# FATIGUE FRACTURE OF A ROTATING CYLINDER

*by* V. P. MUTHUSWAMY *and* R. MOHANSINGH, *Department of Mathematics, Regional Engineering College, Tiruchirapalli, India*

(Communicated by F. C. Auluck, F.N.A.)

(*Received* 25 *January* 1973)

In this investigation, the theory of fatigue fracture of brittle solids, developed by Thomas (1970), is applied to a solid cylinder, rotating about its central axis. Fracture is produced in the cylinder by simple as well as alternating rotation cycles under both atmospheric and hydrostatic pressures. In all the cases, relations between the angular speed $\omega$ and the number of cycles $N$ required to cause fracture and the endurance limits are obtained.

## 1. INTRODUCTION

(i) The material of the cylinder is brittle. It undergoes an adrupt change from its elastic state to a state of plastic equilibrium, immediately prior to fracture. It fractures without the necessity of increasing the angular speed beyod the point required to produce the initial plastic deformation.

(ii) A simple rotation cycle is one in which the angular speed $\omega$ increases continually in numerical value from zero to a maximum value and then decreases continually to the initial value zero. The maximum numerical value of $\omega$, of this rotation cycle, is called the peak load of the cycle. An alternating rotation cycle is one in which the body completes simple cycles in opposite directions alternately.

(iii) The quantities $\tau_{ij}$ are the components of the elastic stress field which is determined by the peak load of the simple cycle. $\tau_1, \tau_2, \tau_3$ (where $\tau_1 \geqslant \tau_2 \geqslant \tau_3$) are the principal stresses of this stress field. A function

$$Q(\tau) = (\tau_1 - \tau_3) - 2\varpi \qquad \ldots(1.1)$$

where $\varpi$ is a modulus of the material, is defined.

(iv) If fracture is produced during the $N$th cyclic load, the equation

$$(\sqrt{1+k^2}+k)\, s_1 - (\sqrt{1+k^2}-k)\, s_3 = 2K - 2\alpha(N-1)\, Q(\tau) \qquad \ldots(1.2)$$

will be satisfied by the stress field in the plastic state preceding fracture The quantities $k$ and $K$ are the slip moduli and $\alpha$ and $\varpi$ the fatigue moduli; $s_1$ and $s_3$

are the algebraically greatest and the least of the principal stresses in the plastic field. The function $\Omega(\tau)$ is already defined in eqn. ( 1.1). If $N = 1$, eqn. (1.2) reduces to

$$(\sqrt{1 + k^2} + k)\, s_1 - (\sqrt{1 + k^2} - k)\, s_3 = 2K \qquad \ldots(1.3)$$

which gives the fracture problem corresponding to a single loading.

(v) The moduli $k$, $\varpi$ and $K$ are subject to the inequality

$$(\sqrt{1 + k^2} + k)\, \varpi < K \qquad \ldots(1.4)$$

where the modulus $k$ is small and $K$ is very large when measured in conventional units.

(vi) The cylinder is assumed to be long enough so that the condition of plane strain exists.

(vii) *Endurance limit*—The magnitude $L$ of the peak load satisfies the inequality $L_1 < L \leqslant L_2$ for loads which are just sufficient to cause fracture during the $N$th loading operation. The upper limit $L_2$ is the magnitude of the peak load when fracture is produced by a single loading; the lower limit $L_1$ is referred to as the endurance limit.

## 2. Formulation of the Problem

The cylindrical coordinates $r$, $\theta$, $z$ are employed such that the $z$-axis lies along the central axis of the cylinder; $r$, $\theta$, $z$ are denoted by $x^1$, $x^2$, $x^3$ respectively for the convenience of writing the equations in tensor notation.

In addition to the condition of fatigue fracture (1.2), the following relations are satisfied by the stress and displacement fields induced in the cylinder, prior to fracture.

The material is incompressible, i.e.

$$g^{ij} e_{ij} = 0 \qquad \ldots(2.1)$$

where $e_{ij}$ are the components of the strain tensor and $g^{ij}$ the contravariant components of the metric tensor and the comma denotes covariant differentiation. The strain tensor is defined by

$$e_{ij} = \tfrac{1}{2}\,(u_{i,j} + g_{j,i})$$

where $u_i$ are the components of plastic deformation.

