HEAT TRANSFER IN A VARIABLE VISCOSITY FLUID DUE TO A GRADUALLY ACCELERATED PLATE

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The heat transfer accompanying a gradually accelerated flat plate in a variable viscosity fluid has been studied. The temperature of the plate is assumed to vary linearly with time, while that far away from it is supposed constant. The thermal boundary layer thickness and the rate of heat transfer at the plate are found to increase with an increase in the Prandtl number, for small values of the non-Newtonian parameter The Nusselt number is seen to increase with increasing values of the parameter. With increasing dissipation of heat, however, the rate of heat transfer from the wall to the fluid registers a fall.

INTRODUCTION

The study of unsteady flow in non-Newtonian fluids have often revealed quite interesting behaviour of the fluid and one is tempted to examine the nature of the heat transfer associated with such unsteady flows both from academic and practical points of view. Many technically important non-Newtonian fluids like suspensions and slurries of rotund and plate-like particles, carbopol and clay suspensions are devoid of shear elasticity and are of variable viscosity which depend on the flow invariants Rath (1969-70) studied the unsteady flow in such a fluid generated by the gradual acceleration of a flat plate. Rath and Mahapatra (1970) considered the heat transfer due to such a flow in a Newtonian fluid. Here we have extended the study to the heat transfer in a non-Newtonian fluid of the type indicated above for small values of the non-Newtonian parameter.

BASIC EQUATIONS AND FORMULATION OF THE PROBLEMS

The fluid considered is governed by the equation

$$\tau_{ij} = -p\delta_{ij} + 2\mu d_{ij} \qquad \text{...(1)}$$

where τ_{ij} and d_{ij} are the stress and the rate of strain tensor, p the hydrostatic pressure and δ_{ij} the Kronecker's delta. Further

$$d_{ij} = \frac{1}{2}\left(\frac{\partial V_i}{\partial x_j} + \frac{\partial V_j}{\partial x_i}\right) \qquad \text{...(2)}$$

where V_i is the velocity component in the i-direction and

$$\mu = \mu\,(I_1, I_2, I_3) \qquad \ldots(3)$$

with

$$I_1 = d_{11} + d_{22} + d_{33} \qquad \ldots(4)$$

$$I_2 = -\,(d_{11}\,d_{22} + d_{22}\,d_{33} + d_{33}\,d_{11}) + d_{12}^2 + d_{23}^2 + d_{31}^2 \qquad \ldots(5)$$

and

$$I_3 = \det\, d_{ij}. \qquad \ldots(6)$$

In our discussion we imagine the plate occupying the position $y = 0$ and that the plate is accelerated from rest with acceleration $At^{1/2}$ in the x direction, A being a constant. The two-dimensional flow of the fluid due to the motion of the plate is supposed to have velocity components u and v in the x and y directions, respectively. The only non-vanishing invariant I_2 is thus found to be $\frac{1}{4}\left(\frac{du}{dy}\right)^2$. We take the first two terms in the expansion.

$$\mu = \mu\,(I_2) = \mu_0 + \mu_1\, I_2 = \mu_0 + \mu_1 \left(\frac{du}{dy}\right)^2 \qquad \ldots(7)$$

where μ_0 is the apparent viscosity and μ_1 is a material constant with dimension $\left[\frac{MT}{L}\right]$. Rath (1969-70) studied the flow pattern of the problem with

$$\frac{u}{At^{1/2}} = e^{-I^2} - 2I\ \mathrm{erfc}\, I + R\left\{2\,I\,(\mathrm{erfc}\, I\,)^3 - \frac{3}{2}\,e^{-I^2}\,(\mathrm{erfc}\, I)^2 + \frac{3}{4}\,e^{-3I^2} - 3\,\frac{\sqrt{3}}{2}\,\mathrm{erfc}\,(\sqrt{3}\,I) + \frac{3}{8}\,(\pi - 2)\,(e^{-I^2} - 2I\ \mathrm{erfc}\ I)\right\} \qquad \ldots(8)$$

where

$$I = \frac{1}{2}\,y\,(\nu t)^{-1/2},\ \nu = \frac{\mu_0}{\rho},\ R = \frac{\mu_1\,A^2}{\rho\,\nu^2}$$

and

$$\mathrm{erfc}\ I = \int_I^\infty e^{-u^2}\,du.$$

The energy equation in the present problem is

$$\rho c\,\frac{\partial T}{\partial t} = k\,\frac{\partial^2 T}{\partial y^2} + \mu\left(\frac{\partial u}{\partial y}\right)^2 \qquad \ldots(9)$$

where ρ, c, k are respectively the density, specific heat and conductivity and are supposed constants. while μ has the value as stated in (7).

