# ON THE ABSOLUTE SUMMABILITY FACTORS OF FOURIER-LAGUERRE EXPANSION

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In this paper the absolute $(C, 1)$ summability factors of the Fourier-Laguerre expansion are considered at the point $t=x$.

## 1. DEFINITIONS AND NOTATIONS

The Fourier-Laguerre expansion correspondiug to a Labesgue-measurable function $f(x)$ in $[0,\infty]$ is

$$f(x) \sim \sum_{n=0}^{\infty} a_n L_n^{(\alpha)}(x) \tag{1.1}$$

where

$$\Gamma(\alpha+1)\binom{n+\alpha}{n} a_n = \int_0^{\infty} e^{-y} y^{\alpha} f(y) L_n^{(\alpha)}(y)\, dy. \tag{1.2}$$

The existence of the above integral is presumed and $L_n^{(\alpha)}(x)$ denotes the $n$ th Laguerre polynomial of order $\alpha > -1$.

A series $\Sigma a_n$ is said to be absolutely summable $(C, 1)$ or $|C, 1|$ summable, if

$$\Sigma \mid \sigma_n - \sigma_{n+1} \mid$$

is convergent.

It is well known that (Kogbetliantz 1925)

$$n(\sigma_n - \sigma_{n+1}) = T_n$$

where $\sigma_n$ and $T_n$ denote the $n$th Cesàro means of order one of the sequences $\{s_n\}$ and $\{na_n\}$ respectively.

## 2. INTRODUCTION

The absolute summability factors for the Fourier series of a Lebesgue-measurable function in $(-\pi, \pi)$ were, for the first time, studied by Prasad

(1931). Since then, a number of researchers such as Izumi and Kawata (1938), Chow (1941), Cheng (1947-48), Prasad and Bhatta (1957) generalized Prasad's result in various directions by establishing a number of interesting theorems. In the same line, recently, Hsiang (1965) has proved the following :

*Theorem*—If

$$\int_0^t |\phi(u)|\, du = O\left(\frac{t}{\log \frac{1}{t}}\right) \qquad \text{as } t \to 0$$

then the series

$$\sum_{n=1}^{\infty} \frac{a_n \cos nx + b_n \sin nx}{\{\log (n+1)\}^{1+\epsilon}}, \quad \epsilon > 0$$

is summable $|C, 1|$ at $t = x$.

In the present paper, we intend to extend the above result of Hsiang to the Fourier–Laguerre expansion.

We write

$$\phi_x(y) = \{\Gamma \alpha+1\}^{-1} [f(y) - f(x)]$$

and

$$\psi_x(u) = f(x \pm u) - f(x).$$

## 3. Theorem

We establish the following theorem :

***Theorem***—For $\alpha > -\frac{1}{2}$ and $\epsilon > 0$, the series

$$\sum_{n=1}^{\infty} \frac{a_n L_n^{(\alpha)}(x)}{\{\log (n+1)\}^{1+\epsilon}}$$

is summable $|C, 1|$ at a point $y = x$ of the interval $[\eta, w]$, $0 < \eta < w < \infty$, provided

$$\int_t^{\delta} y^{\frac{\alpha}{2} - \frac{1}{4}} |\phi_x(y)|\, dy = O\left(\log^{\lambda} \frac{1}{t}\right) \tag{3.1}$$

as $t \to 0$, $0 < \delta < w$, $0 < \lambda < \epsilon$

$$\int_t^{\delta} \frac{|\psi_x(u)|}{u}\, du = O\left(\log^{\lambda} \frac{1}{t}\right) \tag{3.2}$$

as $t \to 0$,

and

$$\int_n^{\infty} e^{-\frac{y}{2}}\, y^{\frac{\alpha}{2} - \frac{13}{12}} |\phi_x(y)|\, dy = O\left(n^{-\frac{1}{2}} \log^{\lambda} n\right) \tag{3.3}$$

as $n \to \infty$.

