P_r^2 + i P_r \Omega}\right)\right\}. \tag{3.3}$$

Hence the non-dimensional temperature is given by

$$\Theta = \frac{\theta}{a} = \exp\left\{ i\Omega T - \frac{\eta}{2}\left(P_r + \sqrt{P_r^2 + iP_r\Omega}\right)\right\},$$

where $$T = \frac{t\,V_0^2}{4\,\nu}\ .$$

Taking the real part we get

$$\Theta = \exp\left\{-\frac{\eta}{2}\left(P_r + R_1 \cos\frac{\gamma}{2}\right)\right\} \cos\left(\Omega T - \frac{\eta}{2} R_1 \sin\frac{\gamma}{2}\right)$$

where

$$R_1 = P_r^{\frac{1}{2}} (P_r^2 + \Omega^2)^{\frac{1}{4}} \text{ and } \gamma = \tan^{-1}\left(\frac{\Omega}{P_r}\right).$$

Again substituting

$$u = \frac{g\beta a}{\omega} e^{i\omega t} F(\eta)$$

and $\theta = ae^{i\omega t} f(\eta)$ in (2.4), we get

$$F'' + F' - \frac{(M + i\Omega)}{4} F = -\frac{\Omega}{4} f \tag{3.5}$$

with boundary conditions

$$F(O) = F(\infty) = 0 \tag{3.6}$$

where,

$$M = \frac{4\nu\sigma\ B_0^2}{\rho V_0^2}\ .$$

Solution of (3.5) satisfying the boundary conditions (3.6) is given by

$$F = \frac{\Omega\left\{M - 2(P_r - 1)\left(P_r + R_1\cos\frac{\gamma}{2}\right) + i(P_r - 1)\left(\Omega + 2R_1 \sin\frac{\gamma}{2}\right)\right\}}{\Delta}$$

$$\times\left[\exp\left\{-\frac{\eta}{2}\left(P_r + R_1\cos\frac{\gamma}{2}\right)\right\}\exp\left\{-\frac{i\eta}{2} R_1 \sin\frac{\gamma}{2}\right\}\right.$$

$$\left. - \exp\left\{-\frac{\eta}{2}\left(1 + R_2 \cos\frac{B}{2}\right)\right\}\exp\left\{-\frac{i\eta}{2} R_2 \sin\frac{\beta}{2}\right\}\right] \tag{3.7}$$

where,

$$R_2 = \{(1+M)^2 + \Omega^2\}^{\frac{1}{4}},\ \beta = \tan^{-1}\left(\frac{\Omega}{1+M}\right)$$

and

$$\Delta = \left\{M - 2(P_r - 1)(P_r + R_1 \cos\frac{\gamma}{2}\right\}^2 + (P_r - 1)^2\left(\Omega + 2R_1 \sin\frac{\gamma}{2}\right)^2$$

Hence the non-dimensional velocity is given by

$$\Phi\,(\eta,\ M)=\frac{u\omega}{g\,\beta\,a}=F(\eta)\,e^{i\omega T}.$$

Taking the real part, we get

$$\begin{aligned}\Phi\,(\eta,\ M)= & \frac{\Omega}{\Delta}\Big[\ \Big\{M-2(P_r-1)\Big(P_r+R_1\cos\frac{\gamma}{2}\Big)\Big\}\\ & \times\Big\langle\exp\Big\{-\frac{\eta}{2}\Big(P_r+R_1\cos\frac{\gamma}{2}\Big)\Big\}\cos\Big(\Omega\,T-\frac{\eta}{2}R_1\sin\frac{\gamma}{2}\Big)\\ & -\exp\Big\{-\frac{\eta}{2}\Big(1+R_2\cos\frac{\beta}{2}\Big)\Big\}\cos\Big(\Omega T-\frac{\eta}{2}R_2\sin\frac{\beta}{2}\Big)\Big\rangle\\ & +(P_r-1)\Big(\Omega+2R_1\sin\frac{\gamma}{2}\Big)\Big\langle\exp\Big\{-\frac{\eta}{2}\Big(1+R_2\cos\frac{\beta}{2}\Big)\Big\}\\ & \times\sin\Big(\Omega\,T-\frac{\eta}{2}R_2\sin\frac{\beta}{2}\Big)-\exp\Big\{-\frac{\eta}{2}\Big(P_r+R_1\cos\frac{\gamma}{2}\Big)\Big\}\\ & \times\sin\Big(\Omega T-\frac{\eta}{2}R_1\sin\frac{\gamma}{2}\Big)\Big\rangle\ \Big]\end{aligned}\tag{3.8}$$

