LAMINAR STEADY FORCED CONVECTION FLOW OF A VISCOUS FLUID IN A CHANNEL

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In the present study, the steady laminar convection problem of heat transfer in the fully developed constant property flow of liquids through a straight channel, with sector of a circle as cross-section is considered. The contribution of heat due to an arbitrary heat source present in the channel is taken. The change in velocity profiles, temperature distribution and in Nusselt number are determined and have been shown graphically, which shows that an increase in angle of the sector causes increase in velocity, temperature and Nusselt number.

1. INTRODUCTION

The attention of many research workers in fluid dynamics has been diverted towards the study of heat transfer in viscous incompressible fluid in the recent years. Some solutions of the problem of heat transfer to a fluid flowing with Poiseuille velocity distribution through a circular pipe has been discussed by various authors including Goldstein (1938). Bhatnagar and Tikekar (1966) have discussed the temperature distribution of a viscous incompressible fluid flowing in a channel bounded by two co-axial circular cylinders, neglecting the effect of viscous dissipation.

The purpose of this paper is to analyse theoretically and numerically the problem of forced convection of a liquid flowing through a straight channel bounded by a sector of a circle, including an angle 2α, as cross-section. In the present work we have imagined the following facts:

(*a*) The flow is laminar and steady and the liquid properties are constant;

(*b*) an arbitrary heat source is present in the channel;

(*c*) the flow is fully developed (both hydrodynamically and thermally);

(*d*) the liquid and wall temperatures increase linearly at the same rate in the direction of flow;

(*e*) the axis of the channel and flow direction are in positive direction of z-axis.

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The problem considered here is a basic and fundamental problem in the heat transfer science and is important from engineering point of view (*e.g.*, temperature distribution in sector shaped pads, which are used in special types of brakes in engineering is required). The fact (*d*) is reasonably satisfied in some nuclear reactor problems and in some counter flow heat exchangers, as it has been remarked by Han (1961). Also the fact (*d*) is of technical importance, and gives rise to the considerable ease with which exact solutions can be obtained. It thus appears that the stated problem is of physical importance.

2. Formulation of the Problem

We take for reference a frame of cylindrical polar system of co-ordinates (r, θ, z), the axis being taken along the length of the tube through which the flow is to be considered, the cross-section of the tube being a sector bounded by two radii $\theta = \pm \alpha$ and the circle $r = a$. Let u, v, w be the components of the velocity in the direction of r, θ, z respectively. In our case the flow is to be taken only along the length of the tube so that u and v are zero.

The equations of motion for a viscous incompressible fluid under the above assumption and under no external forces reduce to

$$0 = -\frac{1}{\rho}\frac{\partial p}{\partial r}, \qquad \ldots(2.1)$$

$$0 = -\frac{1}{\rho r}\frac{\partial p}{\partial \theta}, \qquad \ldots(2.2)$$

$$\frac{\partial w}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2} + \frac{\partial^2 w}{\partial z^2}\right), \qquad \ldots(2.3)$$

and the equation of continuity becomes

$$\frac{\partial w}{\partial z} = 0, \qquad \ldots(2.4)$$

so that $w = w(r, \theta, t)$ only and eqn. (2.3) reduces to

$$\frac{\partial w}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2}\right). \qquad \ldots(2.5)$$

In view of eqns. (2.1) and (2.2) we will have $p = p(z, t)$. As $\partial w/\partial z$ is zero, from eqn. (2.5) we conclude that $\partial p/\partial z$ can only be a function of t or a constant. We therefore assume that, for steady motion,

$$\frac{1}{\rho}\frac{\partial p}{\partial z} = -K, \qquad \ldots(2.6)$$

so that eqn. (2.5) becomes

$$\frac{\partial^2 w}{\partial r^2} + \frac{1}{r}\frac{\partial w}{\partial r} + \frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2} = -\frac{K}{\nu}, \qquad \ldots(2.7)$$

The energy equation neglecting the viscous dissipation under above assumptions reduces to

$$c_1 w = Q + k\left(\frac{\partial^2 T'}{\partial r^2} + \frac{1}{r}\frac{\partial T'}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T'}{\partial \theta^2}\right), \qquad ...(2.8)$$

where

$$c_1 = c_v \frac{\partial T'}{\partial z}, \qquad ...(2.9)$$

and

$$T' = T_1 z + T(r, \theta), \qquad ...(2.10)$$

k is coefficient of thermal conductivity and Q is the heat source intensity.

