ON A POLYNOMIAL OF THE FORM F_4

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(Communicated by P. L. Bhatnagar, F.N.A.)

(*Received* 11 *June* 1974)

In this paper first some generating functions involving the polynomial of the form F_4 are obtained. Later on, some bilinear generating functions are found out a few of which contain the results of Manocha (1968, 1969) and Carlitz (1963) as particular cases.

1. INTRODUCTION

In this paper the main attention is given to the polynomial defined and represented as

$$f_n(\alpha;\ \beta_1, \beta_2;\ x, y) = \frac{(\alpha)_n}{n!} F_4(-n, \alpha + n;\ \beta_1, \beta_2;\ x, y) \qquad \ldots(1.1)$$

where n is a non-negative integer, F_4 is Kampé de Fériet function (Erdélyi 1953, p. 224)

$$F_4(a, b;\ c, d;\ x, y) = \sum_{m,\,n=0}^{\infty} \frac{(a)_{m+n}\,(b)_{m+n}}{m!\,n!\,(c)_m\,(d)_n} x^m y^n$$

$$|\,x^{1/2}\,| + |\,y^{1/2}\,| < 1. \qquad \ldots(1.2)$$

2. GENERATING FUNCTIONS

From the definition (1.1), we obtain below some generating functions. We have

$$\sum_{n=0}^{\infty} f_n(\alpha;\ \beta_1, \beta_2;\ x, y)\, t^n$$

$$= \sum_{n=0}^{\infty} \frac{(\alpha)_n}{n!} \sum_{p+q\leqslant n} \frac{(-n)_{p+q}\,(\alpha+n)_{p+q}}{p!\,q!\,(\beta_1)_p\,(\beta_2)_q} x^p y^q t^n$$

$$= \sum_{n,\,p,\,q=0}^{\infty} \frac{(\alpha)_{n+2p+2q}}{n!\,p!\,q!\,(\beta_1)_p\,(\beta_2)_q} (-xt)^p\,(-yt)^q\, t^n$$

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*The research work of the authors is supported by the C.S.I.R., Government of India.

$$= \sum_{p,q=0}^{\infty} \frac{(\frac{1}{2}\alpha)_{p+q} \left(\frac{\alpha}{2} + \frac{1}{2}\right)_{p+q}}{p!\, q!\, (\beta_1)_p (\beta_2)_q} (-4xt)^p (-4yt)^q$$

$$\times \sum_{n=0}^{\infty} \frac{(\alpha + 2p + 2q)_n}{n!} t^n.$$

From this we arrive at the following generating function

$$\sum_{n=0}^{\infty} f_n(\alpha; \beta_1, \beta_2; x, y)\, t^n$$

$$= (1-t)^{-\alpha} F_4\left(\frac{\alpha}{2}, \frac{\alpha}{2} + \frac{1}{2}; \beta_1, \beta_2; \frac{-4xt}{(1-t)^2}, \frac{-4yt}{(1-t)^2}\right), \qquad ...(2.1)$$

$$|t| + 2(|\sqrt{x}| + |\sqrt{y}|)\,|\sqrt{t}| < 1.$$

By proceeding on lines similar to that given above, we also obtain the following generating functions

$$\sum_{n=0}^{\infty} \frac{(\lambda)_n}{(1-\alpha)_n} f_n(\alpha - n; \beta_1, \beta_2; x, y)\, t^n$$

$$= (1+t)^{-\lambda} F_4\left(\lambda, \alpha; \beta_1, \beta_2; \frac{xt}{1+t}, \frac{yt}{1+t}\right),$$

$$|t^{1/2}| < \frac{1}{(1 + |x^{1/2}| + |y^{1/2}|)}; \qquad ...(2.2)$$

$$\sum_{n=0}^{\infty} \frac{(\beta_1)_n}{(1-\alpha)_n} f_n(\alpha - n, 1 - \beta_1 - n, \beta_2; x, y)\, t^n$$

$$= (1+t)^{-\beta_1} (1+xt)^{-\alpha}\, {}_2F_1\left(\alpha, \beta_1; \beta_2; \frac{yt}{(1+t)(1+xt)}\right),$$