The plastic stress field preceding fracture is denoted by $\sigma_{ij}$ and is determined by the Hencky stress-strain relations

$$\sigma_{ij}^{*} = \chi\, e_{ij} \qquad \ldots(2.2)$$

where $\sigma_{ij}^*$ are the components of the stress deviator tensor and $e_{ij}$ the components of the plastic strain tensor which is identical with its deviator on account of the eqn. (2.1). The proportionality factor $\chi$ is assumed to be a positive function and is regarded as one of the dependent variables of the problem. The deviator stress tensor is given by

$$\sigma_{ij}^* = \sigma_{ij} - \tfrac{1}{3}\, \sigma_{ab}\, g^{ab}\, g_{ij}$$

where $g_{ij}$ are the covariants components of the metric tensor.

The equation of dynamical equilibrium, referred to the fixed frame of reference, is of the form

$$g^{jk}\, \sigma_{ij,k} = \rho\, v_{i,j}\, v^j \qquad \ldots(2.3)$$

where $v_i$ and $v^j$ are respectively the covariant and contravariant components of velocity.

The boundary condition of the problem is

$$\sigma_{11} = 0 \quad \text{at} \quad r = a \qquad \ldots(2.4)$$

where $r$ is the distance of any point from the central axis and $a$ is the outer radius of the cylinder.

Since the cylinder is long enough, the end effects may be neglected and hence

$$\int_0^a 2\pi r\, \sigma_{33}\, dr = 0 \qquad \ldots(2.5)$$

## 3. Analysis

From the symmetry of the problem, with reference to the cylindrical coordinate system, we have

$$\sigma_{12} = \sigma_{23} = \sigma_{31} = 0 \qquad \ldots(3.1)$$

and

$$\sigma_{ij} = \sigma_{ij}\,(r) \text{ for } i = j = 1, 2, 3. \qquad \ldots(3.2)$$

The plastic deformations $u_1$, $u_2$, $u_3$ are assumed to be of the form

$$u_1 = u_1\,(r),\ u_2 = u_2\,(r, z),\ u_3 = u_3\,(r, z). \qquad \ldots(3.3)$$

From plane strain condition, we have

$$e_{33} = C, \text{ a negative constant (Nadai 1950).} \qquad \ldots(3.4)$$

On simplifying eqn. (2.1), we have

$$\frac{\partial u_1}{\partial r} + \frac{u_1}{r} + C = 0 \qquad \ldots(3.5)$$

Since $u_1 = u_1(r)$, this reduces to the ordinary equation

$$\frac{du_1}{dr} + \frac{u_1}{r} + C = 0$$

whose solution is

$$u_1 = -\frac{C}{2} r + \frac{D}{r}$$

Neglecting $D$ in order to keep $u_1$ finite at $r = 0$, and denoting $-C/2$ by $A$, a positive constant, we get $u_1 = Ar$.

On expanding (2.2) with the help of (3.3), we have the following relations:

$$2\sigma_{11} - \frac{\sigma_{22}}{r^2} - \sigma_{33} = 3\chi \frac{\partial u_1}{\partial r} \qquad \ldots(3.6)$$

$$\sigma_{11} - 2\frac{\sigma_{22}}{r^2} + \sigma_{33} = -3\chi \frac{\mu_1}{r} \qquad \ldots(3.7)$$

$$\sigma_{11} + \frac{\sigma_{22}}{r^2} - 2\sigma_{33} = -3C\chi \qquad \ldots(3.8)$$

$$\frac{\partial u_2}{\partial z} = 0 \qquad \ldots(3.9)$$

$$\frac{\partial u_2}{\partial r} - \frac{2u_2}{r} = 0 \qquad \ldots(3.10)$$

$$\frac{\partial u_3}{\partial r} = 0 \qquad \ldots(3.11)$$

From eqn. (3.9), we infer that $u_2$ is independent of $z$ and from eqn. (3.11) that $u_3$ is independent of $r$. Hence

$$u_1 = u_1(r),\ u_2 = u_2(r),\ u_3 = u_3(z).$$

As $u_2 = u_2(r)$, (3.10) reduces to the ordinary equation

$$\frac{du_2}{dr} - \frac{2u_2}{r} = 0$$

whose solution is $u_2 = Br^2$, where $B$ is an arbitrary constant.