We further assume that the fluid temperature far away from the plate is constant, and that the plate temperature is varying linearly with time. The boundary conditions for the eqn. (9) can be written in the form

$$\left.\begin{aligned} T(0, t) &= T_w = T_\infty + \lambda t \\ T(\infty, t) &= T_\infty = \text{constant} \end{aligned}\right\} \qquad \ldots(10)$$

where T_w and T_∞ represent the temperature at the wall and outside the thermal boundary layer.

Method of Solution

The solution of the differential eqn. (9) with the boundary conditions (10) constitutes the solution of the problem of heat transfer due to the motion of a gradually accelerated flat plate.

Using the transformations

$$T - T_\infty = \lambda t \theta(I), \; I = \frac{1}{2} y (\nu t)^{-1/2} \qquad \ldots(11)$$

eqns. (9) and (10) assume the form

$$\theta'' + 2Pr(I\theta' - 2\theta) = -B \operatorname{erfc} I \Big[\operatorname{erfc} I - R \Big\{ (\operatorname{erfc} I)^3 - 2Ie^{-I^2} (\operatorname{erfc} I)^2 + 3e^{-2I^2} \cdot \operatorname{erfc} I - \frac{3}{4}(\pi - 2) \operatorname{erfc} I - \frac{3\sqrt{3}}{2} \operatorname{erfc}(\sqrt{3} I) \Big\} \Big] \qquad \ldots(12)$$

and

$$\theta = 1 \text{ at } I = 0, \; \theta = 0 \text{ as } I \to \infty \qquad \ldots(13)$$

where

$$Pr = \frac{\mu_0 c}{k}, \text{ Prandtl number}$$

$$R = \frac{\mu_1 A^2}{\rho \nu^2}, \text{ non-Newtonian parameter}$$

$$B = \frac{4\mu_0 A^2}{k\lambda} = \frac{4A^2}{\lambda c} Pr, \text{ modified Brinkman number.}$$

and a dash (′) denotes differentiation with respect to I. In (12) we have assumed that R is small justifying our neglecting terms, involving R^2 and higher powers of R. As a closed form solution of eqn. (12) cannot be found out the equation is integrated numerically by the Runge-Kutta step-by-step method.

Discussion of the Results

In this paper we have studied heat transfer due to a gradually accelerated flat plate in a Reiner-Rivlin fluid. For given values of the Prandtl number and Brinkman number we observe that with an increase in the value of the non-Newtonian parameter R, the temperature of the fluid increases at any distance from the plate (Table I). The fall in the temperature of the fluid is ab upt near the wall.

TABLE I

Values of temperature with small variation of R

Values of θ				
$B = 2.0$, $Pr = 1.0$				
η \ R	0	0.05	0.1	0.2
0.0	1.0	1.0	1.0	1.0
0.5	0.31852	0.31897	0.31943	0.32035
1.0	0.06901	0.06924	0.06949	0.06997
1.5	0.00998	0.01002	0.01006	0 01015
2.0	0.00096	0.00096	0.00096	0.00097
2.5	0.00007	0.00005	0.00005	0.00006
3.0	0.00002	0.00000	0.00000	0.00000

The thermal boundary layer thickness seems to decrease with an increase in the value of Pr. For small values of Pr ($= 0.1$), as in gases, the thermal boundary layer thickness, is considerably large compared to the case with larger values of $Pr = 1, 2$ (see Figs. 1 and 2).

Increase in the Brikman number has the effect of increasing the thermal boundary layer thickness. In fluids with small Prandtl number, the temperature rises in the immediate neighbourhood of the wall beyond the wall temperature, but falls with increasing distance from the wall. This rise becomes more marked with an increase in the value of the Brinkman number.