The condition (3.3) can also be replaced by the following conditions

$$\int_{\omega}^{n} e^{-\frac{y}{2}}\, y^{\frac{\alpha}{2}-\frac{3}{4}} \,|\, \phi_x(y) \,|\, dy = O(\log^{\lambda} n) \tag{3.4}$$

$\omega$ being fixed positive constant

and

$$\int_{n}^{\infty} e^{-y}\, y^{\alpha-2} \,|\, \phi_x(y) \,|^2\, dy = O\left(n^{-\frac{3}{2}} (\log n)^2\right) \text{ as } n \to \infty. \tag{3.5}$$

## 4. Lemmas

We need the following lemmas to establish our theorem ;

*Lemma* 1 (*Szegö 1959, p. 175*—Let $\alpha$ be arbitrary and real, $c$ and $\omega$ fixed positive constants. and let $n \to \infty$, then

$$L_n^{(\alpha)}(x) = \begin{cases} x^{-\frac{\alpha}{2}-\frac{1}{4}}\, O\left(n^{\frac{\alpha}{2}-\frac{1}{4}}\right), & \text{if } \frac{c}{n} \leq x \leq \omega \\ O(n^{\alpha}) & \text{, if } o \leq x \leq \frac{c}{n}. \end{cases}$$

*Lemma* 2 (*Szegö 1959, p. 197*)—Let $\alpha$ be an arbitrary and real number, we have

$$L_n^{(\alpha)}(x) = \pi^{-\frac{1}{2}} e^{\frac{x}{2}}\, x^{-\frac{\alpha}{2}-\frac{1}{4}}\, n^{\frac{\alpha}{2}-\frac{1}{4}} \cos\left(2\sqrt{nx} - \frac{\alpha\pi}{2} - \frac{\pi}{4}\right) + 0\left(n^{\frac{\alpha}{2}-\frac{3}{4}}\right),\ x > 0.$$

The bound for the remainder holds uniformly in $[\epsilon, \omega]$.

*Lemma* 3 (*Szegö 1959, p. 238*)—Let $\alpha$ be arbitrary and real $a > 0$, $0 < \eta < 4$. We have for $n \to \infty$

$$\max\ e^{-\frac{x}{2}}\, x^{\frac{\alpha}{2}+\frac{1}{4}} \,|\, L_n^{(\alpha)}(x) \,| \sim \begin{cases} n^{\frac{\alpha}{2}-\frac{1}{4}}, & \text{if } a \leq x \leq (4-\eta)\, n\,; \\ n^{\frac{\alpha}{2}-\frac{1}{12}}, & \text{if } x \geq a. \end{cases}$$

*Lemma* 4—If (3.1) holds, then

$$\int_{0}^{t} y^{\alpha} \,|\, \phi_x(y) \,|\, dy = O\left(t^{\frac{\alpha}{2}+\frac{1}{4}} \log^{\lambda} \frac{1}{t}\right) \quad \text{as } t \to 0.$$

*Proof*· Let

$$\Phi(t) = \int_{t}^{\delta} y^{\frac{\alpha}{2}-\frac{1}{4}} \,|\, \phi_x(y) \,|\, dy,$$

so that

$$\Phi'(t) = -t^{\frac{\alpha}{2}-\frac{1}{4}} \,|\, \phi_x(t) \,|$$

or

$$-\int_{0}^{t} y^{\frac{\alpha}{2}+\frac{1}{4}}\, \Phi'(y) = \int_{0}^{t} y^{\alpha} \,|\, \phi_x(y) \,|\, dy.$$

Now, intergrating by parts the *L. H. S.* and using (3.1), we have

$$\int_0^t y^{\alpha} \mid \phi_x(y) \mid dy = O\left( t^{\frac{\alpha}{2}+\frac{1}{4}} \log^{\lambda} \frac{1}{t} \right).$$

*Lemma* 5—If (3.3) holds then

$$\int_{\omega}^{n} e^{-\frac{y}{2}} y^{\frac{\alpha}{2}-\frac{3}{4}} \mid \phi_x(y) \mid dy = O(\log^{\lambda} n)$$

where $\omega$ is a fixed positive number and $n \rightarrow \infty$.

