MAGNETOHYDRODYNAMIC FLOW BETWEEN TWO DISKS—ONE ROTATING AND THE OTHER AT REST

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(Communicated by F. C. Auluck, F.N.A.)

(*Received* 9 *October* 1973; *after revision* 5 *September* 1974)

The steady flow of a viscous, incompressible and electrically conducting fluid contained between two infinite disks (one rotating and the other at rest) is considered under the influence of a constant magnetic field applied in the direction normal to the disks. The forms for velocity, magnetic field and pressure are taken under the assumption of the existence of similarity solutions. The radial, axial and transverse components of velocity and magnetic fields are obtained for small Reynolds number and are shown graphically. The expressions for pressure coefficient, torque, stream function and the force experienced by the stationary disk are also obtained. The applied field in the axial direction is modified by the interaction of fluid motion and also the fields are induced in the radial and transverse directions. Since it is found that the radial and axial velocity components are not affected by the magnetic field (correct to second order in R), the streamlines are similar to the case when magnetic field is absent.

Introduction

Von Kármán (1921) has discussed the flow of viscous incompressible fluid under the influence of a rotating disk. Following Von Kármán, Batchelor (1951), Stewartson (1953), Lance and Rogers (1962) have discussed the flow between two rotating disks. Lance and Rogers have considered the problem when the ratio of the angular velocities of the two disks, s, is in the range $-1 \leqslant s \leqslant +1$. In particular they have considered the case when $s = 0$, which means that one of the disks (the upper disk in their case) is stationary.

The problem of the flow between a rotating and a stationary disk has been independently solved by Mellor *et al.* (1968) under the assumption of similarity solutions. Narayana and Rudraiah (1972) have presented an analysis in which there is a flow between two disks, one rotating and other stationary with a uniform suction at the stationary disk. They have developed the solution for small suction Reynolds number assuming the solutions in powers of suction Reynolds number and keeping the ratio of rotation and suction Reynolds numbers a constant. The effect of porosity in the problem of Lance and Rogers has been discussed by Gaur (1972).

The magnetohydrodynamic problem corresponding to the Von Kármán problem has been solved by Sychev (1969) and Kakutani (1962). Sychev has assumed the electrical conductivity of the disk to be zero and the applied uniform magnetic field to be in the direction perpendicular to the plane of the disk. Following Batchelor, he has assumed the forms of the velocity, magnetic field and pressure expressions and has solved the magnetohydrodynamic equations. Kakutani has assumed the magnetic Prandtl number to be small so as to neglect any effect of the electric currents flowing in the fluid on the magnetic field and thus the applied magnetic field is essentially undisturbed by the flow field. He has reported that the magnitudes of the radial and the axial components of the velocity decrease rapidly as the hydromagnetic interaction parameter increases, while the velocity gradient of the circumferential component at the disk increases. Srivastava and Sharma (1961) and Lal (1973) have discussed the magnetohydrodynamic flow between two disks, one rotating and other at rest. In both these papers the induced magnetic field is neglected. In fact Lal has attempted to show the effect of the third order terms (which are the terms of order R^2). Srivastava and Sharma have shown that for second order terms there exists only radial and axial flow. Lal in his paper has concluded that for the third order terms the magnetic intensity must be very small such that tanh $M/2 = M/2$ (where M is the Hartmann number). But from his equation (29) it is evident that the conclusion he has given is a restriction imposed on the problem in order to satisfy the boundary condition. Further, the efficiency curve (Fig. 1) drawn by him appears to be in error since L_1 is zero on the boundaries.