From (3.4) we have $\partial u_3/\partial z = C$ and hence $u_3 = Cz + C_1$. We can do away with the constant $C_1$, by a proper choice of the origin of our coordinate system, so that $u_3 = Cz$. Thus we have the following values for the components of plastic deformation:

$$u_1 = Ar,\ u_2 = Br^2,\ u_3 = -2Az$$

where $u_2$ represents a rigid rotation.

Substituting the values of $u_1$ in (3.6), (3.7) and (3.8) we have

$$2\sigma_{11} - \frac{\sigma_{22}}{r^2} - \sigma_{33} = 3A\chi \qquad \ldots(3.12)$$

$$\sigma_{11} - 2\frac{\sigma_{22}}{r^2} + \sigma_{33} = -3A\chi \qquad \ldots(3.13)$$

$$\sigma_{11} + \frac{\sigma_{22}}{r^2} - 2\sigma_{33} = 6A\chi \qquad \ldots(3.14)$$

From these equations we find that

$$\sigma_{11} = \frac{\sigma_{22}}{r^2} \qquad \ldots(3.15)$$

and

$$\sigma_{11} - \sigma_{33} = 3A\chi. \qquad \ldots(3.16)$$

As $3A\chi > 0$, we have $\sigma_{11} > \sigma_{33}$.

On account of (3.1) and (3.2) the equations of equilibrium (2.3) are satisfied for $i = 2$ and 3. For $i = 1$, it reduces to the form

$$\frac{d\sigma_{11}}{dr} - \frac{1}{r}\left(\frac{\sigma_{22}}{r^2} - \sigma_{11}\right) + \rho\omega^2 r = 0. \qquad \ldots(3.17)$$

By virtue of (3.15), the above equation can be written as

$$\frac{d\sigma_{11}}{dr} + \rho\omega^2 r = 0$$

which on integration gives

$$\sigma_{11} = \rho\frac{\omega^2}{2}(a^2 - r^2) \qquad \ldots(3.18)$$

on using the boundary condition (2.4).

The principal values of $\sigma_{ij}$ are given by the determinantal equation

$$|\sigma_{ij} - s\,g_{ij}| = 0. \qquad \ldots(3.19)$$

Equation (3.19) has the roots

$$s_1 = \sigma_{11}, \quad s_2 = \frac{\sigma_{22}}{r^2}, \quad s_3 = \sigma_{33}$$

where $s_1 = s_2 > s_3$.

Substituting these values in eqn. (1.2), we have

$$(\sqrt{1+k^2}+k)\,\sigma_{11} - (\sqrt{1+k^2}-k)\,\sigma_{33} = 2K - 2\alpha\,(N-1)\,\Omega\,(\tau) \qquad \ldots(3.20)$$

Following Hoffman and Sachs (1953) the elastic stresses are found to be

$$\tau_{11} = \frac{\tau_{22}}{r^2} = \rho\,\frac{\omega^2}{2}(a^2 - r^2) \qquad \ldots(3.21)$$

$$\tau_{33} = \rho\,\frac{\omega^2}{2}(a^2 - r^2) - \rho\,\frac{\omega^2 a^2}{4}. \qquad \ldots(3.22)$$

From (3.21) and (3.22) we find that

$$\tau_{11} - \tau_{33} = \rho\,\frac{\omega^2 a^2}{4}$$

and hence

$$\tau_{11} > \tau_{33}. \qquad \ldots(3.23)$$

The principal elastic stresses are given by the determinantal equation

$$|\,\tau_{ij} - \tau g_{ij}\,| = 0$$

which has the solution

$$\tau_1 = \tau_{11}, \tau_2 = \frac{\tau_{22}}{r^2}, \tau_3 = \tau_{33}$$

where

$$\tau_1 = \tau_2 > \tau_3$$

on account of (3.23).