*Proof:* In fact, if we write

$$u(x) = \int_x^{\infty} e^{-\frac{y}{2}} y^{\frac{\alpha}{2}-\frac{13}{12}} \mid \phi_x(y) \mid dy$$

we have

$$u'(x) = -e^{-\frac{x}{2}} x^{\frac{\alpha}{2}-\frac{13}{12}} \mid \phi_x(x) \mid$$

or

$$\int_{\omega}^{n} y^{\frac{1}{3}} u'(y) = -\int_{\omega}^{n} e^{-\frac{y}{2}} y^{\frac{\alpha}{2}-\frac{3}{4}} \mid \phi_x(y) \mid dy.$$

Now

$$\int_{\omega}^{n} y^{\frac{1}{3}} u'(y) = \left[ y^{\frac{1}{3}} u(y) \right]_{\omega}^{n} + O\left( \int_{\omega}^{n} y^{-\frac{2}{3}} u(y)\, dy \right)$$

$$= O\left( n^{\frac{1}{3}} n^{-\frac{1}{2}} \log^{\lambda} n \right) + O(1) \int_{\omega}^{n} y^{-\frac{7}{6}} \log^{\lambda} y \, dy + O(1)$$

$$= O(\log^{\lambda} n) \text{ as } n \rightarrow \infty,$$

since $u(x) = O(x^{-\frac{1}{2}} \log^{\lambda} x)$.

*Lemma* 6—If $S_n(x)$ denotes the sum of the first $(n+1)$ terms of (1.1), then under the hypothesis of the theorem

$$S_n(x) = O(\log^{\lambda} n).$$

*Proof*: We have

$$S_n(x) = (n+1)\{ \overline{\mid \alpha+1} \, A_n^{\alpha} \}^{-1} \int_0^{\infty} e^{-y} y^{\alpha} f(y) \frac{L_n^{(\alpha)}(x) L_{n+1}^{(\alpha)}(y) - L_{n+1}^{(\alpha)}(x) L_n^{(\alpha)}(y)}{x-y} \, dy.$$

Putting $f(y) = 1$. and using orthogonal property of Laguerre polynomials

$$S_n(x) - f(x) = (n+1) \{ \overline{\mid \alpha+1} \, A_n^{\alpha} \}^{-1} \int_0^{\infty} e^{-y} y^{\alpha} [f(y) - f(x)] \frac{L_n^{(\alpha)}(x) L_{n+1}^{(\alpha)}(y) - L_{n+1}^{(\alpha)}(y) L_n^{(\alpha)}(x)}{x-y} \, dy. \tag{4.1}$$

$$= (n+1) (A_n^{\alpha})^{-1} \int_0^{\infty} e^{-y} y^{\alpha} \phi_x(y) \frac{L_{n+1}^{(\alpha)}(x) L_{n+1}^{(\alpha-1)}(y) - L_{n+1}^{(\alpha)}(y) L_{n+1}^{(\alpha-1)}(x)}{x-y} dy$$

by the relation

$$L_n^{(\alpha)}(x)=L_{n+1}^{(\alpha)}(x)-L_{n+1}^{(\alpha-1)}(x)$$

$$=(n+1)\,(A_n^{\alpha})^{-1}\Big[\int_0^{c/n}+\int_{c/n}^{\epsilon'}+\int_{\epsilon'}^{x-\delta}+\int_{x-\delta}^{x+\delta}+\int_{x+\delta}^{\omega}+\int_{\omega}^{n}+\int_n^{\infty}\Big]$$

$$e^{-y}\,y^{\alpha}\,\phi_x(y)\,\frac{L_{n+1}^{(\alpha)}(x)\,L_{n+1}^{(\alpha-1)}(y)-L_{n+1}^{(\alpha-1)}(x)\,L_{n+1}^{(\alpha)}(y)}{x-y}\,dy$$

