

ON GF -STRUCTURE

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In this paper we have considered generalized holomorphic sectional curvature of V_n equipped with H -structure and KH -structure. Some of its properties have been discussed.

1. INTRODUCTION

We consider a C^∞ manifold V_n . Let there exist on V_n a vector valued bilinear function F of class C^∞ such that

$$\bar{X} = a^2 X \quad \dots(1.1)$$

for any arbitrary vector field X , where $F(X) = \bar{X}$ and a is a complex constant. Let us agree to say that F gives to V_n a differential structure briefly GF -structure defined by algebraic equation (1.1). If the given GF -structure is endowed with a Riemannian metric g such that

$$g(\bar{X}, \bar{Y}) = - a^2 g(X, Y) \quad \dots(1.2)$$

then we say that (F, g) gives to V_n a H -structure.

In the sequel arbitrary vector fields are denoted by X, Y, Z, \dots etc.

Let us consider on V_n equipped with H -structure a tensor ' F ' of the type $(0, 2)$ such that

$$'F(X, Y) \stackrel{def}{=} g(\bar{X}, Y) = - g(X, \bar{Y}). \quad \dots(1.3)$$

Then it is easy to verify

$$'F(\bar{X}, Y) = - 'F(X, \bar{Y}) \quad \dots(1.4a)$$

$$'F(\bar{X}, \bar{Y}) = - a^2 'F(X, Y) \quad \dots(1.4b)$$

and

$$'F(X, Y) = - 'F(Y, X). \quad \dots(1.4c)$$

We further identify connexion D to a suitable symmetric connexion with respect to g . If for H -structure

$$(D_X F)(Y) = 0 \tag{1.5}$$

is satisfied, we say that H -structure is KH -structure. Let K be the curvature tensor of V_n . Weyl curvature tensor W , conformal curvature tensor V , conharmonic curvature tensor L and concircular curvature tensor C are respectively given by

$$W(X, Y, Z) = K(X, Y, Z) - \frac{1}{2m-1} \{X \text{ Ric}(Y, Z) - Y \text{ Ric}(X, Z)\} \tag{1.6a}$$

$$\begin{aligned} V(X, Y, Z) = & K(X, Y, Z) - \frac{1}{2(m-1)} \{X \text{ Ric}(Y, Z) - Y \text{ Ric}(X, Z) \\ & - g(X, Z) R(Y) + g(Y, Z) R(X)\} \\ & + \frac{r}{2(m-1)(2m-1)} \{Xg(Y, Z) - Yg(X, Z)\} \end{aligned} \tag{1.6b}$$

$$\begin{aligned} L(X, Y, Z) = & K(X, Y, Z) - \frac{1}{2(m-1)} \{g(Y, Z) R(X) - g(X, Z) R(Y) \\ & + X \text{ Ric}(Y, Z) - Y \text{ Ric}(X, Z)\} \end{aligned} \tag{1.6c}$$

$$C(X, Y, Z) = K(X, Y, Z) - \frac{r}{2m(2m-1)} \{Xg(Y, Z) - Yg(X, Z)\}. \tag{1.6d}$$

For V_n equipped with KH -structure we have,

$$K(X, Y, \bar{Z}) = \overline{K(X, Y, Z)} \tag{1.7a}$$

$$\overline{K(X, Y, \bar{Z})} = a^2 K(X, Y, Z) \tag{1.7b}$$

$$'K(X, Y, \bar{Z}, U) = -'K(X, Y, Z, \bar{U}) \tag{1.7c}$$

$$'K(X, Y, \bar{Z}, \bar{U}) = -a^2 'K(X, Y, Z, U) \tag{1.7d}$$

where

$$'K(X, Y, Z, U) \stackrel{def}{=} g(K(X, Y, Z), U) \tag{1.7e}$$

$$R(\bar{X}, \bar{Y}) = -a^2 R(X, Y). \tag{1.7f}$$

2. GENERALIZED HOLOMORPHIC SECTIONAL CURVATURE OF V_n EQUIPPED WITH H -STRUCTURE

Definition 2.1 — Let V_n ($n = 2m$) be equipped with H -structure. The plane element determined by X, \bar{X} is called a holomorphic section of V_n . The sectional curvature of V_n at a point with regard to a holomorphic section determined by X is called holomorphic sectional curvature with regard to X .