Rawat (1970) has shown that the velocity w under the boundary conditions

$$w(r, \theta) = 0 \text{ when } r = a, \ -\alpha \leqslant \theta \leqslant \alpha, \qquad ...(2.11)$$

and

$$w(r, \theta) = 0 \text{ when } \theta = \pm \alpha, \ 0 \leqslant r \leqslant a, \qquad ...(2.12)$$

is given by

$$w = \frac{2Ka^2}{\alpha \nu} \sum_{m=0}^{\infty} \frac{(-1)^m \cos \lambda_m \theta}{\lambda_m (4 - \lambda_m^2)} \left[\left(\frac{r}{a}\right)^{\lambda_m} - \left(\frac{r}{a}\right)^2\right], \qquad ...(2.13)$$

where

$$\lambda_m = \frac{2m + 1}{2\alpha}\pi, \qquad ...(2.14)$$

substituting (2.13) and (2.10) in (2.8) we have

$$\frac{\partial^2 T}{\partial r^2} + \frac{1}{r}\frac{\partial T}{\partial r} + \frac{1}{r^2}\frac{\partial^2 T}{\partial \theta^2} = \frac{2c_1 Ka^2}{k\nu\alpha} \times \left[\sum_{m=0}^{\infty} \frac{(-1)^m \cos \lambda_m \theta}{\lambda_m (4 - \lambda_m^2)} \left\{\left(\frac{r}{a}\right)^{\lambda_m} - \left(\frac{r}{a}\right)^2\right\}\right] - \frac{Q}{k}, \qquad ...(2.15)$$

which is to be solved under the boundary conditions

$$T(r, \theta) = 0 \text{ when } r = a, \ -\alpha \leqslant \theta \leqslant \alpha, \qquad ...(2.16)$$

and

$$T(r, \theta) = 0 \text{ when } \theta = \pm \alpha, \ 0 \leqslant r \leqslant a. \qquad ...(2.17)$$

From symmetry consideration, (2.17) implies $\partial T/\partial \theta = 0$ when $\theta = 0$.

3. Method of Solution

In previous investigations most of the research workers have taken source of heat generation with constant intensity Q, but in the present investigation Q is taken as any function of r and θ.

Let

$$Q = A_1 f(r) \cdot g(\theta), \qquad \ldots(3.1)$$

where A_1 is constant, $f(r)$ and $g(\theta)$ are functions of r and θ respectively.

Let us define,

$$\bar{T}(r, m) = \int_0^{\alpha} T(r, \theta) \cos \lambda_m \theta \, d\theta, \qquad \ldots(3.2)$$

where $\bar{T}(r, m)$ is the finite cosine transform of $T(r, \theta)$. Multiplying eqn. (2.15) by $\cos \lambda_m \theta$ and integrating within the limit 0 to α, under the boundary condition (2.17), we get

$$\frac{\partial^2 \bar{T}}{dr^2} + \frac{1}{r}\frac{d\bar{T}}{dr} - \frac{\lambda_m^2}{r^2}\bar{T} = \frac{c_1 K a^2}{kv}\left[\frac{(-1)^m}{\lambda_m (4 - \lambda_m^2)} \times \left\{\left(\frac{r}{a}\right)^{\lambda_m} - \left(\frac{r}{a}\right)^2\right\}\right] - \frac{A_1}{k} f(r)\ \bar{g}(m), \qquad \ldots(3.3)$$

where $\bar{g}(m)$ is finite cosine transform of $g(\theta)$, as defined in (3.2), and boundary condition (2.16) reduces to

$$\bar{T}(r, m) = 0, \text{ when } r = a. \qquad \ldots(3.4)$$

The solution of (3.3) may now be written as

$$\bar{T} = B_2 r^{\lambda_m} + B_1 r^{-\lambda_m} + \frac{c_1 K a^4}{kv}\left[\frac{(-1)^m (r/a)^{\lambda_m + 2}}{4\lambda_m (\lambda_m + 1)(4 - \lambda_m^2)} - \frac{(-1)^m (r/a)^4}{\lambda_m (4 - \lambda_m^2)(16 - \lambda_m^2)}\right] + \frac{A_1 \bar{g}(m)}{2k\lambda_m} \cdot F(r), \qquad \ldots(3.5)$$

where

$$F(r) = r^{-\lambda_m} \int r^{1-\lambda_m} f(r)\, dr - r^{\lambda_m} \int r^{1-\lambda_m} f(r)\, dr. \qquad \ldots(3.6)$$

As the temperature and so also its transform are to remain finite on the axis, we must take B_1 equal to zero, also at $r = a$, $\bar{T}(r, m) = 0$, therefore

$$\bar{T} = \frac{c_1 K a^4}{kv} \cdot \frac{(-1)^m}{\lambda_m (4 - \lambda_m^2)}\left[\frac{1}{4(\lambda_m + 1)}\left\{\left(\frac{r}{a}\right)^{\lambda_m + 2} - \left(\frac{r}{a}\right)^{\lambda_m}\right\} + \frac{1}{(16 - \lambda_m^2)}\{(r/a)^{\lambda_m} - (r/a)^4\}\right] + \frac{A_1 \bar{g}(m)}{2k\lambda_m} \times \{F(r) - F(a)(r/a)^{\lambda_m}\}. \qquad \ldots(3.7)$$

The inversion formula for (3.2) is found to be

$$T = \frac{2}{\alpha} \sum_{m=0}^{\infty} \bar{T} \cdot \cos \lambda_m \theta, \qquad \ldots(3.8)$$

and hence from (3.7) we have

$$T = \frac{2c_3}{\alpha} \sum_{m=0}^{\infty} \left[\frac{(-1)^m}{\lambda_m (4 - \lambda_m^2)} \left[\frac{1}{4(\lambda_m + 1)} \{(r/a)^{\lambda_m + 2} - (r/a)^{\lambda_m}\} \right.\right.$$

$$\left. + \frac{1}{(16 - \lambda_m^2)} \{(r/a)^{\lambda_m} - (r/a)^4\} \right] + \frac{c_4 \bar{g}(m)}{2a^2 \lambda_m}$$

$$\left. \times \{F(r) - F(a)(r/a)\} \right] \cos \lambda_m \theta. \qquad \ldots(3.9)$$

where

$$\left.\begin{aligned} &\frac{Ka^2}{\nu} = c_2, \quad \frac{c_1 K a^4}{k\nu} = c_1 c_2 a^2/k = c_3, \\ &\frac{A_1}{c_1 (Ka^2/\nu)} = c_4 \end{aligned}\right\}, \qquad \ldots(3.10)$$

and (3.9) gives the temperature distribution of the fluid in the channel.