In the present paper we have considered the flow between two infinite disks under the influence of a magnetic field applied in the direction perpendicular to the disks taking into account the induced magnetic field which is neglected by earlier authors. The forms for velocity, magnetic field and pressure are taken under the assumption of the existence of similarity solutions and the Reynolds number of the flow is taken to be small. The primary purpose is to study the interaction of the axial magnetic field on the fluid motion. Therefore, the results for velocity, magnetic field, torque etc. are obtained in the simplest form and for that reason our results are mainly confined to the expressions containing Hartmann number and Reynolds number. These results therefore do not contain the terms with electric field in them. We know (Hughes and Young 1966) that the rotation can be produced in the disk when the drag torque is reduced to zero by changing the electric field (from an external source) which on a further change can give the power to the rotating disk to act as a motor. Because the electric field does not occur in our final expression for torque, the role played by the magnetic field as an equivalent rotation is not discussed.

It is interesting to note that the radial and axial velocity components remain unaffected by the applied magnetic field while the transverse velocity is influenced by the magnetic field considerably. The solutions are obtained by using power series in Reynolds number and terms up to the order of R^2 are retained. The velocity and magnetic fields are shown graphically.

1. Basic Equations and Formulation of the Problem

The steady motion of a viscous, incompressible and electrically conducting fluid contained between two infinite disks is considered. The lower disk is at rest and the upper disk is rotating in its own plane with a constant angular velocity. In our analysis we make use of cylindrical polar coordinates (r, θ, z) and take the two disks at $z = 0$ and $z = d$ planes. A constant magnetic field, H_0, is applied in the direction normal to the disks.

In view of rotational symmetry, the magnetohydrodynamic equations for steady motion are

$$u_r \frac{\partial u_r}{\partial r} + u_z \frac{\partial u_r}{\partial z} - \frac{u_\theta^2}{r} = -\frac{1}{\rho}\frac{\partial}{\partial r}\left(p + \frac{\mu_e}{8\pi}\bar{H}^2\right) + \nu\left(\frac{\partial^2 u_r}{\partial r^2} + \frac{1}{r}\frac{\partial u_r}{\partial r} + \frac{\partial^2 u_r}{\partial z^2} - \frac{u_r}{r^2}\right) + \frac{\mu_e}{4\pi\rho}\left(H_r \frac{\partial H_r}{\partial r} + H_z \frac{\partial H_r}{\partial z} - \frac{H_\theta^2}{r}\right) \quad \ldots(1.1)$$

$$u_r \frac{\partial u_\theta}{\partial r} + u_z \frac{\partial u_\theta}{\partial z} + \frac{u_r u_\theta}{r} = \nu\left(\frac{\partial^2 u_\theta}{\partial r^2} + \frac{1}{r}\frac{\partial u_\theta}{\partial r} + \frac{\partial^2 u_\theta}{\partial z^2} - \frac{u_\theta}{r^2}\right) + \frac{\mu_e}{4\pi\rho}\left(H_r \frac{\partial H_\theta}{\partial r} + H_z \frac{\partial H_\theta}{\partial z} + \frac{H_r H_\theta}{r}\right) \quad \ldots(1.2)$$

$$u_r \frac{\partial u_z}{\partial r} + u_z \frac{\partial u_z}{\partial z} = -\frac{1}{\rho}\frac{\partial}{\partial z}\left(p + \frac{\mu_e}{8\pi}\bar{H}^2\right) + \nu\left(\frac{\partial^2 u_z}{\partial r^2} + \frac{1}{r}\frac{\partial u_z}{\partial r} + \frac{\partial^2 u_z}{\partial z^2}\right) + \frac{\mu_e}{4\pi\rho}\left(H_r \frac{\partial H_z}{\partial r} + H_z \frac{\partial H_z}{\partial z}\right) \quad \ldots(1.3)$$

$$u_r \frac{\partial H_r}{\partial r} + u_z \frac{\partial H_r}{\partial z} = \left(H_r \frac{\partial u_r}{\partial r} + H_z \frac{\partial u_r}{\partial z}\right) + \eta\left(\frac{\partial^2 H_r}{\partial r^2} + \frac{1}{r}\frac{\partial H_r}{\partial r} + \frac{\partial^2 H_r}{\partial z^2} - \frac{H_r}{r^2}\right) \quad \ldots(1.4)$$