For V_n equipped with H -structure, the generalized holomorphic sectional curvature k with regard to X is given by

$$k = \frac{{}'K(X, \bar{X}, X, \bar{X})}{a^2g(X, X)g(\bar{X}, \bar{X})} \dots(2.1)$$

Theorem 2.1 — The generalized sectional curvature k of V_n equipped with H -structure is given by

$$k = \frac{{}'W(X, \bar{X}, X, \bar{X})}{a^2g(X, X)g(\bar{X}, \bar{X})} + \frac{1}{(2m-1)} \frac{\text{Ric}(X, X)}{g(X, X)} \dots(2.2a)$$

$$k = \frac{{}'V(X, \bar{X}, X, \bar{X})}{a^2g(X, X)g(\bar{X}, \bar{X})} + \frac{1}{2(m-1)} \left[\frac{a^2\text{Ric}(X, X) - \text{Ric}(\bar{X}, \bar{X})}{a^2g(X, X)} \right] - \frac{r}{2(m-1)(2m-1)} \dots(2.2b)$$

$$k = \frac{{}'L(X, \bar{X}, X, \bar{X})}{a^2g(\bar{X}, \bar{X})g(X, X)} - \frac{1}{2(m-1)} \left[\frac{\text{Ric}(\bar{X}, \bar{X}) - a^2\text{Ric}(X, X)}{a^2g(X, X)} \right] \dots(2.2c)$$

$$k = \frac{{}'C(X, \bar{X}, X, \bar{X})}{a^2g(X, X)g(X, X)} + \frac{r}{2m(2m-1)} \dots(2.2d)$$

PROOF : We define

$${}'W(X, Y, Z, U) \stackrel{def}{=} g(W(X, Y, Z), U) \dots(2.3a)$$

$${}'V(X, Y, Z, U) \stackrel{def}{=} g(V(X, Y, Z), U) \dots(2.3b)$$

$${}'L(X, Y, Z, U) \stackrel{def}{=} g(L(X, Y, Z), U) \dots(2.3c)$$

$${}'C(X, Y, Z, U) \stackrel{def}{=} g(C(X, Y, Z), U). \dots(2.3d)$$

From (1.6a), (1.2), (1.3), (1.4c) and (2.3a) we get

$${}'W(X, \bar{X}, X, \bar{X}) = {}'K(X, \bar{X}, X, \bar{X}) - \frac{1}{2m-1} a^2g(X, X) \text{Ric}(X, X).$$

Substituting in above from (2.1) we get (2.2a). From (1.6b), (1.2), (1.3), (1.4c) and (2.3d) we get

$${}'V(X, \bar{X}, X, \bar{X}) = {}'K(X, \bar{X}, X, \bar{X}) - \frac{1}{2(m-1)} [a^2g(X, X) \text{Ric}(X, X) - g(X, X) \text{Ric}(\bar{X}, \bar{X})] + \frac{ra^2}{2(m-1)(2m-1)} g(X, X) g(X, X).$$

Substituting from (2.1) in above we get (2.2b). From (1.6c), (1.2), (1.3), (1.4c) and (2.3c) we get

$$\begin{aligned} 'L(X, \bar{X}, X, \bar{X}) &= 'K(X, \bar{X}, X, \bar{X}) + \frac{1}{2(m-1)} [g(X, X) \text{ Ric}(\bar{X}, \bar{X}) \\ &\quad - a^2 g(X, X) \text{ Ric}(X, X)]. \end{aligned}$$

Substituting from (2.1) in above we get (2.2c). From (1.6d), (1.2), (1.3), (1.4c) and (2.3d) we get

$$'C(X, \bar{X}, X, \bar{X}) = 'K(X, \bar{X}, X, \bar{X}) - \frac{ra^2}{2m(2m-1)} g(X, X) g(X, X).$$

Substituting from (2.1) in above we get (2.2d).