The mean velocity, mean temperature and mixed mean temperature are respectively defined as

$$w_M = \frac{1}{A} \int_D w dA \; ; \quad T_M = \frac{1}{A} \int_D T dA \; ; \quad T'_M = \frac{1}{A w_M} \int_D w \cdot T dA, \qquad \ldots(3.11)$$

where A = Area of the sector $= a^2 \alpha$,

and D = Domain of the cross-section of the channel.

Using (2.13), we get

$$w_M = \frac{1}{A} \int_0^a \int_{-\alpha}^{\alpha} w \, r \, dr \, d\theta$$

$$= \frac{2c_2}{a^2 \alpha^2} \left[\sum_{m=0}^{\infty} \frac{(-1)^m}{\lambda_m (4 - \lambda_m^2)} \left\{ \int_{-\alpha}^{\alpha} \cos \lambda_m \theta \, d\theta \right\} \right.$$

$$\left. \times \int_0^a \left\{ \left(\frac{r}{a}\right)^{\lambda_m} - (r/a)^2 \right\} r \, dr \right]$$

$$= \frac{c_2}{2\alpha^2} \sum_{m=0}^{\infty} \frac{1}{\lambda_m^2 (\lambda_m + 2)^2},$$

$$= c_2 \, . \, c_2' \quad \text{(say)}, \qquad \ldots(3.12)$$

where

$$c_2' = \frac{1}{2\alpha^2} \sum_{m=0}^{\infty} \frac{1}{\lambda_m^2 (\lambda_m + 2)^2} \, . \qquad \ldots(3.13)$$

Using (3.9), we obtain

$$T_M = \frac{1}{a^2 \alpha} \int_0^a \int_{-\alpha}^{\alpha} T \, . \, r \, d\theta \, dr$$

$$= \frac{4c_3}{\alpha^2} \sum_{m=0}^{\infty} \left[\frac{1}{\lambda_m^2 (4 - \lambda_m^2)} \left[\frac{1}{4 (\lambda_m + 1)} \left(\frac{1}{\lambda_m + 4} - \frac{1}{\lambda_m + 2} \right) \right. \right.$$

$$\left. + \frac{1}{6 (\lambda_m + 2) (\lambda_m + 4)} \right] + \frac{c_4 \bar{g} (m) (-1)^m}{2a^4 \lambda_m^2}$$

$$\left. \times \int_0^a r \, \{F(r) - F(a) (r/a)^{\lambda_m}\} \, dr \right], \qquad \ldots(3.14)$$

and

$$T_M' = \frac{1}{A w_M} \int_0^a \int_{-\alpha}^{\alpha} w \, . \, T r \, d\theta \, dr$$

$$= \frac{1}{a^2 \alpha c_2 c_2'} \sum_{m=0}^{\infty} \int_0^a \left[\frac{2c_3}{\alpha} \left[\frac{(-1)^m}{\lambda_m (4 - \lambda_m^2)} \right. \right.$$

$$\times \left\{ \frac{1}{4 (\lambda_m + 1)} \{(r/a)^{\lambda_m + 2} - (r/a)^{\lambda_m}\} \right.$$

$$\left. \left. + \frac{1}{(16 - \lambda_m^2)} \{(r/a)^{\lambda_m} - (r/a)^4\} \right\} \right.$$

$$\left. + \frac{c_4 \bar{g} (m)}{2a^2 \lambda_m} \{F(r) - F(a) (r/a)^{\lambda_m}\} \right]$$

$$\times \left[\frac{2c_2 (-1)^m}{\lambda_m (4 - \lambda_m^2) \alpha} \{(r/a)^{\lambda_m} - (r/a)^2\} \right] r \, dr \Big] \alpha,$$

since

$$\int_{-\alpha}^{\alpha} \cos \lambda_{m_1} \theta \,.\, \cos \lambda_{m_2} \theta \; d\theta = 0 \text{ when } m_1 \neq m_2$$

$$= \alpha \text{ when } m_1 = m_2.$$

Therefore

$$T'_M = \frac{4c_3}{c'_2 \alpha^2} \sum_{m=0}^{\infty} \left[\frac{1}{\lambda_m (4 - \lambda_m^2)} \left\{ \frac{1}{4 (\lambda_m + 1)} \left(\frac{1}{2\lambda_m + 4} - \frac{1}{\lambda_m + 6} \right. \right. \right.$$

$$\left. - \frac{1}{2\lambda_m + 2)} + \frac{1}{\lambda_m + 4)} \right) + \frac{1}{16 - \lambda_m^2} \left(\frac{1}{2\lambda_m + 2} - \frac{1}{\lambda_m + 6} \right.$$

$$\left. \left. - \frac{1}{\lambda_m + 4} + \frac{1}{8} \right) \right\} + \frac{(-1)^m c_4 \bar{g}(m)}{2a^3 \lambda_m} \int_0^a \{F(r) - F(a)(r/a)^{\lambda_m}\}$$

$$\times \{(r/a)^{\lambda_m + 1} - (r/a)^2\} \, dr \Big] \times \frac{1}{\lambda_m (4 - \lambda_m^2)}. \qquad \ldots(3.15)$$