$$=\sum_{i=1}^{7} I_i,\ \text{say}.$$

Now, using Lemma 1 and Lemma 4, we have for $\alpha>-\frac{1}{2}$

$$I_1=O(n^{1-\epsilon})\int_0^{c/n} e^{-y}\,y^{\alpha}\,|\phi_x(y)|$$

$$\Big[n^{\frac{\alpha}{2}-\frac{1}{4}}n^{\epsilon-1}+n^{\frac{\alpha}{2}-\frac{3}{4}}\,n^{\alpha}\Big]\,dy$$

$$=O\Big(n^{\frac{n}{2}+\frac{1}{4}}\Big)\int_0^{c/n} y^{\alpha}\,|\phi_x(y)|\,dy$$

$$=O\Big(n^{\frac{\alpha}{2}+\frac{1}{4}}.\,n^{-\frac{\alpha}{2}-\frac{1}{4}}\log^{\lambda} n\Big)$$

$$=O\,(\log^{\lambda} n).$$

Again, with the help of Lemma 1, we have

$$I_2=O\,(n^{1-\epsilon})\int_{c/n}^{\epsilon'} e^{-y}\,y^{\alpha}\,|\phi_x(y)|\,\Big[n^{\frac{\alpha}{2}-\frac{1}{4}}.\,n^{\frac{\alpha}{2}-\frac{3}{4}}\,y^{-\frac{\epsilon}{2}+\frac{1}{4}}\,,\,n^{\frac{\epsilon}{2}-\frac{3}{4}}.\,n^{\frac{\epsilon}{2}-\frac{1}{4}}\,y^{-\frac{\alpha}{2}-\frac{1}{4}}\Big]\,dy$$

$$=O\Big[\int_{c/n}^{\epsilon'} y^{\frac{\epsilon}{2}-\frac{1}{4}}\,|\phi_x(y)|\,dy\Big]$$

$=O\,(\log^{\lambda} n)$ as $n\to\infty$, by the hypothesis (3.1).

Consider

$$I_3=(n+1)\,(A_n^{\epsilon})^{-1}\int_{\epsilon'}^{x-\delta} e^{-y}\,y^{\epsilon}\,|\phi_x(y)|$$

$$\frac{L_{n+1}^{(\alpha)}(x)\,L_{n+1}^{(\alpha-1)}(y)-L_{n+1}^{(\epsilon-1)}(x)\,L_{n+1}^{(\alpha)}(y)}{x-y}\,dy$$

$$=I_{3,1}-I_{3,2},\ \text{say}.$$

Now, using lemma 2 and noting that $x$ is confined to a fixed positive number, we have

$$I_{3,1}=O(n^{1-\epsilon})\int_{\epsilon'}^{x-\delta}\frac{e^{-y}\,y^{\epsilon}\,|\phi_x(y)|}{x-y}\,O\Big(n^{\frac{\epsilon}{2}-\frac{1}{4}}\Big)\,.$$

$$\Big[\pi^{-\frac{1}{2}}\,e^{y/2}\,y^{-\frac{\epsilon}{2}+\frac{1}{4}}\,n^{\frac{\alpha}{2}-\frac{3}{4}}\cos\Big\{2\sqrt{ny}-\frac{\alpha\pi}{2}+\frac{\pi}{4}\Big\}+O\Big(n^{\frac{\epsilon}{2}-\frac{5}{4}}\Big)\Big]dy$$

$$= O(1) \int_{\epsilon'}^{x-\delta} \frac{e^{-y/2} y^{\frac{\alpha}{2}+\frac{1}{4}}}{x-y} \mid \phi_x(y) \mid \cos\left\{ 2\sqrt{ny} - \frac{2\alpha-1}{4}\pi \right\} dy$$

$$+ O\left( n^{-\frac{1}{2}} \right) \int_{\epsilon'}^{x-\delta} \frac{e^{-y} y^{\alpha} \mid \phi_x(y) \mid}{x-y} dy + o(1)$$

$$= o(1),$$

because the first term tends to zero by Riemann-Lebesgue theorem and the second term tends to zero in the limit as $n \to \infty$.

