STEADY FLOW OF A VISCO-ELASTIC LIQUID ABOUT A POROUS ROTATING DISK

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The flow of a visco-elastic fluid of Walters B′-fluid model around an infinite porous rotating disk has been studied by Perturbation technique. Drag on the disk is observed to be larger than in the Newtonian case. The non-Newtonian effect decreases with an increase in the speed of rotation.

Introduction

Various authors have studied the flow about a rotating disk ever since it was indicated by Von Karman (1921) and later investigated more thoroughly by Cochran (1934). The study has immense practical utility especially when the fluid is non-Newtonian. As quite a large number of fluids of industrial importance have their constitutive equation as proposed by Walters, we consider the rotation of an infinite and porous disk in a fluid of Walters B′-model. The parameter characteristic of the fluid being supposed sufficiently small ($\ll 1$), we use it as a perturbation parameter and obtain sets of differential equations governing the flow. The solutions of the first two sets only have been considered as it is presumed that the remaining sets influence but very weakly the real state of motion of the fluid. In obtaining the solutions of the two sets of which the first is a set of non-linear differential equations, recourse is taken to the Runge-Kutta technique of numerical integration.

Formulation of the Problem

We assume that a porous disk of infinite radius has been rotating about a fixed axis through its centre, for sufficiently large time, with a constant angular velocity ω. The constitutive equation for a fluid of Walters B′-model is

$$P^{ik} = -p\delta^{ik} + 2\mu g^{ik} - 2K_0 \overset{\nabla}{g}{}^{ik} \qquad \ldots(1)$$

$$\overset{\nabla}{g}{}^{ij} = g^{ij}_{,k}\, v^k - g^{ik}\, v^j_{,k} - g^{kj}\, v^i_{,k} + g^{ij}\, v^k_{,k}$$

where P^{ik} is the stress tensor, g^{ij} the rate of strain tensor given by

$$2g^{ik} = v_{i,k} + v_{k,i}$$

and v^i a velocity vector. A comma denotes covariant differentiation with respect to the symbol following it. It is also supposed that a non-Newtonian fluid of Walters B'-model occupies the space $0 < Z < \infty$ where the disk is in the plane $Z = 0$. Assuming that the constant velocity of suction (or injection as the case may be) at the disk is $w = w_0$, we have to solve the Navier Stokes equations of motion and the equation of continuity for the fluid in cylindrical polar coordinate system with u, v, w as the radial, azimuthal and axial velocity component respectively (taking $\partial/\partial\theta = 0$ due to axial symmetry) under the following boundary conditions:

$$\left.\begin{array}{llll} u = 0, & v = r\omega, & w = w_0, & \text{at } z = 0 \\ \text{and} & & & \\ u = 0, & v = 0 & & \text{at } z \to \infty \end{array}\right\} \quad \ldots(2)$$

The Von-Kármán similarity transformations,

$$\eta = Z\,(\omega/\nu)^{1/2}, \quad u = r\omega F(\eta), \quad v = r\omega G(\eta),$$

$$w = (\nu w)^{1/2}\, H(\eta), \quad p = \mu\omega\, p(\eta)$$

reduce the equations of motion, continuity and the boundary conditions (2) to

$$\left.\begin{array}{l} F^2 - G^2 + HF' = F'' - \lambda\,(HF''' - 2H'F') \\ 2FG + HG' = G'' - \lambda\,(HG''' - 6F'G') + 2\lambda\, RFG \\ P' = H'' - HH' + 2\lambda\,(HF'' + 6FF') \\ H' = -2F \end{array}\right\} \quad \ldots(3)$$

and

$$\left.\begin{array}{llll} F(0) = 0, & G(0) = 1, & H(0) = RS, & P(0) = 0 \\ F(\infty) = 0, & G(\infty) = 0 & & \end{array}\right\} \quad \ldots(4)$$

(where a dash denotes differentiation with respect to η)

with $\lambda = \dfrac{K_0\,\omega}{\mu}$, $R = \dfrac{\nu}{r^2\,\omega}$ and $RS = \dfrac{w_0}{(\nu\omega)^{1/2}}$ = Porosity parameter.