The overall heat transfer rate at the boundary is obtainable by applying Gauss theorem Milne-Thomson (1960), to the energy equation as follows:

$$q = \left(\frac{c_1 w_M}{k} - \frac{1}{Ak} \int_D Q \, dA \right) kA$$

$$= \left(c_1 w_M - \frac{A_1}{a^2 \alpha} \int_0^a \int_{-\alpha}^{\alpha} f(r) \,.\, g(\theta) \, r \, d\theta \; dr \right) a^2 \alpha$$

$$= - c_1 c_2 \left(\frac{c_4}{a^2 \alpha} \int_0^a \int_{-\alpha}^{\alpha} f(r) \; g(\theta) \, r \, d\theta \; dr - c'_2 \right) a^2 \alpha. \qquad \ldots(3.16)$$

Therefore, the heat transfer coefficient h and the Nusselt number N_u, based on mixed mean temperature, are

$$h = - q / S T'_M$$

$$= c_1 c_2 \left(\frac{c_4}{a^2 \alpha} \int_0^a \int_{-\alpha}^{\alpha} f(r) \; g(\theta) \; r \, dr \, d\theta - c'_2 \right) \frac{a^2 \alpha}{S T'_M}, \qquad \ldots(3.17)$$

where S = length of circumference of the cross-section

$$= 2a \; (1 + \alpha),$$

and Nusselt number $N_u = h\,De/K$

where De = equivalent hydraulic diameter

$$= \frac{4A}{S}.$$

Substituting the value of h

$$N_u = \frac{1}{c_3'}\left(\frac{c_4}{a^2\alpha}\int_0^a\int_{-\alpha}^{\alpha} f(r)\,g(\theta)\,r\,d\theta\,dr - c_2'\right)\cdot\left(\frac{\alpha}{1+\alpha}\right)^2. \qquad ...(3.18)$$

4. Special Cases

If we suppose that heat source intensity,

$$Q = A_1 r^{\sigma}\cos\delta\theta,\ \sigma \geqslant 0 \text{ and } \delta\theta \leqslant 0.$$

Then in this particular case we have

$$F(R) = \frac{-2\lambda_m r^{\sigma+2}}{(\sigma+2)^2 - \lambda_m^2} \qquad ...(4.1)$$

and

$$g(m) = \frac{(-1)^m \lambda_m \cos\delta\alpha}{\lambda_m^2 - \delta^2}. \qquad ...(4.2)$$

Substituting (4.1) and (4.2) in (3.9), we get required temperature in this particular case,

$$T(r,\theta) = \frac{2c_3}{\alpha}\sum_{m=0}^{\infty}\left[\frac{(-1)^m}{\lambda_m(4-\lambda_m^2)}\left[\frac{1}{4(\lambda_m+1)}\{(r/a)^{\lambda_m+2} - (r/a)^{\lambda_m}\}\right.\right.$$
$$\left. + \frac{1}{(16-\lambda_m^2)}\{(r/a)^{\lambda_m} - (r/a)^4\}\right] + \frac{c_4(-1)^m\lambda_m\cos\delta\alpha\cdot a^{\sigma}}{(\lambda_m^2-\delta^2)\{(\sigma+2)^2-\lambda_m^2\}}$$
$$\left.\times\{(r/a)^{\lambda_m} - (r/a)^{\sigma+2}\}\right]\cos\lambda_m\theta. \qquad ...(4.3)$$

Mean temperature T_M, mixed mean temperature T_M', heat transfer rate q and Nusselt number N_u, in this case, are found as following:

$$T_M = \frac{4c_3}{\alpha^2}\sum_{m=0}^{\infty}\left[\frac{1}{\lambda_m^2(4-\lambda_m^2)}\left\{\frac{(-1)}{2(\lambda_m+1)(\lambda_m+2)(\lambda_m+4)}\right.\right.$$
$$\left. + \frac{1}{6(\lambda_m+2)(\lambda_m+4)}\right\} + \frac{c_4 a^{\sigma}\cos\delta\alpha}{\{(\sigma+2)^2-\lambda_m^2\}(\lambda_m^2-\delta^2)}$$
$$\left.\times\left\{\frac{1}{\lambda_m+2} - \frac{1}{\lambda_m+4}\right\}\right] \qquad ...(4.4)$$

$$T'_M = \frac{4c_3}{c_2'\alpha^2}\sum_{m=0}^{\infty}\left[\frac{1}{\lambda_m(4-\lambda_m^2)}\left\{\frac{1}{4(\lambda_m+1)}\left(\frac{1}{2\lambda_m+4}-\frac{1}{\lambda_m+6}-\frac{1}{2\lambda_m+2}\right.\right.\right.$$