Corollary 2.1 — The generalized holomorphic sectional curvature k of V_n equipped with H -structure in the direction of a unit vector X is given by

$$a^2 k = 'K(X, \bar{X}, X, \bar{X}) \quad \dots(2.4a)$$

$$k = \frac{'W(X, \bar{X}, X, \bar{X})}{a^2} + \frac{1}{2m-1} \text{ Ric}(X, X) \quad \dots(2.4b)$$

$$\begin{aligned} k &= \frac{'V(X, \bar{X}, X, \bar{X})}{a^2} + \frac{1}{2(m-1)} \left[\frac{a^2 \text{ Ric}(X, X) - \text{ Ric}(\bar{X}, \bar{X})}{a^2} \right] \\ &\quad - \frac{r}{2(m-1)(2m-1)} \quad \dots(2.4c) \end{aligned}$$

$$k = \frac{'L(X, \bar{X}, X, \bar{X})}{a^2} + \frac{1}{2(m-1)} \left[\frac{a^2 \text{ Ric}(X, X) - \text{ Ric}(\bar{X}, \bar{X})}{a^2} \right] \quad \dots(2.4d)$$

$$k = \frac{'C(X, \bar{X}, X, \bar{X})}{a^2} + \frac{r}{2m(2m-1)}. \quad \dots(2.4e)$$

PROOF : Putting for $g(X, X)$ unity in eqns. (2.1), (2.2a), (2.2b), (2.2c) and (2.2d), we get (2.4a), (2.4b), (2.4c), (2.4d) and (2.4e).

Theorem 2.2 — The condition that the V_n equipped with H -structure be of constant generalized holomorphic sectional curvature is

$$\begin{aligned} &3a^4 'K(X, Y, Z, U) - 3a^2 'K(\bar{X}, \bar{Y}, Z, U) - 3a^2 'K(X, Y, \bar{Z}, \bar{U}) \\ &\quad + 3 'K(\bar{X}, \bar{Y}, \bar{Z}, \bar{U}) + a^2 'K(\bar{Y}, \bar{Z}, X, U) - a^2 'K(\bar{X}, \bar{Z}, Y, U) \\ &\quad + a^2 'K(Z, Y, \bar{U}, \bar{X}) - a^2 'K(Z, X, \bar{U}, \bar{Y}) \\ &\quad + a^2 'K(\bar{U}, Y, Z, \bar{X}) - a^2 'K(\bar{U}, X, Z, \bar{Y}) \\ &\quad + a^2 'K(\bar{Y}, U, X, \bar{Z}) - a^2 'K(\bar{X}, U, Y, \bar{Z}) \\ &= -4a^4 k [2 'F(X, Y) 'F(Z, U) \\ &\quad - a^2 g(X, Z) g(Y, U) + a^2 g(Y, Z) g(X, U) \\ &\quad + 'F(Y, Z) 'F(U, X) - 'F(X, Z) 'F(U, Y)]. \quad \dots(2.5) \end{aligned}$$

PROOF : We know that the necessary and sufficient condition that the V_n equipped with H -structure be of constant generalized holomorphic sectional curvature with respect to any vector field X is that eqn. (2.1) is satisfied for an arbitrary vector field X . This statement is equivalent to

$$\begin{aligned}
 & 'K(X, \bar{Y}, Z, \bar{U}) + 'K(Y, \bar{X}, Z, \bar{U}) + 'K(X, \bar{Y}, U, \bar{Z}) \\
 & + 'K(Y, \bar{X}, U, \bar{Z}) + 'K(Z, \bar{Y}, X, \bar{U}) + 'K(Y, \bar{Z}, X, \bar{U}) \\
 & + 'K(Z, \bar{Y}, U, \bar{X}) + 'K(Y, \bar{Z}, U, \bar{X}) + 'K(U, \bar{Y}, Z, \bar{X}) \\
 & + 'K(Y, \bar{U}, Z, \bar{X}) + 'K(U, \bar{Y}, X, \bar{Z}) + 'K(Y, \bar{U}, X, \bar{Z}) \\
 & = -4a^4k[g(X, Y)g(Z, U) + g(X, Z)g(Y, U) \\
 & + g(Y, Z)g(X, U)]. \qquad \dots(2.6)
 \end{aligned}$$