$$u_r \frac{\partial H_\theta}{\partial r} + u_z \frac{\partial H_\theta}{\partial z} + \frac{u_\theta H_r}{r} = \left(H_r \frac{\partial u_\theta}{\partial r} + H_z \frac{\partial u_\theta}{\partial z} + \frac{H_\theta u_r}{r}\right) + \eta\left(\frac{\partial^2 H_\theta}{\partial r^2} + \frac{1}{r}\frac{\partial H_\theta}{\partial r} + \frac{\partial^2 H_\theta}{\partial z^2} - \frac{H_\theta}{r^2}\right) \quad \ldots(1.5)$$

$$u_r \frac{\partial H_z}{\partial r} + u_z \frac{\partial H_z}{\partial z} = \left(H_r \frac{\partial u_z}{\partial r} + H_z \frac{\partial u_z}{\partial z}\right) + \eta\left(\frac{\partial^2 H_z}{\partial r^2} + \frac{1}{r}\frac{\partial H_z}{\partial r} + \frac{\partial^2 H_z}{\partial z^2}\right) \quad \ldots(1.6)$$

$$\frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{\partial u_z}{\partial z} = 0 \qquad \ldots(1.7)$$

$$\frac{\partial H_r}{\partial r} + \frac{H_r}{r} + \frac{\partial H_z}{\partial z} = 0 \qquad \ldots(1.8)$$

where u_r, u_θ, u_z are the components of the velocity field $\bar{u}$ and H_r, H_θ, H_z the components of the magnetic field $\bar{H}$ in radial, transversal and axial directions respectively. ρ is the density of the fluid, μ_e the magnetic permeability, ν the kinematic viscosity, p the pressure, $\eta \left(= \frac{1}{4\pi\mu_e\sigma}\right)$ the magnetic diffusivity and σ the electrical conductivity of the fluid.

Also, the current density $\bar{j}$ is given by

$$\bar{j} = \sigma(\bar{E} + \mu_e \bar{u} \times \bar{H})$$

and

$$\text{curl } \bar{H} = 4\pi\bar{j}$$

where E is the electric field.

From these, we have

$$\text{curl } \bar{H} = 4\pi\sigma(\bar{E} + \mu_e \bar{u} \times \bar{H}). \qquad \ldots(1.9)$$

For the present problem, we have

$$\left.\begin{array}{l} u_r = u_r(r, z),\ u_\theta = u_\theta(r, z),\ u_z = u_z(z) \\ H_r = H_r(r, z)\ H_\theta = H\ (r, z),\ H_z = H_z(z). \end{array}\right\} \qquad \ldots(1.10)$$

Let us introduce the following dimensionless quantities

$$\left.\begin{array}{l} R(\text{Reynolds number}) = \dfrac{\omega d^2}{\nu} \\[2ex] p_m = \dfrac{\eta}{\nu} \\[2ex] \text{and} \quad \xi = \dfrac{z}{d} \\[2ex] \text{so that} \quad 0 \leqslant \xi \leqslant 1. \end{array}\right\} \qquad \ldots(1.11)$$

We seek the similarity solutions of equations (1.1) to (1.8) in the form

$$\left.\begin{aligned} &u_r = \omega r F(\xi),\ u_\theta = \omega r G(\xi),\ u_z = d\omega H(\xi),\\ &H_r = \left(\frac{4\pi\rho}{\mu_e}\right)^{1/2} \omega r f(\xi),\ H_\theta = \left(\frac{4\pi\rho}{\mu_e}\right)^{1/2} \omega r g(\xi),\\ &H_z = \left(\frac{4\pi\rho}{\mu_e}\right)^{1/2} \omega d h(\xi),\\ &\frac{1}{\rho}\left(p + \frac{\mu_e}{8\pi}\bar{H}^2\right) = -\,\omega\nu\rho(\xi) + \frac{\omega\nu\lambda}{d^2} r^2 \end{aligned}\right\} \qquad \ldots(1.12)$$

where λ is an absolute constant.