SOLUTION OF THE EQUATIONS

As the non-Newtonian character of a fluid is a deviation of small extent from the Newtonian behaviour of fluids in general, one can write the velocity and pressure functions as,

$$\left.\begin{array}{l} F = \sum\limits_{n=0}^{\infty} \lambda^n F_n(\eta), \quad G = \sum\limits_{n=0}^{\infty} \lambda^n G_n(\eta), \quad H = \sum\limits_{n=0}^{\infty} \lambda^n H_n(\eta) \\ P = \sum\limits_{n=0}^{\infty} \lambda^n P_n(\eta). \end{array}\right\} \quad \ldots(5)$$

For small values of λ, say $\lambda \ll 1$, one is justified in neglecting terms involving λ^2, λ^3, etc., in the above expansions as the terms would have at best vanishingly little effect on the character of the flow. Insertion of (5), with $n = 1$ into (3) and (4) and equating coefficient functions of similar powers of λ from both sides of the equations lead to the following two sets of equations followed by their appropriate boundary conditions.

First set

$$\left.\begin{aligned} F_0'' &= F_0^2 - G_0^2 + H_0F_0' \\ G_0'' &= 2F_0G_0 + H_0G_0' \\ H_0' &= -2F_0 \\ P_0' &= H_0'' - H_0H_0'. \end{aligned}\right\} \qquad \ldots(6)$$

Boundary conditions

$$F_0(0) = 0, \quad G_0(0) = 1, \quad H_0(0) = RS, P_0(0) = 0$$

$$F_0(\infty) = 0, \; G_0(\infty) = 0.$$

Second set

$$\left.\begin{aligned} F_1'' &= 2F_0F_1 - 2G_0G_1 + H_1F_0' + H_0F_1' + H_0F_0''' - 2H_0'F_0'' \\ G_1'' &= 2(F_0G_1 + F_1G_0) + H_0G_1' + H_1G_0' + H_0G_0''' \\ &\quad - 6F_0'G_0' - 2RF_0G_0 \\ H_1' &= -2F_1 \\ P_1' &= -H_0H_0''' + 3H_0''H_1' + H_1'' - H_0H_1' - H_1H_0' \end{aligned}\right\} \qquad \ldots(7)$$

Boundary conditions

$$F_1(0) = 0, \; G_1(0) = 0, \quad H_1(0) = 0, \; P_1(0) = 0$$

$$F_1(\infty) = 0, \quad G_1(\infty) = 0.$$

One can immediately see that the first is a set of simultaneous non-linear differential equations while the second set (and as a matter of fact any subsequent set if obtained) is a linear one. One of the most suitable techniques for the solution of such sets of equations is the Runge-Kutta technique of numerical integration as indicated in Ralston and Wilf (1960). We have integrated the equations accordingly carrying out the computations, on an IBM 1130 with the step length as 0·1. The results exhibited through Tables I and II give us some interesting informations which have been included in the following paragraphs.

TABLE I

R	RS	$-G_0'(0)$	$-G_1'(0)$
	−·03	·626	·664
	−·02	·624	·664
·05	−·01	·619	·661
	0	·612	·659
	·02	·607	·658
	·03	·603	·656

TABLE II

RS	R	$-G_0'(0)$	$-G_1'(0)$
	·01		·665
	·05		·659
	·1		·653
0	·2	·615	·641
	·3		·629
	·4		·617
	·5		·605
	·1		·656
−·02	·2	·624	·646
	·3		·634
	·1		·650
·03	·2	·607	·638
	·3		·626

DISCUSSION

Drag on the disk is larger in the visco-elastic fluid than in the Newtonian case. While drag does not respond to any small change in the rotation of the disk in a Newtonain fluid, there is a distinct response in the visco-elastic case. Drag seems to increase with an increase in the suction (Table 1). The viscoelastic fluid rotates slower than a Newtonian fluid, the plane of motion utmost effected being $\eta = 0 \cdot 5$. Both the radial velocity which is everywhere away from the axis and the axial velocity which is directed towards the disk increase in comparison to a Newtonian fluid upto $\eta = 0 \cdot 2$, but farther away they decrease. The visco-elastic effect decreases with an increase in either the speed of rotation or of suction at the disk. The radial velocity which is always away from the axis attains a maximum abruptly near $\eta = 1$. The visco-elastic parameter helps in lowering the pressure everywhere in the fluid region except very near the disk where there occurs a slight increase in the pressure.

References

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