Barring Y, U in above and using (1.1), (1.3) we get

$$\begin{aligned}
 & a^4 'K(X, Y, Z, U) + a^2 'K(\bar{Y}, \bar{X}, Z, U) + a^2 'K(X, Y, \bar{U}, \bar{Z}) \\
 & + 'K(\bar{Y}, \bar{X}, \bar{U}, \bar{Z}) + a^4 'K(Z, Y, X, U) + a^2 'K(\bar{Y}, \bar{Z}, X, U) \\
 & + a^2 'K(Z, Y, \bar{U}, \bar{X}) + 'K(\bar{Y}, \bar{Z}, \bar{U}, \bar{X}) + a^2 'K(\bar{U}, Y, Z, \bar{X}) \\
 & + a^2 'K(\bar{Y}, U, Z, \bar{X}) + a^2 'K(\bar{U}, Y, X, \bar{Z}) + a^2 'K(\bar{Y}, U, X, \bar{Z}) \\
 & = -4a^4k[F(Y, X)'F(U, Z) - a^2g(X, Z)g(Y, U) \\
 & + 'F(Y, Z)'F(U, X)]. \qquad \dots(2.7)
 \end{aligned}$$

Taking skew-symmetric part of eqn. (2.7) in X, Y we get

$$\begin{aligned}
 & 2a^4 'K(X, Y, Z, U) + 2a^2 'K(\bar{Y}, \bar{X}, Z, U) + 2a^2 'K(X, Y, \bar{U}, \bar{Z}) \\
 & + 2'K(\bar{Y}, \bar{X}, \bar{U}, \bar{Z}) + a^4 'K(Z, Y, X, U) - a^4 'K(Z, X, Y, U) \\
 & + a^2 'K(\bar{Y}, \bar{Z}, X, U) - a^2 'K(\bar{X}, \bar{Z}, Y, U) + a^2 'K(Z, Y, \bar{U}, \bar{X}) \\
 & - a^2 'K(Z, X, \bar{U}, \bar{Y}) + 'K(\bar{Y}, \bar{Z}, \bar{U}, \bar{X}) - 'K(\bar{X}, \bar{Z}, \bar{U}, \bar{Y}) \\
 & + a^2 'K(\bar{U}, Y, Z, \bar{X}) - a^2 'K(\bar{U}, X, Z, \bar{Y}) + a^2 'K(\bar{Y}, U, Z, \bar{X}) \\
 & - a^2 'K(\bar{X}, U, Z, \bar{Y}) + a^2 'K(\bar{U}, Y, X, \bar{Z}) - a^2 'K(\bar{U}, X, Y, \bar{Z}) \\
 & + a^2 'K(\bar{Y}, U, X, \bar{Z}) - a^2 'K(\bar{X}, U, Y, \bar{Z}) \\
 & = -4a^4k[2'F(X, Y)'F(Z, U) - a^2g(X, Z)g(Y, U) \\
 & + a^2g(Y, Z)g(X, U) + 'F(Y, Z)'F(U, X) \\
 & - 'F(X, Z)'F(U, Y)].
 \end{aligned}$$

Using Bianchi's first identity in above we get (2.5).

Note 2.1: For $a = \pm i$ expressions (2.1), (2.2a), (2.2b), (2.2d) and (2.5) hold for almost Hermite manifold.

3. GENERALIZED HOLOMORPHIC SECTIONAL CURVATURE OF V_n EQUIPPED WITH KH -STRUCTURE

Theorem 3.1 — For V_n equipped with KH -structure the generalized holomorphic sectional curvature is given by

$$k = \frac{K(X, \bar{X}, X, \bar{X})}{a^2 g(X, X) g(X, X)} \quad \dots(3.1a)$$

$$k = \frac{W(X, \bar{X}, X, \bar{X})}{a^2 g(X, X) g(X, X)} + \frac{1}{2m-1} \frac{\text{Ric}(X, X)}{g(X, X)} \quad \dots(3.1b)$$

$$k = \frac{V(X, \bar{X}, X, \bar{X})}{a^2 g(X, X) g(X, X)} + \frac{1}{m-1} \frac{\text{Ric}(X, X)}{g(X, X)} - \frac{r}{2(m-1)(2m-1)} \quad \dots(3.1c)$$

$$k = \frac{L(X, \bar{X}, X, \bar{X})}{a^2 g(X, X) g(X, X)} + \frac{1}{m-1} \frac{\text{Ric}(X, X)}{g(X, X)} \quad \dots(3.1d)$$

$$k = \frac{C(X, \bar{X}, X, \bar{X})}{a^2 g(X, X) g(X, X)} + \frac{r}{2m(2m-1)} \quad \dots(3.1e)$$

PROOF: By putting the value of $R(\bar{X}, \bar{X})$ from (1.7f) in (2.1), (2.2a), (2.2b), (2.2c) and (2.2d) where required we obtain (3.1a), (3.1b), (3.1c), (3.1d) and (3.1e).

