UNSTEADY HEAT TRANSFER FOR FLOW OVER A FLAT PLATE WITH SUCTION

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Solution of the energy equation for viscous flow over a porous flat plate is considered when constant suction is applied at the plate. For time $t < 0$, there is the usual steady velocity boundary layer and neglecting viscous dissipation, no thermal boundary layer. At $t=0$, a temperature boundary layer is initiated without altering the velocity, and subsequeut temperature distribution is studied.

1. INTRODUCTION

One of the simplest solution of Navier-Stokes equation is obtained by considering the steady flow of a viscous fluid past a wall of infinite extent when a condition of suction is applied at the wall (Schlichting 1960). The solution presumably approximates the flow for downstream of the leading edge of flat plate with suction and is therefore termed asymptotic suction profile. No solution is possible if a condition of blowing is instead imposed.

If for time $t < 0$ the fluid and plate have the same temperature, the thermal boundary layer is absent if viscous dissipation is neglected.

At $t=0$ the plate temperature is changed so that for $t>0$ the plate is maintained at a constant temperature which is different from the temperature of the fluid at infinity, The applied heating is assumed to be sufficiently small for the consequent changes in the density of the fluid to have no effect upon the steady velocity distribution. The flow parameters are also assumed to be small so that the effects of viscous dissipation may be neglected. The resulting energy equation is solved by Laplace transform technique.

2. EQUATIONS OF MOTION

Consider a semi-infinite porous flat plate immersed in an incompressible fluid which, at infinity, has constant velocity U_∞ parallel to the plate. There is a constant suction velocity $V_\infty < 0$ normal to the plate, For time $t < 0$, the plate and fluid have the same temperature T_∞, At $t=0$ the plate temperature is changed so that for $t>0$ the plate is maintained at a constant temperature $T_w \neq T_\infty$.

The momentum and continuity equations for the boundary layer problem described above are

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} \tag{2.1}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2.2}$$

Here the coordinates (x,y) are measured along and normal to the plate respectively and (u, v) represents the velocity components in these directions; the kinematic viscosity ν is assumed to be constant. These equations are independent of time t since changes in density, associated with changes in the plate temperature are assumed to be so small as not to affect the velocity boundary layer already established upon the plate.

The boundary conditions are

$$\left.\begin{array}{lll} y=0: & u=0, & v=v_0=\text{const}<0 \\ y=\infty: & u=U_\infty. & \end{array}\right\} \tag{2.3}$$

If we assume that velocity is independent of the current length x then the solution of eqns. (2.1) and (2.2) under the boundary condiitions (2.3) is

$$\left.\begin{array}{l} u(y) = U_\infty [1 - \exp. \{v_0 y/\nu\}] \\ v(x, y) = v_0 < 0. \end{array}\right\} \tag{2.4}$$

This simple solution is an exact solution of the complete Navier-Stokes equations, Schlichting (1960).

The energy equation satisfied by the temperature T is

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} \tag{2.5}$$

where α, the thermal diffusivity, is assumed to be constant. The heating due to viscous dissipation is neglected in the formulation of (2.5) as the flow parameters are assumed to be small. The boundary conditions for T are

$$\left.\begin{array}{lll} T=T_\infty, & y \geq 0, & t=0 \\ T=T_w, & y=0, & t>0 \\ T=T_\infty, & y \to \infty, & t>0. \end{array}\right\} \tag{2.6}$$

Here T_w and T_∞ are the constant plate and free stream temperature.

The flow distribution is independent of x, the distance parallel to wall and the temperature of the plate also independent of x, we look for the temperature which is independent of x. So putting $\frac{\partial T}{\partial x}=0$ in (2.5), we have

$$\frac{\partial T}{\partial t}+v_0\frac{\partial T}{\partial y}=\alpha\frac{\partial^2 T}{\partial y^2}\,. \tag{2.7}$$

3. Solution of Energy Equation

To solve eqn. (2.7) for the temperature we introduce the following dimensionless variables

$$\theta=\frac{T-T_\infty}{T_w-T_\infty}\,,\ \eta=\frac{|v_0|\,y}{\nu}$$

and

$$\tau=\frac{|v_\infty|^2\,t}{\nu} \tag{3.1}$$

and obtain

$$\frac{\partial\theta}{\partial\tau}+\frac{v_0}{|v_0|}\frac{\partial\theta}{\partial\eta}=\frac{1}{\sigma}\frac{\partial^2\theta}{\partial\eta^2} \tag{3.2}$$

where $\sigma\ \frac{\nu}{\alpha}$ is Prandtl Number. For suction at the wall $v_0<0$, therefore we have

$$\frac{\partial^2\theta}{\partial\eta^2}+\sigma\frac{\partial\theta}{\partial\eta}-\sigma\frac{\partial\theta}{\partial\tau}=0\,. \tag{3.3}$$

The bounndary conditions (2.6) become

$$\left.\begin{array}{lll}\theta=0, & \eta\geqslant 0\,, & \tau=0\\ \theta=1, & \eta=0\,, & \tau>0\\ \theta=0, & \eta\to\infty, & \tau>0\end{array}\right\}\,. \tag{3.4}$$

Equation (3.3) is a linear equation with constant coefficients and is thus amenable to solution by means of Laplace transform. Defining the transform of $\theta(\eta,\tau)$ as

$$\theta_s=\int_0^\infty e^{-s\tau}\,\theta(\eta,\tau)\,d\tau$$

then allows the transformation of (3.3) as

$$\frac{d^2\theta_s}{d\eta^2}+\sigma\frac{d\theta_s}{d\eta}-\sigma_s\,\theta_s=0 \tag{3.5}$$

The boundary conditions (3.4) give

$$\left.\begin{array}{ll} \eta=0 : & \theta_s = \dfrac{1}{s} \\ \eta\rightarrow\infty : & \theta_s \rightarrow 0 \end{array}\right\} . \tag{3.6}$$

Solving equation (3.5), we have

$$\theta_s = c_1 \exp\left[\left\{-\frac{\sigma}{2}+\sqrt{\frac{\sigma^2}{4}+\sigma s}\right\}\eta\right] + c_2 \exp\left[\left\{-\frac{\sigma}{2}-\sqrt{\frac{\sigma^2}{4}+\sigma s}\right\}\eta\right] \tag{3.7}$$

where c_1 and c_2 are the constants of integration.