$$|t| < 1, |xt| < 1, |t| < \frac{1}{(|y| - |x| - 1)}; \qquad ...(2.3)$$

$$\sum_{n=0}^{\infty} \frac{(\beta_1)_n (\beta_2)_n}{(\alpha)_{2n}} f_n(1 - \alpha - 2n; 1 - \beta_1 - n, 1 - \beta_2 - n; x, y)\, t^n$$

$$= (1-xt)^{-\beta_2} (1-yt)^{-\beta_1}\, {}_2F_1\left(\beta_1, \beta_2; \alpha; \frac{-t}{(1-xt)(1-yt)}\right),$$

$$|xt| < 1, |yt| < 1, |t| < \frac{1}{(|x| + |y| - 1)}. \qquad ...(2.4)$$

3. Special Cases

Let us consider the polynomial $f_n(\beta + \beta^1 - 1; \beta, \beta^1; x(1-y), y(1-x))$, we have by definition (1.1)

$$f_n(\beta + \beta^1 - 1; \beta, \beta^1; x(1-y), y(1-x))$$
$$= \frac{(\beta+\beta^1-1)_n}{n!} F_4(-n, \beta+\beta^1+n-1; \beta, \beta^1; x(1-y), y(1-x)) \quad ...(3.1)$$

which on using (Erdélyi 1953, p. 238)

$$F_4(\alpha, \gamma+\gamma^1-\alpha-1; \gamma, \gamma^1; x(1-y), y(1-x))$$
$$= {}_2F_1(\alpha, \gamma+\gamma^1-\alpha-1; \gamma; x)\, {}_2F_1(\alpha, \gamma+\gamma^1-\alpha-1; \gamma^1; y),$$

gives

$$f_n(\beta + \beta^1 - 1; \beta, \beta^1; x(1-y), y(1-x))$$
$$= \frac{(\beta+\beta^1-1)_n}{n!}\, {}_2F_1(-n, \beta+\beta^1+n-1; \beta; x)\, {}_2F_1(-n, \beta+\beta^1+n-1; \beta^1; y). \quad ...(3.2)$$

Now replacing β, β^1, x and y by $\beta+1$, β^1+1, $\frac{1-x}{2}$ and $\frac{1+y}{2}$, respectively and using the definitions (Rainville 1960, p. 254) for Jacobi polynomials

$$P_n^{(\alpha,\beta)}(x) = \frac{(1+\alpha)_n}{n!}\, {}_2F_1\left(-n, 1+\alpha+\beta+n; 1+\alpha; \frac{1-x}{2}\right)$$

and

$$P_n^{(\alpha,\beta)}(x) = \frac{(-1)^n (1+\beta)_n}{n!}\, {}_2F_1\left(-n, 1+\alpha+\beta+n; 1+\beta; \frac{1+x}{2}\right)$$

it becomes

$$f_n\left(\beta+\beta^1+1; \beta+1, \beta^1+1; \frac{(1-x)(1-y)}{4}, \frac{(1+x)(1+y)}{4}\right)$$
$$= \frac{(\beta+\beta^1+1)_n (-1)^n n!}{(1+\beta)_n (1+\beta^1)_n} P_n^{(\beta,\beta^1)}(x)\, P_n^{(\beta,\beta^1)}(y). \quad ...(3.3)$$

Now, suitably specializing the parameters in (2.1), (2.3) and (2.4) in accordance with the relation (3.3), we get

$$\sum_{n=0}^{\infty} \frac{(1+\beta+\beta^1)_n\, n!}{(1+\beta)_n (1+\beta^1)_n} P_n^{(\beta,\beta^1)}(x)\, P_n^{(\beta,\beta^1)}(y)\, t^n$$
$$= (1+t)^{-(1+\beta+\beta^1)} F_4\Big(\tfrac{1}{2}(1+\beta+\beta^1), \tfrac{1}{2}(2+\beta+\beta^1); \beta+1, \beta^1+1;$$
$$\frac{(x-1)(y-1)t}{(1+t)^2}, \frac{(x+1)(y+1)t}{(1+t)^2}\Big). \quad ...(3.4)$$

$$\sum_{n=0}^{\infty} \frac{n!\, t^n}{(1+\beta^1)_n} P_n^{(\beta-n,\beta^1)}(x)\, P_n^{(\beta-n,\beta^1)}(y)\, t^n$$