Corollary 3.1 — The generalized holomorphic sectional curvature k of V_n equipped with KH -structure in the direction of a unit vector X is given by

$$a^2 k = K(X, \bar{X}, X, \bar{X}) \quad \dots(3.2a)$$

$$k = \frac{W(X, \bar{X}, X, \bar{X})}{a^2} + \frac{1}{2m-1} \text{Ric}(X, X) \quad \dots(3.2b)$$

$$k = \frac{V(X, \bar{X}, X, \bar{X})}{a^2} + \frac{1}{m-1} \text{Ric}(X, X) - \frac{r}{2(m-1)(2m-1)} \quad \dots(3.2c)$$

$$k = \frac{L(X, \bar{X}, X, \bar{X})}{a^2} + \frac{1}{m-1} \text{Ric}(X, X) \quad \dots(3.2d)$$

$$k = \frac{C(X, \bar{X}, X, \bar{X})}{a^2} + \frac{r}{2m(2m-1)} \quad \dots(3.2e)$$

PROOF: By putting $g(X, X)$ unity in (3.1a), (3.1b), (3.1c), (3.1d) and (3.1e) we get (3.2a), (3.2b), (3.2c), (3.2d) and (3.2e) respectively.

Theorem 3.2 — The condition that V_n equipped with KH -structure be of constant generalized holomorphic sectional curvature with respect to any vector X is

$$\begin{aligned}
 4 \ 'K(X, Y, Z, U) = & -k [2 \ 'F(X, Y) \ 'F(Z, U) - a^2 g(X, Z) g(Y, U) \\
 & + a^2 g(Y, Z) g(X, U) + \ 'F(Y, Z) \ 'F(U, X) \\
 & - \ 'F(X, Z) \ 'F(U, Y)]. \quad \dots(3.3)
 \end{aligned}$$

PROOF : Substituting from (1.7d) and (1.7e) in (2.5) and using algebraic identities satisfied by $\ 'K(X, Y, Z, U)$ we obtain (3.3).

Theorem 3.3 — If V_n equipped with KH -structure be of constant generalized holomorphic sectional curvature. Then

$$Ric(Y, Z) = -k^2 \left(\frac{m+1}{2} \right) a^2 g(Y, Z).$$

PROOF : From (3.3), (1.3) and (1.4c) we have

$$\begin{aligned}
 4 \ 'K(X, Y, Z, U) = & -k [2 \ 'F(X, Y) g(\bar{Z}, U) - a^2 g(X, Z) g(Y, U) \\
 & + a^2 g(Y, Z) g(X, \bar{U}) - \ 'F(Y, Z) g(\bar{X}, U) + \ 'F(X, Z) g(\bar{Y}, U)]. \quad \dots(3.4)
 \end{aligned}$$

Contracting U we get

$$\begin{aligned}
 4K(X, Y, Z) = & -k [2 \ 'F(X, Y) \bar{Z} - a^2 g(X, Z) Y + a^2 g(Y, Z) X \\
 & - \ 'F(Y, Z) \bar{X} + \ 'F(X, Z) \bar{Y}].
 \end{aligned}$$

Contracting X and using (1.1) we get (3.4).

Note 3.1 : For $a = \pm i$, Theorem (3.1), Theorem (3.2), hold for Kähler manifold. Also for $a = \pm i$ equation (3.4) represents Einstein manifold.

Theorem 3.4 — If $\ 'k$ is the generalized section curvature of V_n equipped with $\ 'H$ -structure of constant generalized holomorphic sectional curvature k with respect to two unit orthogonal vector fields P, Q , then

$$\ 'k = -\frac{1}{4a^2} k [3 \cos^2 \theta - a^2] \quad \dots(3.5)$$

where θ is the angle between \bar{P} and Q .