The rate of heat transfer at the plate is given by

$$Q\,(T)=\frac{2\nu q\,(t)}{aKV_0}=P_r\cos\Omega\,T+R_1\cos\Big[\,\Omega T+\frac{\gamma}{2}\Big]\tag{3.9}$$

where

$$q(t)=-K\,\Big(\frac{\partial\theta}{\partial y}\Big)_{y=0}.$$

The drag coefficient at the plate is given by

$$\begin{aligned}\tau_\omega{}^*= & \frac{2\omega\,\tau_\omega}{\rho V_0\,g\,\beta a}=\frac{\Omega}{\Delta}\Big[\Big\{M-2(P_r-1)\Big(P_r+R_1\cos\frac{\gamma}{2}\Big)\Big\}\\ & \times\Big\langle R_2\cos\Big(\Omega T+\frac{\beta}{2}\Big)-R_1\cos\Big(\Omega T+\frac{\gamma}{2}\Big)-(P_r-1)\cos\Omega T\Big\rangle\\ & +(P_r-1)\Big(\Omega+2R_1\sin\frac{\gamma}{2}\Big)\Big\langle R_1\sin\Big(\Omega T+\frac{\gamma}{2}\Big)-R_2\sin\Big[\,\Omega T+\frac{\beta}{2}\Big]\\ & +(P_r-1)\sin\Omega T\Big\rangle\Big]\end{aligned}\tag{3.10}$$

where,

$$\tau_\omega=\rho\nu\Big[\frac{\partial u}{\partial y}\Big]_{y=0}.$$

Figs. 1 and 2 show the variation of dimensionless temperature and velocity with η respectively for $\Omega T=\frac{\pi}{2}$ and for different values of M and

P_r. The variations of rate of transfer at the plate and skin friction with T, for different values of P_r and M, are shown in figs. 3 and 4 respectively.

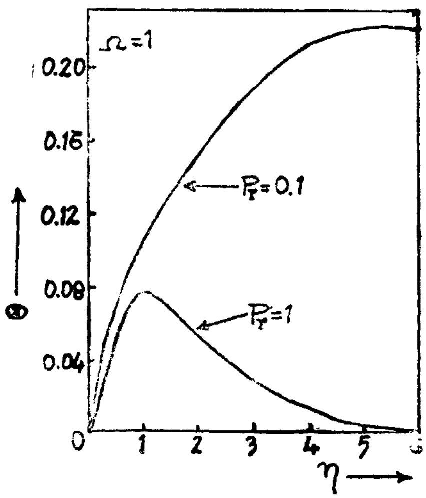

FIG. 1. Temperature profile versus η.

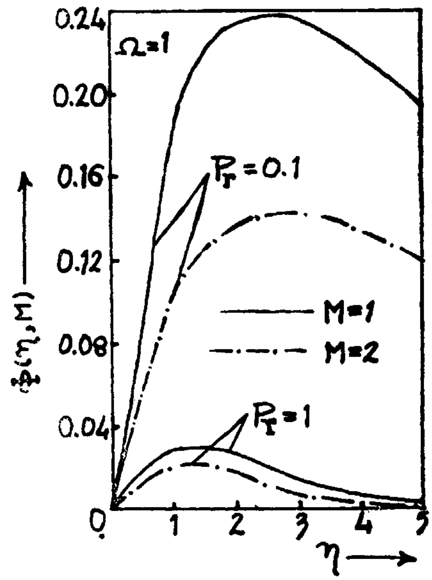

FIG. 2. Velocity profile versus η.

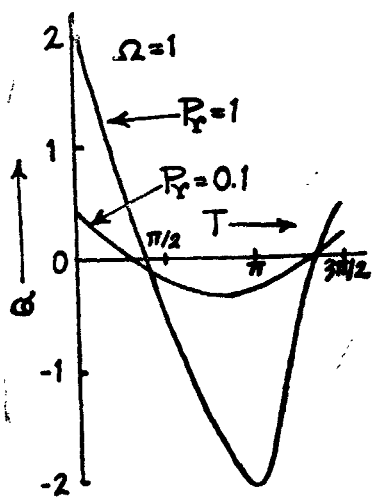

FIG. 3. Rate of heat transfer versus T.

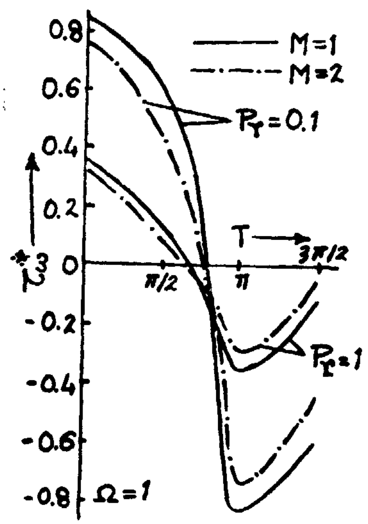

FIG. 4. Skin friction versus T.