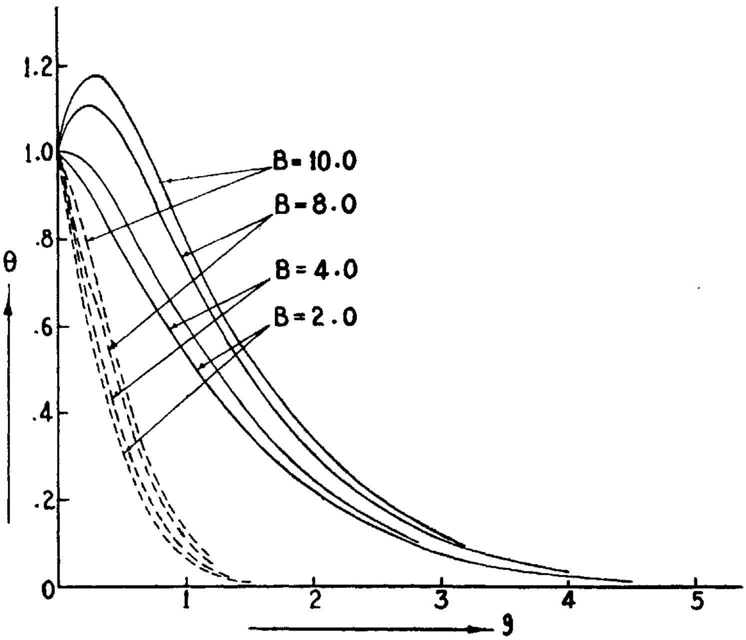

FIG. 1. Temperature profiles for different values of Pr and B. ($R = 0$). ($Pr = 0.1$, ———; $Pr = 1.0$ — — — —).

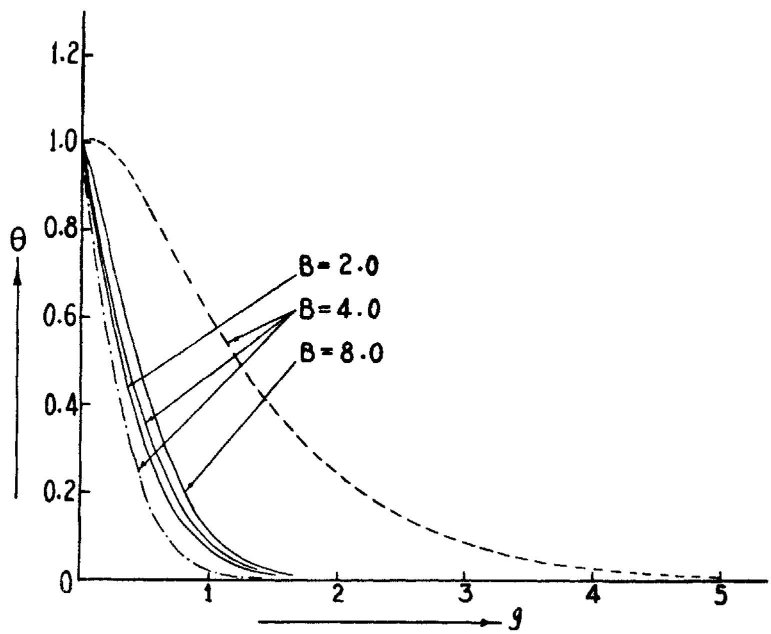

FIG. 2. Temperature profiles for different values of Pr and B ($Pr = 0.1$, — — —; $Pr = 1.0$, ———; $Pr = 2.0$, —.—.—).

Nusselt Number

According to the Newton's law if α is the coefficient of heat transfer and q the quantity of heat exchanged per unit area and unit time, we have

$$q = \alpha (T_w - T_\infty) = \alpha\lambda t = -k \left(\frac{\partial T}{\partial y} \right)_{y=0}$$

$$= -k \left\{ \lambda t \, \theta' \, \frac{1}{2} \, (\nu t)^{-1/2} \right\}_{I=0}$$

then

$$\alpha = -\frac{1}{2} k\theta'(0) (\nu t)^{-1/2}.$$

Thus we obtain a non-dimensional heat transfer coefficient and

$$Nu = \frac{2\alpha}{k} (\nu t)^{1/2} = -\theta'(0).$$

It is seen from the Table II that the heat transfer occurs from the wall to the fluid at a rate (characterized by Nusselt number $= -\theta'(0)$) which decreases with an increase in the Brinkman number, but increases with an increase either in the non-Newtonian parameter or the Prandtl number.

TABLE II

Table showing the variation of Nusselt number

Pr	R	B	$-\theta'(0)$
1.0	0.2	2.0	1.9384
1.0	0.2	4.0	1.6202
1.0	0.2	8.0	0.9837
1.0	0	2.0	1.9347
1.0	0.05	2.0	1.9356
1.0	0.1	2.0	1.9366
0.1	0.05	4.0	0.1552
1.0	0.05	4.0	1.6146
2.0	0.05	4.0	2.6364

REFERENCES

Rath, R. S. (1960-70). The motion of a gradually accelerated flat plate in a non-Newtonian fluid. *Sambalpur University J.*, **3**, No. 1, 1-5.

Rath, R. S., and Mahapatra, B. S. (1970). A note on heat transfer due to the motion of a gradually accelerated plate. *Indian J. Phys.*, **44**, 463-65.