$$\left.+\frac{1}{\lambda_m+4}\right)+\frac{1}{16-\lambda_m^2}\left(\frac{1}{2\lambda_m+2}-\frac{1}{\lambda_m+6}-\frac{1}{\lambda_m+4}+\frac{1}{8}\right)\Bigg\}$$

$$+\frac{c_4\lambda_m a^\sigma\cos\delta\alpha}{\{(\sigma+2)^2-\lambda_m^2\}(\lambda_m^2-\delta^2)}\times\left(\frac{1}{2\lambda_m+2}-\frac{1}{\lambda_m+\sigma+4}\right.$$

$$\left.\left.-\frac{1}{\lambda_m+4}+\frac{1}{\sigma+6}\right)\right]\cdot\frac{1}{\lambda_m(4-\lambda_m^2)}$$

$$= c_3c_3' \qquad \text{(say)}, \qquad \ldots(4.5)$$

where

$$c_3' = \frac{4}{c_2'\alpha^2}\sum_{m=0}^{\infty}\left[\frac{1}{(4-\lambda_m^2)\lambda_m}\left\{\frac{1}{4(\lambda_m+1)}\left(\frac{1}{2\lambda_m+4}-\frac{1}{\lambda_m+6}-\frac{1}{2\lambda_m+2}\right.\right.\right.$$

$$\left.+\frac{1}{\lambda_m+4}\right)+\frac{1}{16-\lambda_m^2}\left(\frac{1}{2\lambda_m+2}-\frac{1}{\lambda_m+6}-\frac{1}{\lambda_m+4}+\frac{1}{8}\right)\Bigg\}$$

$$+\frac{c_4\lambda_m a^\sigma\cos\delta\alpha}{\{(\sigma+2)^2-\lambda_m^2\}(\lambda_m^2-\delta^2)}\times\left(\frac{1}{2\lambda_m+2}-\frac{1}{\lambda_m+\sigma+4}\right.$$

$$\left.\left.-\frac{1}{\lambda_m+4}+\frac{1}{\sigma+6}\right)\right]\times\frac{1}{\lambda_m(4-\lambda_m^2)}\cdot$$

$$q = -c_1c_2\left(\frac{2c_4a^\sigma\sin\delta\alpha}{(\sigma+2)\delta\alpha}-c_2'\right)\cdot a^2\alpha \qquad \ldots(4.6)$$

and

$$N_u = \frac{1}{c_3'}\left(\frac{2c_4a^\sigma\sin\delta\alpha}{(\sigma+2)\delta\alpha}-c_2'\right)\cdot\left(\frac{\alpha}{1+\alpha}\right)^2. \qquad \ldots(4.7)$$

5. Discussion

Taking angle of the sector α to be $\pi/2$ and Q to be $A_1 r^2\cos 2/3\theta$, the change in velocity w and in temperature $T(r,0)$, for different values of r/a is determined, and taking $A_1 = c_1 K/\nu$, are shown in Figs. 1 and 2 respectively. From these figures we observe that velocity and temperature are maximum on the axis of the sector and decrease gradually as 0 increases. The velocity increases as r/a increase from 0 to 0·5 and is maximum when $r/a = 0{\cdot}5$, for all values of θ, and an opposite effect is observed as r/a increases from 0·5 to 1. Also it is symmetrical about $r/a = 0{\cdot}5$ Similarly temperature increases as r/a varies from 0 to 0·7 and then decrease as r/a varies from 0·7 to 1. It is asymmetrical.