It is seen that temperature, velocity and skin friction decrease with p_r while the rate of heat transfer at the plate increases. Figs. 2 and 4 show that the effect of magnetic field is to decrease the velocity and skin friction.

Further inspection reveals, that as the Prandtl number increases the effectiveness of magnetic field in reducing the velocity and skin friction decreases.

4. Solution of the Equations For Oscillatory Heat Flux Variation

In this case substituting

$$\theta = \frac{q(t)}{K} \cdot \frac{\nu}{V_0} h(\eta) \frac{b\nu}{V_0} e^{i\omega t} h(\eta) \tag{4.1}$$

in equation (2.5) and solving the resulting equation we get

$$\Theta^* = \frac{\theta}{\theta_1} = \frac{\exp\left\{-\frac{\eta}{2}\left(P_r + R_1 \cos\frac{\gamma}{2}\right)\right\}}{\Delta_1}\left[P_r \cos\left(\Omega T - \frac{\eta}{2} R_1 \sin\frac{\gamma}{2}\right) + R_1 \cos\left(\Omega T - \frac{\gamma}{2} - \frac{\eta}{2} R_1 \sin\frac{\gamma}{2}\right)\right] \tag{4.2}$$

where

$$\theta_1 = \frac{2b\nu}{V_0} \text{ and } \Delta_1 = \left(P_r + R_1 \cos\frac{\gamma}{2}\right)^2 + R_1^2 \sin^2\frac{\gamma}{2}. \tag{4.2}$$

Again substituting

$$u = \frac{\nu g \beta}{V_0 \omega}\left(\frac{q}{k}\right) F(\eta) = \frac{\nu b g \beta}{V_0 \omega} e^{i\omega t} F(\eta) \tag{4,3}$$

and

$$\theta = \frac{\nu b}{V_0} e^{i\omega t} h(\eta)$$

in equation (2.4) and solving the resulting equation, we get

$$\Phi^*(\eta, M) = \frac{u\omega}{\theta_1 g \beta} = \frac{\Omega}{\Delta \Delta_1}\Big[\Big\langle \Big\{M - 2(P_r - 1)\Big(P_r + R_1 \cos\frac{\gamma}{2}\Big)\Big\} \times \Big(P_r + R_1 \cos\frac{\gamma}{2}\Big) + R_1(P_r - 1)\Big(\Omega + 2R_1 \sin\frac{\gamma}{2}\Big)\sin\frac{\gamma}{2}\Big\rangle$$

$$\times \Big\langle \exp\Big\{-\frac{\eta}{2}\Big(P_r + R_1 \cos\frac{\gamma}{2}\Big)\Big\}\cos\Big(\Omega T - \frac{\eta}{2} R_1 \sin\frac{\gamma}{2}\Big) - \exp\Big\{-\frac{\eta}{2}\Big(1 + R_2 \cos\frac{\beta}{2}\Big)\Big\}\cos\Big(\Omega T - \frac{\eta}{2} R_2 \sin\frac{\beta}{2}\Big)\Big\rangle$$

$$+ \Big\langle (P_r - 1)\Big(P_r + R_1 \cos\frac{\gamma}{2}\Big)\Big(\Omega + 2R_1 \sin\frac{\gamma}{2}\Big) - R_1\Big\{M - 2(P_r - 1)\Big(P_r + R_1 \cos\frac{\gamma}{2}\Big)\Big\}\sin\frac{\gamma}{2}\Big\rangle$$

$$\times \langle \exp \left\{ -\frac{\eta}{2}\left[1+R_2 \cos \frac{\beta}{2}\right]\right\} \sin \left[\Omega T-\frac{\eta}{2} R_2 \sin \frac{\beta}{2}\right]$$
$$-\exp \left\{-\frac{\eta}{2}\left[P_r+R_1 \cos \frac{\gamma}{2}\right]\right\} \sin \left[\Omega T-\frac{\eta}{2} R_1 \sin \frac{\gamma}{2}\right] \rangle \Big] . \quad (4.4)$$

The plate temperature difference is given by

$$\Theta_0{}^* = \frac{\bar{T}_\omega(t)-\bar{T}_\infty}{\theta_1} = \frac{1}{\Delta_1}\left[P_r \cos \Omega T + R_1 \cos \left[\Omega T - \frac{\gamma}{2}\right]\right] . \quad (4.5)$$