$$= (1-t)^{\beta}\left[1 - \frac{(x-1)(y-1)\,t}{4}\right]^{-1-\beta-\beta^1}$$

$$\times\, {}_2F_1\left(1+\beta+\beta^1, -\beta; \beta^1+1; \frac{-(x+1)(y+1)\,t}{(1-t)\,[4-(x-1)(y-1)\,t]}\right) \quad ...(3.5)$$

$$\sum_{n=0}^{\infty} \frac{n!}{(-\beta-\beta^1)_n} P_n^{(\beta-n,\beta^1-n)}(x)\, P_n^{(\beta-n,\beta^1-n)}(y)\, t^n$$

$$= \left[1 - \frac{(x+1)(y+1)\,t}{4}\right]^{\beta}\left[1 - \frac{(x-1)(y-1)\,t}{4}\right]^{\beta^1}$$

$$\times\, {}_2F_1\left[-\beta, -\beta^1; -\beta-\beta^1; \frac{-t}{\left[1-\frac{(x+1)(y+1)\,t}{4}\right]\left[1-\frac{(x-1)(y-1)\,t}{4}\right]}\right] \quad ...(3.6)$$

The results (3.4) and (3.6), have already been obtained by Bailey (1935) and Manocha (1968) respectively.

Again, if in (3.6) we replace x, y and t by $(x+1)/(x-1)$, $-(y+1)/(y-1)$ and $-(x-1)(y-1)/t$ respectively and then use the relation (Rainville 1960, p. 256)

$$P_n^{(\alpha,\beta)}(x) = (-1)^n P_n^{(\beta,\alpha)}(x) \quad ...(3.7)$$

we get

$$\sum_{n=0}^{\infty} \frac{n!}{(-\beta-\beta^1)_n} (x-1)^n (y-1)^n P_n^{(\beta-n,\beta^1-n)}\left(\frac{x+1}{x-1}\right)$$

$$\times\, P_n^{(\beta^1-n,\beta-n)}\left(\frac{y+1}{y-1}\right) t^n$$

$$= (1-xt)^{\beta}(1-yt)^{\beta^1}\, {}_2F_1\left(-\beta, -\beta^1; -\beta-\beta^1; \frac{(x-1)(y-1)\,t}{(1-xt)(1-yt)}\right). \quad ...(3.8)$$

(3.8) is due to Carlitz (1963).

Finally, using the confluence principle in (2.2), we obtain

$$\sum_{n=0}^{\infty} \frac{1}{(1-\alpha)_n} f_n(\alpha-n; \beta_1, \beta_2; x, y)\, t^n$$

$$= e^{-t}\, \psi_2(\alpha, \beta_1, \beta_2, xt, yt) \quad ...(3.9)$$

where ψ_2 is defined as (Erdélyi 1953, p. 225)

$$\psi_2(\alpha, \gamma, \gamma^1, x, y) = \sum_{m,n=0}^{\infty} \frac{(\alpha)_{m+n}}{m!\,n!\,(\gamma)_m (\gamma^1)_n} x^m y^n. \quad \ldots(3.10)$$

$$\text{Let } F = e^{-t}\, \psi_2(\alpha, \beta_1, \beta_2, xt, yt). \quad \ldots(3.11)$$

Now differentiating (3.11), partially with respect to x, y and t and eliminating ψ_2 therein, we obtain the following recurrence relation

$$x\frac{\partial F}{\partial x} + y\frac{\partial F}{\partial y} - t\frac{\partial F}{\partial t} = tF. \quad \ldots(3.12)$$

$$\text{Since } F = \sum_{n=0}^{\infty} \frac{1}{(1-\alpha)_n} f_n(\alpha - n; \beta_1, \beta_2; x, y)\, t^n$$

eqn. (3.12) yields

$$x\frac{\partial}{\partial x} f_n(\alpha - n; \beta_1, \beta_2; x, y) + y\frac{\partial}{\partial y} f_n(\alpha - n; \beta_1, \beta_2; x, y)$$
$$- n f_n(\alpha - n; \beta_1, \beta_2; x, y) = (-\alpha + n)\, f_{n-1}(\alpha - (n-1); \beta_1, \beta_2; x, y). \quad \ldots(3.13)$$