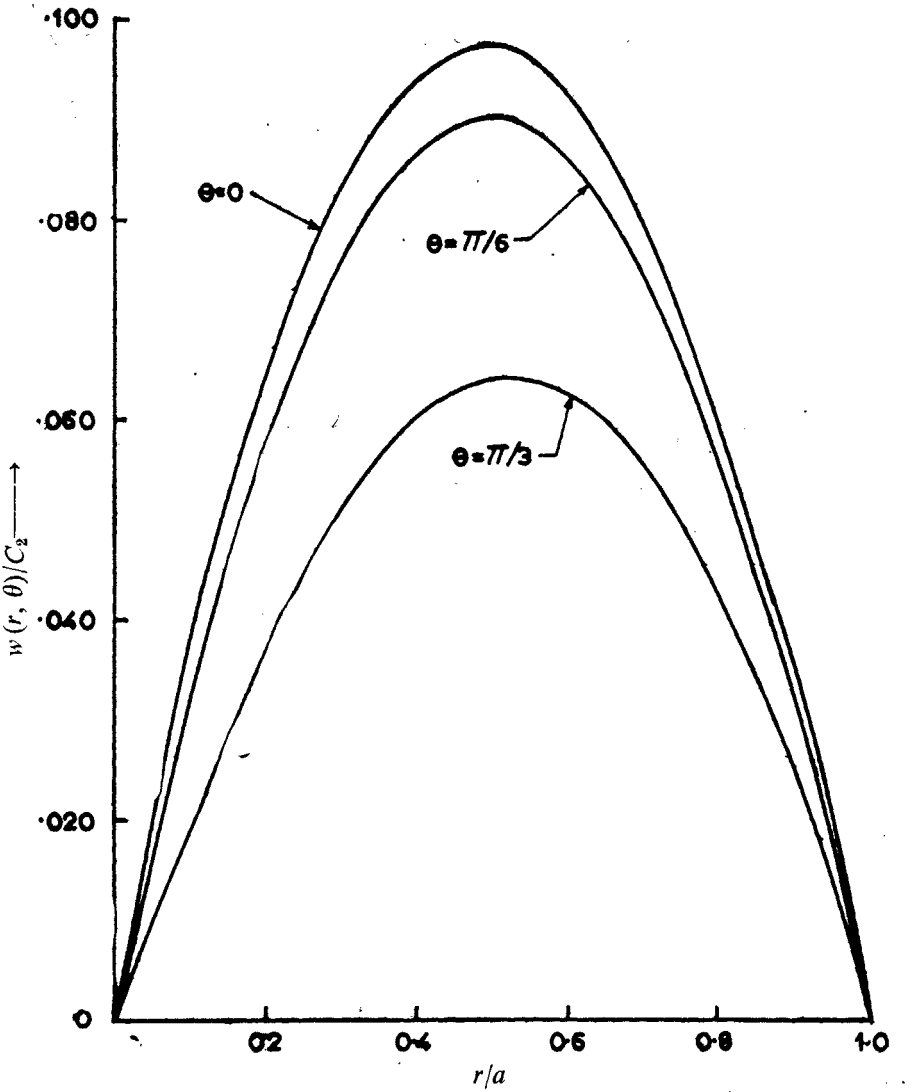

FIG. 1. Velocity Profiles of the fluid when r/a varies for different values of θ ($\alpha = \pi/2$).

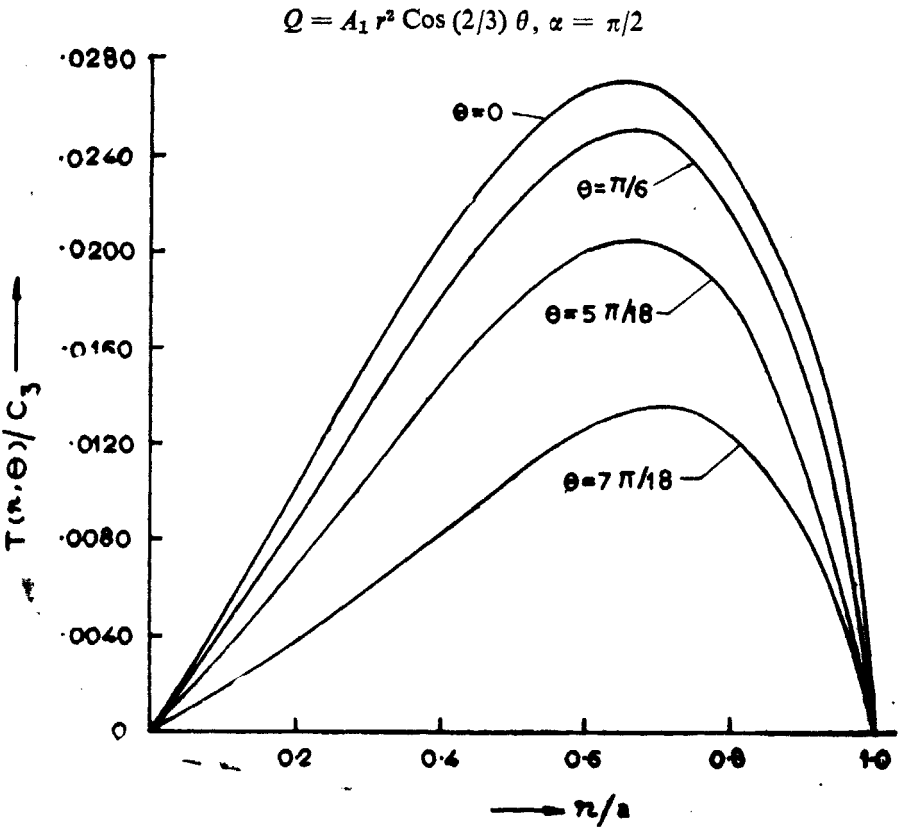

FIG. 2. Temperature distribution of the fluid when r/a varies for different values of θ.

In the following discussion, we consider heat source intensity to be constant, which can be obtained by taking σ and δ zero in (4.3) to (4.7).

Thus on putting σ and δ zero in eqns. (4.5), (4.6), and (4.7) various numerical values of mixed mean temperature T'_M/c_3, heat transfer rate $q/(c_1 Ka^4/\nu)$ and Nusselt number N_u are calculated for different values of c_4 and α, and are shown in Figs. 3, 4 and 5 respectively.

From Figs. 3, 4 we note that mixed mean temperature increases as α increases but heat transfer rate q decreases. Also from Fig. 5 it is interesting to note that Nusselt number increases as α increases and it decreases as heat source intensity c_4 increases, when heat transfer rate is negative. But when heat transfer rate is positive, it increases with increase in heat source intensity.