For θ_s to be zero at $\eta=\infty$ the constant c_1 must be equal to zero. Evaluating the other constant from the boundary conditions (3.6) we have

$$\theta_s = \frac{1}{s}\exp\left[\left\{-\frac{\sigma}{2}-\sqrt{\frac{\sigma^2}{4}+\sigma s}\right\}\eta\right] \tag{3.8}$$

On taking the inverse Laplace transform of (3.8), we have using standard results, Erdélyi (1954).

$$\theta(\eta,\tau) = \frac{1}{2}\left\{\text{Erfc}\left[\frac{\eta\sqrt{\sigma}}{2\sqrt{\tau}}+\frac{\sqrt{\sigma\tau}}{2}\right] + \theta^{-\sigma\eta}\,\text{Erfc}\left[\frac{\sigma\sqrt{\sigma}}{2\sqrt{\tau}}-\frac{\sqrt{\sigma\tau}}{2}\right]\right\} \tag{3.9}$$

where Erfc (x) is complementary error function.

Fig. 1 gives temperature distribution at $\tau = 1$ for Prandtl numbers $\sigma = 1, 4, 9$. As σ increases the thermal boundary layer region contracts. Fig. 2 gives the temperature distribution for Prandtl number $\sigma = 1$ at various intervals of time $\tau = 1, 2, 3, 4, 5$. As τ increases the temperature profiles assumes a steady state.

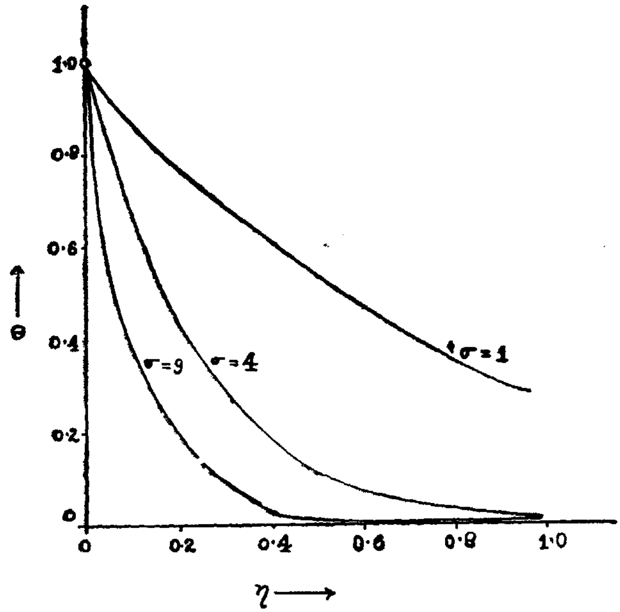
4. Heat Transfer

The heat transfer from the plate to the fluid may be computed by the application of Fourier's Law

$$Q = -k\left(\frac{\partial T}{\partial y}\right) y = 0.$$

Introducing the variables of equation (3.1), the expression for Q becomes

$$Q = -\frac{k\,|\,v_0\,|\,(T_w - T_\infty)}{\nu}\left(\frac{\partial\theta}{\partial\eta}\right)_{\eta=0}. \tag{4.1}$$



FIG, 1. Distribution of temperature at $\tau=1$.

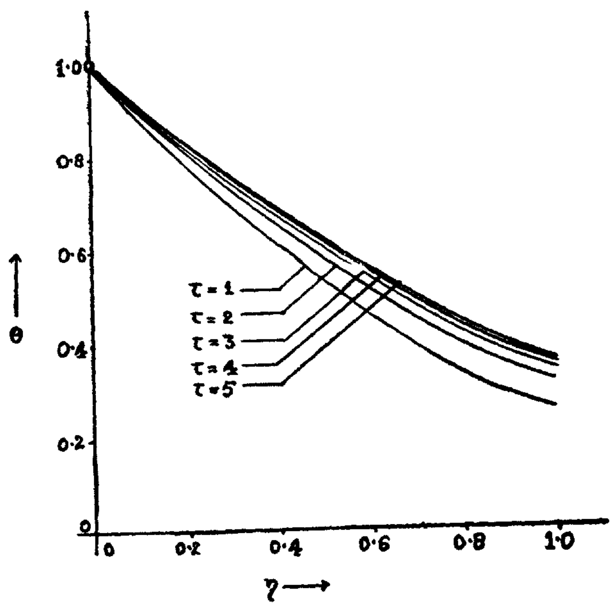


FIG. 2. Distribution of temperature for $\sigma=1$.

From (3.9) and (4.1) we have

$$Q = \frac{\sigma k \mid v_0 \mid (T_w - T_\infty)}{2\nu} \left\{ 1 + \mathrm{Erf}\left(\frac{\sqrt{\sigma\tau}}{2} \right) + \right.$$

$$\left. + \frac{1}{\sqrt{\sigma\tau}} e^{-\frac{\tau\sigma}{4}} \right\} \tag{4.2}$$

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