From (1.1) to (1.8) and (1.10) to (1.12), we have

$$R(F^2 + HF' - G^2) + 2\lambda = F'' + R(f^2 + hf' - g^2) \qquad \ldots(1.13)$$

$$R(2FG + HG') = G'' + R(2fg + hg') \qquad \ldots(1.14)$$

$$RHH' = P' + H'' + Rhh' \qquad \ldots(1.15)$$

$$RH\begin{pmatrix} f' \\ g' \\ h' \end{pmatrix} = Rh\begin{pmatrix} F' \\ G' \\ H' \end{pmatrix} + \rho_m\begin{pmatrix} f'' \\ g'' \\ h'' \end{pmatrix} \qquad \ldots(1.16)$$

and

$$2\begin{pmatrix} F \\ f \end{pmatrix} + \begin{pmatrix} H' \\ h' \end{pmatrix} = 0. \qquad \ldots(1.17)$$

The boundary conditions are

$$\left.\begin{aligned} &\left.\begin{aligned} &F = 0,\quad G = 0,\quad H = 0,\\ &f = 0,\quad g = 0,\quad h = \chi, \end{aligned}\right\} \text{at } \xi = 0\\ &\text{and}\\ &\left.\begin{aligned} &F = 0,\quad G = 1,\quad H = 0,\\ &f = 0,\quad g = 0,\quad h' = 0 \end{aligned}\right\} \text{at } \xi = 1 \end{aligned}\right\} \qquad \ldots(1.18)$$

where dashes denote differentiation with respect to ξ, $\chi = \dfrac{H_0}{\omega d}\left(\dfrac{\mu_e}{4\pi\rho}\right)^{1/2}$ and the disks are assumed to be infinitely conducting.

The boundary conditions on magnetic field at $\xi = 1$ are obtained from the equation (1.9) as follows :

$\bar{E}$ is the electric field with components E_r, E_θ and E_z in the radial, transversal and axial directions respectively. Hence, writing the axial component of equation (1.9), we get

$$E_z = \frac{1}{4\pi\sigma}\left(\frac{\partial H_\theta}{\partial r} + \frac{H_\theta}{r}\right) + \mu_e(u_\theta H_r),$$

so that

$$E_z\Big|_{z=d-} = \frac{1}{4\pi\sigma}\left(\frac{\partial H_\theta}{\partial r} + \frac{H_\theta}{r}\right)_{z=d-} + \mu_e(u_\theta H_r)_{z=d-}$$
$$= \frac{1}{4\pi\sigma}\left(\frac{\partial H_\theta}{\partial r} + \frac{H_\theta}{r}\right)_{z=d-}$$

as $H_r = 0$ at $z = d^-$ because there is no flow in the radial direction at the plate and hence there will be no induced field in that direction. The symbols $z = d^+$ and $z = d^-$ have been used for the upper and lower surfaces of the disk at $z = d$.

Also,

$$E_z \mid_{z=d+} = 0.$$

But,

$$E_z \mid_{z=d+} = E_z \mid_{z=d-}$$

and hence we have

$$\frac{\partial H_\theta}{\partial r} + \frac{H_\theta}{r} = 0, \text{ at } \xi = 1$$

which results in the boundary condition

$$g = 0 \text{ at } \xi = 1.$$

Further,

$$\frac{\partial H_r}{\partial r} + \frac{H_r}{r} + \frac{1}{d}\frac{\partial H_z}{\partial \xi} = 0 \text{ throughout the space.}$$

But at $\xi = 1$, we have $H_r = 0$

so that

$$\frac{\partial H_z}{\partial \xi} = 0 \text{ at } \xi = 1,$$

which results in boundary condition

$$h' = 0 \text{ at } \xi = 1.$$

2. Method of Solution

In case of small Reynolds number we can assume the unknown functions in equation (1.10) in ascending powers of R as

$$A = A^{(0)} + RA^{(1)} + R^2A^{(2)} + \ldots \qquad \ldots(2.1)$$

where A represents F, G, H, f, g, h, λ and P.