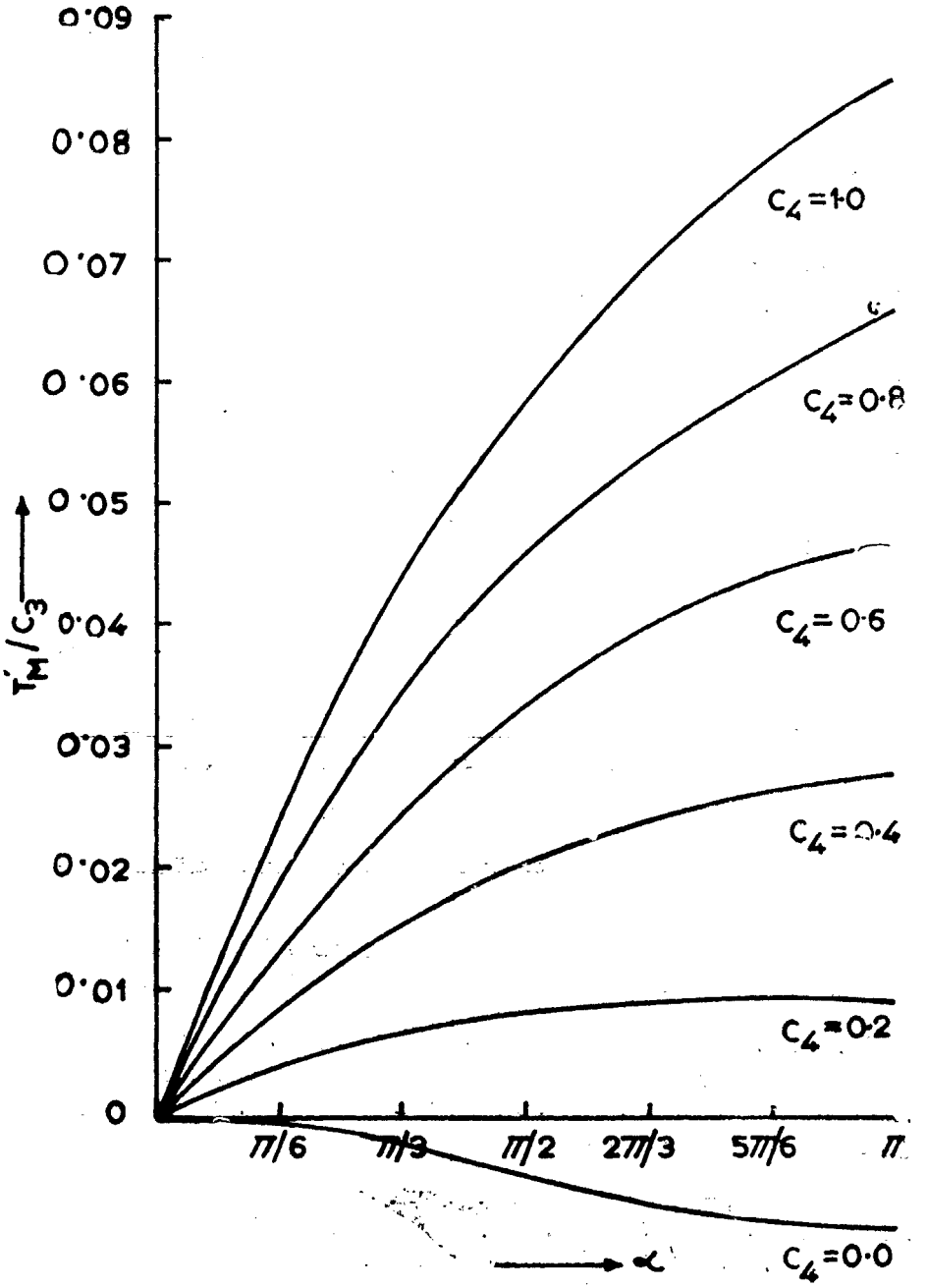

FIG. 3. Mixed mean temperature T'_M/C_3 *versus* α, angle of the sector, with heat source intensity $C_4 = (A_1/C_1C_2)$ as parameters.