Also we rewrite (3.9) as

$$e^t \sum_{n=0}^{\infty} \frac{1}{(1-\alpha)_n} f_n(\alpha - n; \beta_1, \beta_2; x, y)\, t^n$$

$$= \psi_2(\alpha, \beta_1, \beta_2, xt, yt).$$

Or

$$\sum_{n=0}^{\infty} \sum_{k=0}^{n} \frac{t^n}{(n-k)!\,(1-\alpha)_k} f_k(\alpha - k; \beta_1, \beta_2; x, y)$$

$$= \sum_{n=0}^{\infty} \sum_{p=0}^{n} \frac{(\alpha)_n x^n}{(n-p)!\,p!(\beta_1)_{n-p}\,(\beta_2)_p} \left(\frac{y}{x}\right)^p t^n \quad \ldots(3.14)$$

which gives

$$\sum_{p=0}^{n} \frac{1}{(n-p)!\,p!\,(\beta_1)_{n-p}\,(\beta_2)_p} \left(\frac{y}{x}\right)^p$$

$$= \frac{x^{-n}}{(\alpha)_n} \sum_{k=0}^{n} \frac{1}{(n-k)!\,(1-\alpha)_k} f_k(\alpha - k; \beta_1, \beta_2; x, y) \quad \ldots(3.15)$$

now

$$\psi_2(\lambda, \beta_1, \beta_2, xt, yt) = \sum_{n=0}^{\infty} (\lambda)_n \left[\sum_{p=0}^{n} \frac{1}{(n-p)!\, p!\, (\beta_1)_{n-p} (\beta_2)_p} \times \left(\frac{y}{x}\right)^p\right] (xt)^n$$

$$= \sum_{n=0}^{\infty} \frac{(\lambda)_n t^n}{(\alpha)_n} \sum_{k=0}^{n} \frac{1}{(n-k)!\,(1-\alpha)_k} f_k(\alpha - k; \beta_1, \beta_2; x, y) \text{ by (3.15)}$$

$$= \sum_{n,k=0}^{\infty} \frac{(\lambda)_{n+k}\, t^{n+k}}{(\alpha)_{n+k}\, n!} \frac{1}{(1-\alpha)_k} f_k(\alpha - k; \beta_1, \beta_2; x, y)$$

$$= \sum_{k=0}^{\infty} \frac{(\lambda)_k\, t^k}{(\alpha)_k\,(1-\alpha)_k} f_k(\alpha - k; \beta_1, \beta_2; x, y) \sum_{n=0}^{\infty} \frac{(\lambda+k)_n}{n!\,(\alpha+k)_n} t^n,$$

from which it follows that

$$\sum_{n=0}^{\infty} \frac{(\lambda)_n}{(\alpha)_n\,(1-\alpha)_n} f_n(\alpha - n; \beta_1, \beta_2; x, y)\; {}_1F_1(\lambda+n; \alpha+n; t)\, t^n$$

$$= \psi_2(\lambda, \beta_1, \beta_2, xt, yt). \qquad \text{...(3.16)}$$

Putting $\lambda = \alpha + m$ in (3.16) and using Kummer's transformation

$${}_1F_1(\alpha; \beta; x) = e^x\; {}_1F_1(\beta - \alpha; \beta; -x),$$

we get

$$\sum_{n=0}^{\infty} \frac{1}{(1-\alpha)_n} f_n(\alpha - n; \beta_1, \beta_2; x, y)\; L_m^{(\alpha+n-1)}(-t)\, t^n$$

$$= \frac{(\alpha)_m}{m!} e^{-t}\, \psi_2(\lambda, \beta_1, \beta_2, xt, yt). \qquad \text{...(3.17)}$$

4. Bilinear Generating Functions

We now prove the following bilinear generating functions

$$\sum_{n=0}^{\infty} \frac{(\lambda)_n}{(1-\alpha)_n} f_n(\alpha - n; \beta_1, \beta_2; x, y)\; {}_2F_1(\lambda+n, \gamma; \mu; z)\, t^n$$

(*equation continued on p.* 1332)

$$= (1+t)^{-\lambda} F_E\left(\lambda, \lambda, \lambda, \gamma, \alpha, \alpha; \mu, \beta_1, \beta_2; \frac{z}{1+t}, \frac{xt}{1+t}, \frac{yt}{1+t}\right)$$

$$|z| < 1, \ |t| < \frac{1-|z|}{1+|x|+|y|+2|xy|} \qquad \ldots(4.1)$$

and where F is one of the Saran's (1954) functions.