Substituting these in equations (1.13) to (1.17) and equating the coefficients of like powers of R, we get the following sets of differential equations :

$$\left.\begin{aligned}
&F^{(0)''} = 2\lambda^{(0)}\\
&F^{(1)''} - (F^{(0)^2} + H^{(0)}F^{(0)'} - G^{(0)^2})\\
&\qquad = 2\lambda^{(1)} - (f^{(0)^2} + h^{(0)}f^{(0)'} - g^{(0)^2})\\
&F^{(2)''} - 2F^{(0)}F^{(1)} - (H^{(0)}F^{(1)'} + H^{(1)}F^{(0)'}) + 2G^{(0)}G^{(1)}\\
&\qquad = 2\lambda^{(2)} - 2f^{(0)}f^{(1)} - (h^{(0)}f^{(1)'} + h^{(1)}f^{(0)'}) + 2g^{(0)}g^{(1)}.
\end{aligned}\right\} \quad \ldots(2.2)$$

$$\left.\begin{aligned}
&G^{(0)''} = 0\\
&G^{(1)''} - 2F^{(0)}G^{(0)} - G^{(0)}H^{(0)'} = -2f^{(0)}g^{(0)} - h^{(0)}g^{(0)'}\\
&G^{(2)''} - 2(F^{(0)}G^{(1)} + F^{(1)}G^{(0)}) - (H^{(0)}G^{(1)'} + H^{(1)}G^{(0)'})\\
&\quad = -2(f^{(0)}g^{(1)} + f^{(1)}g^{(0)}) - (h^{(0)}g^{(1)'} + h^{(1)}g^{(0)'}).
\end{aligned}\right\} \quad \ldots(2.3)$$

$$\left.\begin{aligned}
&P^{(0)'} + H^{(0)''} = 0\\
&P^{(1)'} + H^{(1)''} - H^{(0)'}H^{(0)} + h^{(0)}h^{(0)'} = 0\\
&P^{(2)'} + H^{(2)''} - (H^{(0)}H^{(1)'} + H^{(1)}H^{(0)'})\\
&\qquad + (h^{(0)}h^{(1)'} + h^{(1)}h^{(0)'}) = 0.
\end{aligned}\right\} \quad \ldots(2.4)$$

$$\left.\begin{aligned}
&f^{(0)''} = 0\\
&P_m f^{(1)''} - H^{(0)}f^{(0)'} + h^{(0)}F^{(0)'} = 0\\
&P_m f^{(2)''} - (H^{(0)}f^{(1)'} + H^{(1)}f^{(0)'})\\
&\qquad + (h^{(0)}F^{(1)'} + h^{(1)}F^{(0)'}) = 0.
\end{aligned}\right\} \quad \ldots(2.5)$$

$$\left.\begin{aligned}
&g^{(0)''} = 0\\
&P_m g^{(1)''} - H^{(0)}g^{(0)'} + G^{(0)'}h^{(0)} = 0\\
&P_m g^{(2)''} - (H^{(0)}g^{(1)'} + H^{(1)}g^{(0)'})\\
&\qquad + (G^{(0)'}h^{(1)} + G^{(1)'}h^{(0)}) = 0.
\end{aligned}\right\} \quad \ldots(2.6)$$

$$\left.\begin{aligned}
&h^{(0)''} = 0\\
&P_m h^{(1)''} - H^{(0)}h^{(0)'} + H^{(0)'}h^{(0)} = 0\\
&P_m h^{(2)''} - (H^{(0)}h^{(1)'} + H^{(1)}h^{(0)'})\\
&\qquad + (H^{(0)'}h^{(1)} + H^{(1)'}h^{(0)}) = 0.
\end{aligned}\right\} \quad \ldots(2.7)$$