PROOF : We define the generalized section curvature $\ 'k$

$$\ 'k \stackrel{def}{=} \frac{\ 'K(P, Q, P, Q)}{a^2}. \quad \dots(3.6)$$

From (3.6) and (3.3) we get

$$4a^2 \ 'k = -k [3\{ \ 'F(P, Q) \}^2 - a^2]$$

i.e. $4a^2 'k = - k [3\{g(\bar{P}, Q)\}^2 - a^2]$

i.e. $'k = - \frac{k}{4a^2} [3 \cos^2 \theta - a^2].$

Note 3.2 : For $a = \pm i$ (3.6) becomes

$$\left. \begin{aligned} k \leq 4 'k \leq 4k, & \quad k > 0 \\ k \geq 4 'k \geq 4k, & \quad k < 0. \end{aligned} \right\} \quad \text{(Yano 1965)}$$

Corollary 3.2 — If

$$'l(Y, Z) \stackrel{def}{=} (C_1^1 L)(Y, Z) \quad \dots(3.7)$$

$$\alpha \stackrel{def}{=} C_1^1 l \quad \dots(3.8)$$

$$'l(Y, Z) \stackrel{def}{=} g(l(X), Y). \quad \dots(3.9)$$

Then

$$'l(Y, Z) = \frac{ka^2m(m+1)}{2(m-1)} g(Y, Z) \quad \dots(3.10a)$$

$$\alpha = \frac{ka^2m^2(m+1)}{m-1}. \quad \dots(3.10b)$$

PROOF : From (1.6c) and (3.7) we obtain

$$'l(Y, Z) = - \frac{r}{2(m-1)} g(Y, Z). \quad \dots(3.11)$$

From (3.4) on contraction generalized scalar curvature r is given by

$$r = - ka^2m(m+1). \quad \dots(3.12)$$

From (3.11) and (3.8) we have

$$\alpha = - \frac{r}{2(m-1)} 2m. \quad \dots(3.13)$$

From (3.11), (3.12) and (3.13) we obtain (3.10a) and (3.10b).

Theorem 3.5 — If V_n equipped with KH -structure be of constant generalized holomorphic sectional curvature k we have

$$\begin{aligned} 'W(X, Y, Z, U) = & - \frac{1}{4}k [2 'F(X, Y) 'F(Z, U) - 'F(Y, Z) 'F(X, U) \\ & + 'F(X, Z) 'F(Y, U)] + \frac{3ka^2}{4(2m-1)} [g(Y, Z) g(X, U) \\ & - g(X, Z) g(Y, U)] \quad \dots(3.14a) \end{aligned}$$

$$\begin{aligned}
 'V(X, Y, Z, U) = & -\frac{1}{4}k [2 'F(X, Y) 'F(Z, U) - 'F(Y, Z) 'F(X, U) \\
 & + 'F(X, Z) 'F(Y, U)] + \frac{3ka^2}{4(2m-1)} [g(Y, Z) \\
 & - g(X, U) - g(X, Z) g(Y, U)] \quad \dots(3.14b)
 \end{aligned}$$

$$\begin{aligned}
 'L(X, Y, Z, U) = & -\frac{1}{4}k [2 'F(X, Y) 'F(Z, U) - 'F(Y, Z) 'F(X, U) \\
 & + 'F(X, Z) 'F(Y, U)] + \frac{ka^2(m+3)}{4(m-1)} [g(Y, Z) g(X, U) \\
 & - g(X, Z) g(Y, U)] \quad \dots(3.14c)
 \end{aligned}$$

$$\begin{aligned}
 'C(X, Y, Z, U) = & -\frac{1}{4}k [2 'F(X, Y) 'F(Z, U) - 'F(Y, Z) 'F(X, U) \\
 & + 'F(X, Z) 'F(Y, U)] + \frac{3ka^2}{4(2m-1)} [g(Y, Z) g(X, U) \\
 & - g(X, Z) g(Y, U)]. \quad \dots(3.14d)
 \end{aligned}$$

Consequently for V_n equipped with KH -structure of constant generalized holomorphic sectional curvature $'W(X, Y, Z, U)$, $'V(X, Y, Z, U)$, $'L(X, Y, Z, U)$ and $'C(X, Y, Z, U)$ are all skew symmetric in Z and U . Also they are all symmetric in two pair of slots (X, Y) , (Z, U) .

PROOF : Substituting the values from (3.3), (3.4), (3.12) in (1.6a), (1.6b), (1.6c) and (1.6d) we obtain (3.14a), (3.14b), (3.14c) and (3.14d).

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