Similarly

$$I_{3,2} = o(1).$$

Thus

$$I_3 = o(1).$$

Proceeding on the same lines

$$I_5 = o(1).$$

Now, with the help of the first part of Lemma 3 and noting that $x$ is confined to a fixed positive number, we get

$$I_6 = O(n^{1-\alpha}) \int_{\omega}^{n} \frac{e^{-y} y^{\alpha} \mid \phi_x(y)}{x-y} [n^{\frac{\alpha}{2}-\frac{1}{4}} \,.\, e^{y/2} y^{-\frac{\alpha}{2}+\frac{1}{4}} n^{\frac{\alpha}{2}-\frac{3}{4}}$$

$$- n^{\frac{\alpha}{2}-\frac{3}{4}} . \, e^{y/2} y^{-\frac{\alpha}{2}-\frac{1}{4}} n^{\frac{\alpha}{2}-\frac{1}{4}}] \, dy$$

$$= O(1) \int_{\omega}^{n} e^{-y/2} y^{\frac{\alpha}{2}-\frac{3}{4}} \mid \phi_x(y) \mid dy + O(1) \int_{\omega}^{n} e^{-y/2} y^{\frac{\alpha}{2}-\frac{5}{4}} \mid \phi_x(y) \mid dy$$

$$= O(1) \int_{\omega}^{n} e^{-y/2} y^{\frac{\alpha}{2}-\frac{3}{4}} \mid \phi_x(y) \mid dy$$

$$= O(\log^{\lambda} n), \text{ by Lemma 5.}$$

In the interval $n \leq y < \infty$, the second part of Lemma 3 tells us that

$$e^{-x/2} x^{\frac{\alpha}{2}+\frac{1}{12}} \mid L_n^{(\alpha)}(x) \mid = O(n^{\frac{\alpha}{2}-\frac{1}{4}}).$$

Now, using formula (4.1), we have

$$I_7 = O(n^{1-\alpha}) \int_{n}^{\infty} \frac{e^{-y} y^{\alpha} \mid \phi_x(y) \mid}{y-x} [L_{n+1}^{(\alpha)}(x) \, L_n^{(\alpha)}(y) - L_n^{(\alpha)}(x) \, L_{n+1}^{(\alpha)}(y)] \, dy$$

$$= I_{7,1} - I_{7,2}, \text{ say.}$$

With the help of the above result, we get

$$I_{7,1} = O(n^{1-\alpha}) \int_{n}^{\infty} \frac{e^{-y} y^{\alpha} \mid \phi_x(y) \mid}{y-x} O(n^{\frac{\alpha}{2}-\frac{1}{4}}) \, e^{y/2} y^{-\frac{\alpha}{2}-\frac{1}{12}} n^{\frac{\alpha}{2}-\frac{1}{4}} \, dy$$

$$= O(n^{\frac{1}{2}}) \int_{n}^{\infty} e^{-y/2} \, y^{\frac{\alpha}{2}-\frac{13}{12}} \mid \phi_x(y) \mid dy$$

$$= O(\log^{\lambda} n), \text{ by the hypothesis (3.3).}$$

If the condition (3.3) be replaced by the conditions (3.4) and (3.5), the treatment of $I_6$ will be the same as given already, while the treatment of $I_7$ is given below. Here we use Schwarz's inequality and orthogoeal property of the Laguerre polynomials and obtain

$$I_{7,1}=O\ (n^{1-\epsilon})\int_n^\infty e^{-y}\, y^{\alpha-1}\,|\,\phi_x\,(y)\,|\,O\,(n^{\frac{\epsilon}{2}-\frac{1}{4}})\,|\,L_n^{(\alpha)}\,(y)\,|\,dy$$

$$=O\ (n^{\frac{3}{4}-\frac{\epsilon}{2}})\Big[\Big\{\int_n^\infty e^{-y}\, y^{\epsilon-2}\,|\,\phi_x\,(y)\,|^2\,dy\Big\}^{\frac{1}{2}}.\Big\{\int_n^\infty e^{-y}\, y^{\alpha}\,(L_n^{(\alpha)}(\,y\,))^2\Big\}^{\frac{1}{2}}\Big]$$

$=O\,(\log^\lambda n)$, because the last integral is $O\,(n^\epsilon)$.