$$\left.\begin{aligned}
&2F^{(0)} + H^{(0)'} = 0\\
&2F^{(1)} + H^{(1)'} = 0\\
&2F^{(2)} + H^{(2)'} = 0.
\end{aligned}\right\} \quad \ldots(2.8)$$

$$\left.\begin{aligned} 2f^{(0)} + h^{(0)\prime} &= 0 \\ 2f^{(1)} + h^{(1)\prime} &= 0 \\ 2f^{(2)} + h^{(2)\prime} &= 0 \end{aligned}\right\} \qquad \ldots(2.9)$$

and so on.

The boundary conditions are

$$\left.\begin{aligned} & F^{(n)}(0) = 0 = G^{(n)}(0) = H^{(n)}(0) \\ & f^{(n)}(0) = 0 = g^{(n)}(0),\ h^{(0)}(0) = \chi,\ h^{(n+1)}(0) = 0 \\ & F^{(n)}(1) = 0,\ G^{(0)}(1) = 1,\ G^{(n+1)}(1) = 0 = H^{(n)}(1) \\ & f^{(n)}(1) = 0 = g^{(n)}(1) = h^{(n)\prime}(1) \end{aligned}\right\} \qquad \ldots(2.10)$$

where $n = 0, 1, 2, \ldots$.

The solutions of above sets of differential equations [except eqn. (2.4)], on retaining the term up to $O(R^2)$, give

$$F(\xi) = -\frac{R}{60}\,\xi(1-\xi)(4-5\xi-5\xi^2) \qquad \ldots(2.11)$$

$$G(\xi) = \xi + \frac{M^2}{12}\,\xi(1-\xi)(1-2\xi) - \frac{R^2}{6300}\,\xi(1-\xi)$$

$$(8 + 8\xi + 8\xi^2 + 43\xi^3 - 20\xi^4 - 20\xi^5) \qquad \ldots(2.12)$$

$$H(\xi) = \frac{R}{30}\,\xi^2(1-\xi)^2(2+\xi) \qquad \ldots(2.13)$$

$$f(\xi) = \frac{\chi R^2}{60\rho_m}\,\xi^2(1-\xi)^2(2+\xi) \qquad \ldots(2.14)$$

$$g(\xi) = \frac{\chi R}{2\rho_m}\,\xi(1-\xi) \qquad \ldots(2.15)$$

$$h(\xi) = \chi - \frac{\chi R^2}{360\rho_m}\,\xi^3(8 - 9\xi + 2\xi^3) \qquad \ldots(2.16)$$

where M (Hartmann number) $= \dfrac{R\chi}{\rho_m^{1/2}}$.

3. Expressions for Pressure Coefficient, Torque, Stream Function and the Force Experienced by the Stationary Disk

From eqns. (1.12) and (2.4) the pressure coefficient is obtained as

$$c_p = \frac{\bar{p}(r_0, 1) - \bar{p}(r, 1)}{\rho\omega\nu}\cdot\frac{d}{r_0} = \frac{3R}{20}\left(1 - \frac{r^2}{r_0^2}\right) \qquad \ldots(3.1)$$

where $\bar{p}(r, \xi) = \left(p + \frac{\mu_e}{8\pi} \bar{H}^2\right)$ and c_p is the coefficient of pressure on the upper disk of radius r_0 when the edge effects are neglected.