To prove (4.1), we start with the left-hand side.

$$\sum_{n=0}^{\infty} \frac{(\lambda)_n}{(1-\alpha)_n} f_n(\alpha - n; \beta_1, \beta_2; x, y)\, {}_2F_1(\lambda+n, \gamma; \mu; z)\, t^n$$

$$= \sum_{n=0}^{\infty} \frac{(\lambda)_n (\alpha-n)_n}{(1-\alpha)_n\, n!} F_4(-n, \alpha; \beta_1, \beta_2; x, y)\, t^n \sum_{r=0}^{\infty} \frac{(\lambda+n)_r (\gamma)_r}{r!\,(\mu)_r} z^r$$

$$= \sum_{n, r=0}^{\infty} \frac{(\lambda)_{n+r} (\gamma)_r}{n!\, r!\, (\mu)_r} (-1)^n t^n z^r \sum_{p+q \leqslant n} \frac{(-n)_{p+q} (\alpha)_{p+q}}{p!\, q!\, (\beta_1)_p (\beta_2)_q} x^p y^q$$

$$= \sum_{n, p, q, r=0}^{\infty} \frac{(\lambda)_{n+p+q+r} (\alpha)_{p+q} (\gamma)_r}{n!\, p!\, q!\, r!\, (\beta_1)_p (\beta_2)_q (\mu)_r} (xt)^p (yt)^q z^r (-t)^n$$

$$= \sum_{r, p, q=0}^{\infty} \frac{(\lambda)_{r+p+q} (\gamma)_r (\alpha)_{p+q}}{r!\, p!\, q!\, (\mu)_r (\beta_1)_p (\beta_2)_q} z^r (xt)^p (yt)^q \sum_{n=0}^{\infty} \frac{(\lambda+r+p+q)_n}{n!} (-t)^n$$

$$= (1+t)^{-\lambda} F_E\left(\lambda, \lambda, \lambda, \gamma, \alpha, \alpha; \mu, \beta_1, \beta_2; \frac{z}{1+t}, \frac{xt}{1+t}, \frac{yt}{1+t}\right)$$

which proves (4.1).

By suitably specializing the parameters in (4.1) and using the definition for Jacobi function of second kind we can express it as follows:

$$\sum_{n=0}^{\infty} \binom{m+n}{m} \frac{n!}{(1-\alpha)_n} Q_{m+n}^{(\nu-n, \delta-n)}(z)\, f_n(\alpha-n; \beta_1, \beta_2; x, y)\, t^n$$

$$= 2^{m+\nu+\delta} \frac{\Gamma(m+\nu+1)\, \Gamma(m+\delta+1)}{\Gamma(2m+\nu+\delta+2)\, (z-1)^{m+\nu+1} (z+1)^{\delta}}$$

$$\times [1 + \tfrac{1}{2}(z+1)t]^{-(m+1)} F_E\Big(m+1, m+1, m+1, m+\nu+1, \alpha, \alpha;$$

$$\nu + 2m + \delta + 2, \beta_1, \beta_2; \frac{4}{2(1-z)+(1-z^2)t}, \frac{x(z+1)t}{2+(1+z)t}, \frac{y(z+1)t}{2+(1+z)t}\Big) \qquad \ldots(4.2)$$

Next, if we replace λ by $\mu + m$ in (4.2) and use Euler's transformation

$$ {}_2F_1(a, b; c; x) = (1 - x)^{-a} \, {}_2F_1\left(a, c - b; c; \frac{-x}{1-x}\right) $$

we have

$$ \sum_{n=0}^{\infty} \frac{(\mu + m)_n}{(1 - \alpha)_n} f_n(\alpha - n; \beta_1, \beta_2; x, y) \, {}_2F_1\left(-m - n, \gamma; \mu; \frac{-z}{1-z}\right) t^n $$

$$ = (1 - z)^{\gamma} (1 + t)^{-\mu - m} $$

$$ \times F_E\left(\mu + m, \mu + m, \mu + m, \gamma, \alpha, \alpha; \mu, \beta_1, \beta_2; \frac{z}{1+t}, \frac{xt}{1+t}, \frac{yt}{1+t}\right). \quad \ldots(4.3) $$