Similarly

$$I_{7,2}=O\,(\log^\lambda n).$$

Thus

$$I_7\ =O\,(\log^\lambda\ n).$$

Finally, using lemma 2 and making simplifications, we have

$$I_4\ =\frac{1}{2}\ x^{\frac{1}{2}}\left(\pi^{-\frac{1}{2}}\,e^{x/2}\,x^{-\frac{\alpha}{2}-\frac{1}{4}}\right)^2\int_{x-\delta}^{x+\delta} e^{-y}\,y^{\epsilon-\frac{1}{2}}\left|\phi_x\,(y)\right|$$

$$\times\left|\frac{\sin\{2\sqrt{n}\ (\sqrt{x}-\sqrt{y})\}}{\sqrt{x}-\sqrt{y}}\right|dy+O\,(1)$$

$$=O\left[\int_{x-\delta}^{x-(1/n)}+\int_{x-(1/n)}^{x+(1/n)}+\int_{x+(1/n)}^{x+\delta}\right]e^{-y}y^{\alpha-\frac{1}{2}}\left|\phi_x\,(y)\right|$$

$$\times\left|\frac{\sin\{2\sqrt{n}\,(\sqrt{x}-\sqrt{y})\}}{\sqrt{x}-\sqrt{y}}\right|dy+O\,(1)$$

$$=I_{4,1}+I_{4,2}+I_{4,3}+O(1).$$

Now substituting $x-y=u$, we get

$$I_{4,2}=O\ \left[n^{\frac{1}{2}}\int_0^{(1/n)}\left|\psi_x\,(u)\right|du\ \right]$$

$=O(n^{\frac{1}{2}}\,n^{-1}\,\log^\lambda n)$, by the hypothesis (3.2)

$=O\,(\log^\lambda n)$, as $n\to\infty$.

Also

$$I_{4,3}=O\ \left[\int_{x+(1/n)}^{x+\delta}\frac{|\,\phi_x\,(y)\,|}{|\,x-y\,|}\,(x^{\frac{1}{2}}+y^{\frac{1}{2}})\,dy\ \right]$$

$$=O\ \left[\int_{(1/n)}^{\delta}\frac{|\,\psi_x\,(u)\,|}{u}\,du\ \right]$$

$=O\,(\log^\lambda n)$. by the hypothesis (3.2).

Similarly

$$I_{4,1} = O(\log^{\lambda} n).$$

Thus

$$I_4 = O(\log^{\lambda} n).$$

Thus the lemma is proved.

## 5. Proof of the Theorem

By Abel's transformation and Lemma 6, we have writing

$$t_n(x) = \frac{1}{n+1} \sum_{\nu=1}^{n} \nu\, a_\nu\, L_\nu^{(\alpha)}(x)$$

$$\sum_{n=1}^{m} \left| \frac{T_n(x)}{n} \right| = \sum_{n=1}^{m} \frac{1}{n} \cdot \left| \frac{1}{n+1} \sum_{\nu=1}^{n} \frac{\nu\, A_\nu(x)}{\{\log(\nu+1)\}^{1+\epsilon}} \right|$$

$$\leqslant \sum_{\nu=1}^{m} \left| \Delta\left( \frac{1}{\{\log(\nu+1)\}^{1+\epsilon}} \right) \right| (\nu+1) \mid t_\nu(x) \mid \sum_{n=\nu+1}^{m} \left( \frac{1}{n} - \frac{1}{n+1} \right)$$

$$\sum_{n=1}^{m} \frac{\mid t_n(x) \mid}{n\{\log(n+1)\}^{1+\epsilon}}$$

$$= \sum_{\nu=1}^{m} \mid t_\nu(x) \mid \frac{1}{(\nu+1)\{\log(\nu+1)\}^{2+\epsilon}} + O(1)$$

$$= O\left( \sum_{\nu=1}^{m-1} \frac{\log^{\lambda}(\nu+1)}{(\nu+1)\{\log(\nu+1)\}^{2+\epsilon}} \right) + O(1)$$

$$= O(1) \quad \text{as } m \to \infty.$$


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