The torque exerted by the fluid on the rotating upper disk of radius r_0, when edge effects are neglected, is

$$T_u = -\frac{2\pi\rho\nu}{d}\int_0^{r_0}\left(\frac{\partial u_\theta}{\partial\xi}\right)_{\xi=1} r^2 dr = -\frac{\pi\rho\nu\omega}{2d} G'(1)\, r_0^4$$

$$= -\frac{\pi\rho\nu\omega}{2d}\left(1 + \frac{M^2}{12} + \frac{3R^2}{700}\right) r_0^4. \qquad \ldots(3.2)$$

Similarly, the torque exerted by the fluid on the stationary lower disk of radius r_0, is

$$T_L = \frac{2\pi\rho\nu}{d}\int_0^{r_0}\left(\frac{\partial u_\theta}{\partial\xi}\right)_{\xi=0} r^2 dr = \frac{\pi\rho\nu\omega}{2d} G'(0)\, r_0^4$$

$$= \frac{\pi\rho\nu\omega}{2d}\left(1 + \frac{M^2}{12} - \frac{2R^2}{1575}\right) r_0^4. \qquad \ldots(3.3)$$

From (3.2) and (3.3) we find that as the Hartmann number increases the torque exerted by the fluid on the rotating disk decreases while the torque exerted on the stationary disk increases. We define a stream function ψ as

$$u_r = -\frac{1}{rd}\frac{\partial\psi}{\partial\xi}, \quad u_z = \frac{1}{r}\frac{\partial\psi}{\partial r}.$$

From (1.12), (2.11) and (2.13), we get

$$\psi = \frac{R}{30}\,\omega d r^2 \xi^2 (1-\xi)^2 (2+\xi). \qquad \ldots(3.4)$$

The pressure field is independently determined from eqns. (2.4) and (2.13). For this, we have

$$P(\xi) = \Pi - \frac{R\xi}{30}(1-\xi)(4 - 5\xi - 5\xi^2) \qquad \ldots(3.5)$$

where Π is the value of $P(\xi)$ at either disk.

4. Numerical Discussion

Profiles for the velocity components in the radial and axial directions, transverse direction, three components of the magnetic field and the streamlines have been plotted in Figs. 1 to 4.

The velocity profiles in the radial and axial directions are drawn in Fig. 1 for different Reynolds numbers. Since the lower disk is stationary there is a back flow in the neighbourhood of this disk for the radial component of the velocity. This back flow decreases as the Reynolds number decreases while in the neighbourhood of the rotating upper disk the magnitude of the radial component of the velocity increases with the increase in the Reynolds number. The effect of the magnetic field on the radial and axial components is very small, actually the effect is only seen in the terms of $O(R^3)$. The expressions for $F(\xi)$ and $H(\xi)$, in the absence of magnetic field are in full agreement with those obtained by Lance and Rogers (1962) for the case when $s = 0$ and the lower plate is considered to be stationary instead of the upper disk.

In Fig. 2, transverse velocity profiles have been plotted for $R = 0.2$ and different values of Hartmann number. It can be seen that the increase in Hartmann number

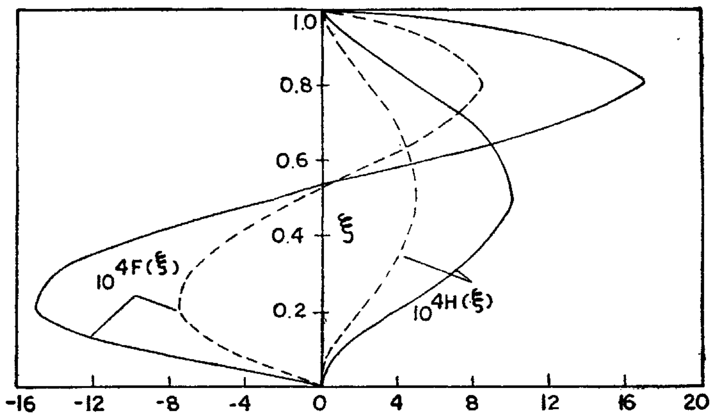

FIG. 1. Velocity profiles in the radial and axial directions ($R = 0.2$ ——, $R = 0.1$ - - - -).