Now, if we make replacements for γ, μ, z and t by $-\gamma - m$, $-\mu - \gamma - 2m$, $\frac{2}{1+z}$ and $\frac{1}{2}(z - 1)\,t$ respectively, and use the definition for Jacobi polynomial (4.3) becomes (Rainville 1960, p. 255)

$$ \sum_{n=0}^{\infty} \binom{m+n}{m} \frac{(-1)^n n!}{(1-\alpha)_n} P_{m+n}^{(\gamma - n, \mu - n)}(z) \, f_n^{(\alpha - n; \beta_1, \beta_2; x, y)} t^n $$

$$ = \frac{(1 + \gamma + \mu + m)_m}{m!} \left(\frac{z+1}{2}\right)^m \left(\frac{z-1}{z+1}\right)^{-\gamma} \rho^{\mu + \gamma + m} $$

$$ \times F_E\Big(-\mu - \gamma - m, -\mu - \gamma - m, -\mu - \gamma - m, -\gamma - m, \alpha, \alpha; $$

$$ -\mu - \gamma - 2m, \beta_1, \beta_2; \frac{2}{\rho(1+z)}, \frac{-x(1-z)\,t}{2\rho}, \frac{-y(1-z)\,t}{2\rho}\Big) \quad \ldots(4.4) $$

where $\rho = 1 + \frac{1}{2}(z - 1)\,t$.

If we suitably express (5.4) in terms of function F_4, we arrive at a known result due to Manocha (1969).

Next we prove the following generating function :

$$ \sum_{n=0}^{\infty} \frac{(\lambda)_n (\mu)_n}{(1+\alpha)_n (1+\beta)_n} P_n^{(\alpha, \beta)}(x) \, F_4(\lambda + n, \mu + n; \gamma, \delta; z, u) \, t^n $$

$$ = F_c\left(\lambda, \mu; 1 + \alpha, 1 + \beta, \gamma, \delta; \frac{t(x-1)}{2}, \frac{t(x+1)}{2}, z, u\right), $$

$$ t^{1/2} < \frac{2^{1/2}(1 - |z|^{1/2})}{|x - 1|^{1/2} + |x + 1|^{1/2}} \quad \ldots(4.5) $$

where F_c is defined as (Slater 1966, p. 228)

$$F_c\,[a, b; c_1, c_2, \ldots, c_n; x_1, x_2, \ldots, x_n)$$

$$= \sum_{m_1, \ldots m_n=0}^{\infty} \frac{(a)_{m_1+m_2+\cdots+m_n} (b)_{m_1+\cdots+m_n} x_1^{m_1} \ldots x_n^{m_n}}{m! \ldots m_n! (c_1)_{m_1} \ldots (c_m)_{m_n}} \qquad \ldots(4.6)$$

where for convergence

$$\left| x_1^{1/2} \right| + \left| x_2^{1/2} \right| + \ldots + \left| x_n^{1/2} \right| < 1.$$

The proof of (4.5) can be developed on the lines similar to that of (4.1).

If we put $\lambda = -m$ in (4.5), we get

$$\sum_{n=0}^{m} \frac{(-m)_n (\mu)_n}{(1+\alpha)_n (1+\beta)_n} P_n^{(\alpha,\beta)}(x)\, F_4(-(m-n), \mu+n; \gamma, \delta; z, u)\, t^n$$

$$= F_c\left(-m, \mu; 1+\alpha, 1+\beta, \gamma, \delta; \frac{t(x-1)}{2}, \frac{t(x+1)}{2}, z, u\right)$$

or

$$\sum_{n=0}^{\infty} \frac{(\mu)_n\, m!}{(1+\alpha)_n (1+\beta)_n (\mu-m+2n)_{m-n}} P_n^{(\alpha,\beta)}(x)$$

$$\times f_{m-n}(\mu-m+2n; \gamma, \delta; z, u)\, t^n$$

$$= F_c\left(-m, \mu; 1+\alpha, 1+\beta, \gamma, \delta; \frac{-t(x-1)}{2}, \frac{-t(x+1)}{2}, z, u\right). \qquad \ldots(4.7)$$

ACKNOWLEDGEMENT

The authors are highly thankful to Dr. H. L. Manocha for his helpful suggestions.

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