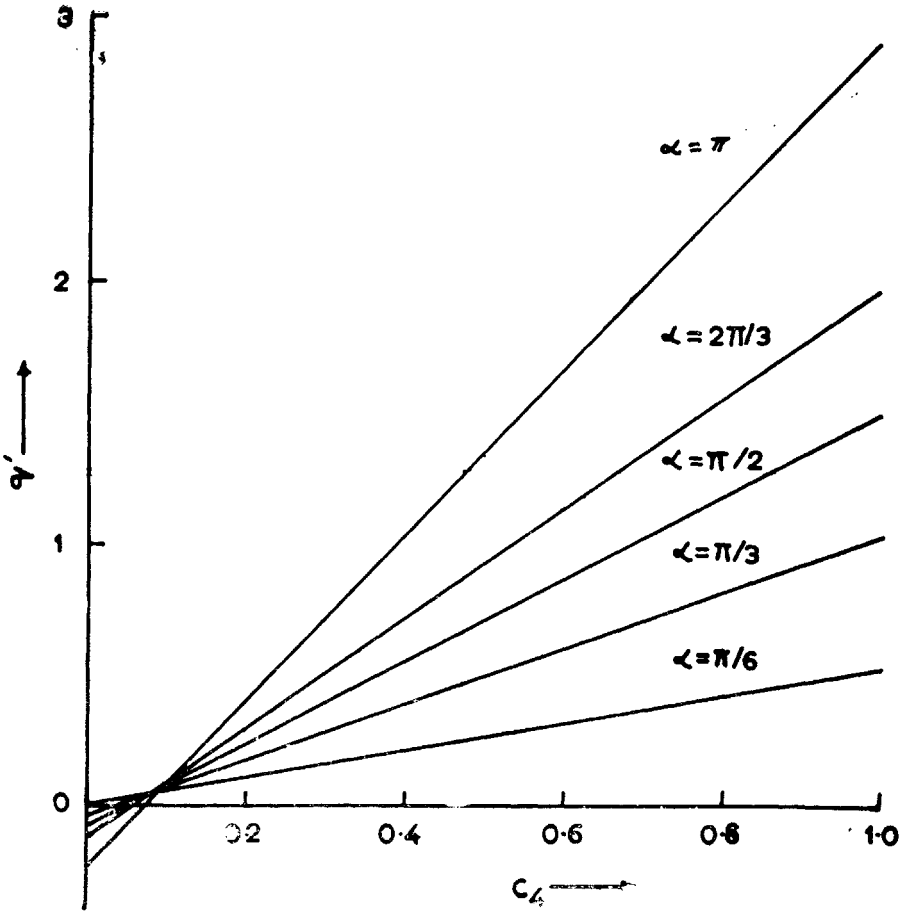

FIG. 4. Heat transfer rate $q' = (-qv/C_1 Ka^4)$ *versus* heat source intensity $C_4 = (A_1/C_1C_2)$, with angle of the sector α as parameters.

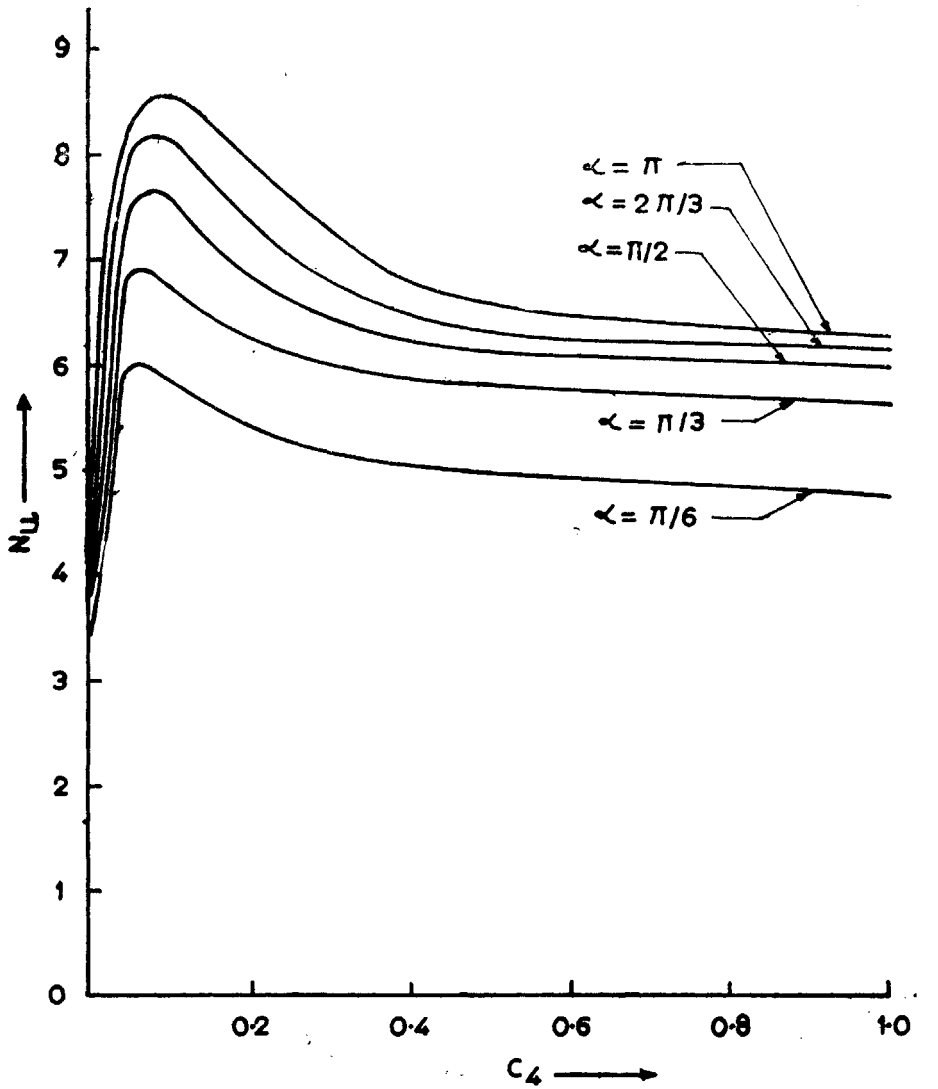

FIG. 5. Nusselt number N_u *versus* heat source intensity $C_4 = (A_1/C_1C_2)$, with angle of the sector α as parameters.

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