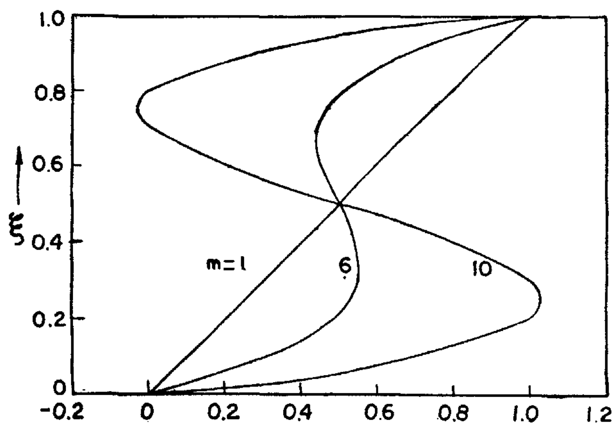

FIG. 2. Transverse velocity profiles at different Hartmann numbers and $R = 0.2$.

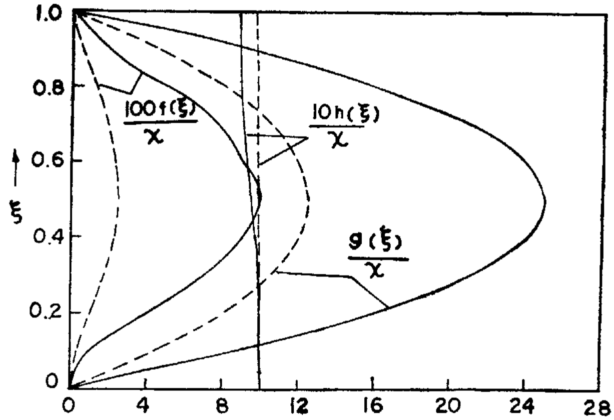

FIG. 3. Profiles for magnetic fields ($P_m = 0.001$; $R = 0.2$ ——, $R = 0.1$ - - - -).

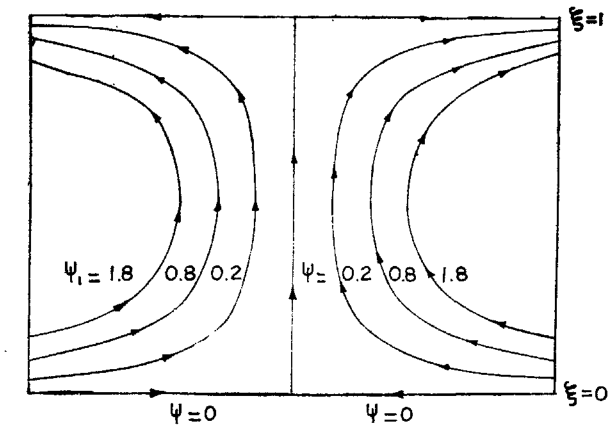

FIG. 4. Stream lines for $R = 0.2$.

changes the flow pattern considerably. When $M = 1$, we can see that the effect of the magnetic field is almost absent while M is raised to 6 and then to 10 we have very prominent change in the velocity pattern in the transverse direction.

Magnetic field profiles in the radial, transverse and axial directions are drawn in Fig. 3 at different Reynolds numbers. The effect of the interaction of the flow of the conducting fluid on the magnetic field is very apparent. The applied field in the axial direction is modified by this flow and also the fields are induced in the radial and transverse directions. The pattern also reveals that as the Reynolds number increases the effect on the magnetic field also increases.

The streamline patterns $\left(\Psi = \dfrac{30\psi}{\omega d^3}\right)$ are drawn in Fig. 4. Since the radial and axial velocity components are not affected by the magnetic field (correct to second order in R), the streamlines are similar to the case when magnetic field is absent.

ACKNOWLEDGEMENT

The authors are extremely grateful to Professor P. L. Bhatnagar for his encouragement.

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