The skin friction at the plate is given by

$$\tau'_\omega = \frac{2\omega\,\tau_\omega}{\rho V_0\,\theta_1\, g\beta} = \frac{\Omega}{\Delta\,\Delta_1}\Big[\langle\Big\{M-2(P_r-1)\Big(P_r+R_1 \cos\frac{\gamma}{2}\Big)\Big\}\Big(P_r+R_1 \cos\frac{\gamma}{2}\Big)$$
$$+R_1(P_r-1)\left(\Omega+2R_1 \sin\frac{\gamma}{2}\right) \sin \frac{\gamma}{2}\rangle \langle R_2 \cos\left(\Omega T+\frac{\beta}{2}\right)$$
$$-(P_r-1)\cos\Omega T - R_1 \cos\left(\Omega T+\frac{\gamma}{2}\right)\rangle$$
$$+\langle (P_r-1)\left(P_r+R_1 \cos\frac{\gamma}{2}\right)\left(\Omega+2R_1 \sin\frac{\gamma}{2}\right]$$
$$-R_1\left\{M-2(P_r-1)\left[P_r-R_1 \cos\frac{\gamma}{2}\right]\right\} \sin\frac{\gamma}{2}\rangle \langle R_1 \sin\left[\Omega T+\frac{\gamma}{2}\right]$$
$$+(P_r-1)\sin\Omega T - R_2 \sin\left(\Omega T+\frac{\beta}{2}\right)\rangle\Big] . \quad (4.6)$$

The dimensionless temperature Θ^* and velocity $\Phi^*(\eta, M)$ are plotted, respectively, in Figs. 5 and 6 for $\Omega T=\pi/2$. Figs. 7 and 8 show the variation

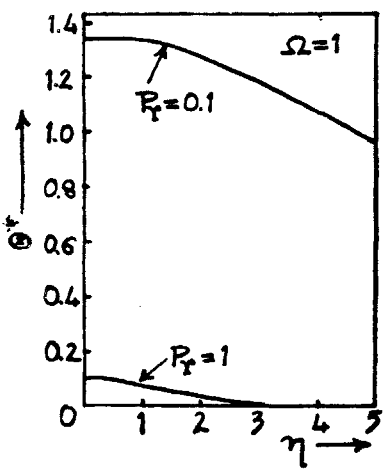

Fig. 5. Temperature profile versus η.

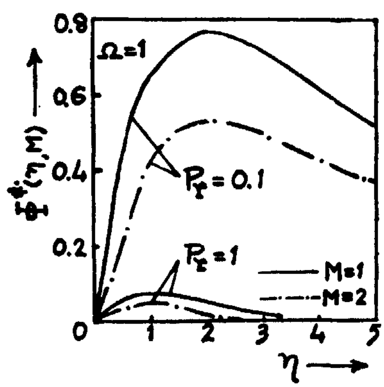

Fig. 6. Velocity profile versus η.

of dimensionless plate temperature difference Θ_0^* and drag coefficient τ'_ω with T for $\Omega=1$ and $P_r = 0.1, 1$.

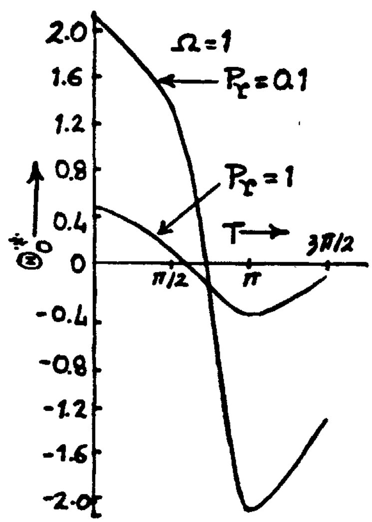

Fig. 7. Plate temperature difference versus T.

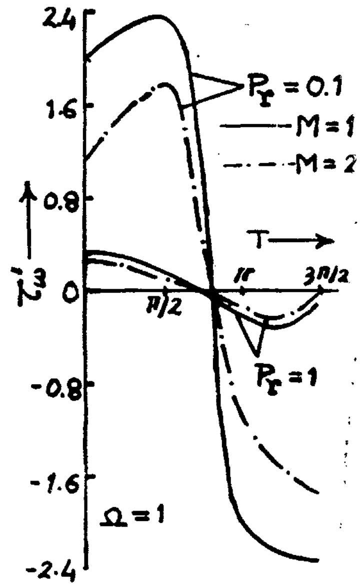

Fig. 8. Skin friction versus T.

It is seen that the temperature, velocity, plate temperature difference and skin friction decrease as P_r increases. It is also seen that for any fixed value of Prandtl number, the velocity and skin friction decrease with magnetic field, and as Prandtl number increases, effect of magnetic field decreases.

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