Hence (1.1) becomes, on using (3.21) and (3.22)

$$\Omega\,(\tau) = \rho\,\frac{\omega^2 a^2}{4} - 2\varpi. \qquad \ldots(3.24)$$

Substituting this value of $\Omega\,(\tau)$ in eqn. (3.20), we get

$$(\sqrt{1+k^2}+k)\,\sigma_{11} - (\sqrt{1+k^2}-k)\,\sigma_{33} = 2K - 2\alpha\,(N-1)\left(\rho\,\frac{\omega^2 a^2}{4} - 2\varpi\right). \qquad \ldots(3.25)$$

From (3.18) and (3.25), we obtain

$$\sigma_{33} = \frac{\sqrt{1+k^2}+k}{\sqrt{1+k^2}-k} \cdot \rho\frac{\omega^2}{2}(a^2-r^2) - \frac{2}{\sqrt{1+k^2}-k}$$

$$\times\left[K - \alpha(N-1)\left(\rho\frac{\omega^2 a^2}{4} - 2\varpi\right)\right]. \qquad \ldots(3.26)$$

Substituting this value of $\sigma_{33}$ in the end condition (2.5) and integrating, we find that the angular speed $\omega$ prior to fracture is given by

$$\omega^2 = \frac{8[K + 2\alpha\omega\ (N-1)]}{\rho a^2[(\sqrt{1+k^2}+k)+2\alpha(N-1)]}. \qquad \ldots(3.27)$$

Equation (3.27) can be written as

$$\left(\rho\frac{Ma^2}{4} - 2\varpi\right)\left[(\sqrt{1+k^2}+k) + 2\alpha(N-1)\right]$$

$$= 2\left[K - \varpi(\sqrt{1+k^2}+k)\right] \qquad \ldots(3.28)$$

where $M = \omega^2$. It represents a hyperbola in the variables $M$ and $N$. The second factor on the left side of equation (3.28) is always positive. Also the right-hand member is positive because of (1.4).

Hence

$$\rho\frac{Ma^2}{4} - 2\varpi > 0.$$

Therefore, the endurance limit $L_1$ has the value

$$L_1 = M^{1/2} = \frac{2}{a}\left(\frac{2\varpi}{\rho}\right)^{\frac{1}{2}}.$$

From eqn. (3.27), we can deduce the value of $\omega$ which will cause fracture in a single loading operation. On setting $N=1$, in (3.27), we observe that

$$\omega^2 = \frac{8K}{\rho a^2(\sqrt{1+k^2}+k)}.$$

The nature of the graph of eqn. (3.28) for $N > 1$ is shown in Fig. 1. Endurance limit $L_1$ is given by the asymptote $M = \frac{8\varpi}{\rho a^2}$ of the hyperbola. Also it could be seen from the figure that cycles with $M < \frac{8\varpi}{\rho a^2}$ will not cause fracture in the material.

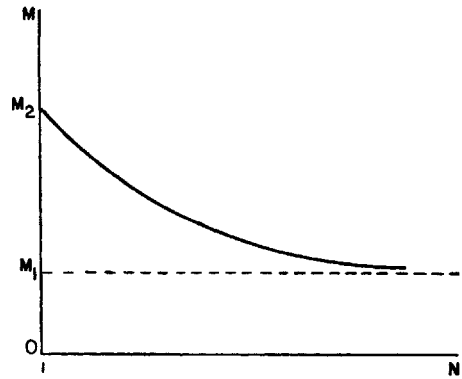


FIG. 1. Graph of simple rotation cycles.

$M_1 = \dfrac{8\varpi}{\rho a^3}$ = square of the endurance limit, $L_1$

$M_2 = \dfrac{8K}{\rho a^2(\sqrt{1+k^2}+k)}$ = square of the magnitude of peak load, $L_2$ when fracture is produced by a single loading.

## 4. CYLINDER ROTATING UNDER HYDROSTATIC PRESSURE

Denoting the hydrostatic pressure by $p$, we have

$$\sigma^1_{ij} = \sigma_{ij} - p\, g_{ij} \qquad \ldots(4.1)$$

where $\sigma_{ij}$ and $\sigma^1_{ij}$ are the stresses under atmospheric and hydrostatic pressures respectively. From (4.1) we get

$$\left.\begin{aligned} \sigma^1_{11} &= \sigma_{11} - p \\ \frac{\sigma^1_{22}}{r^2} &= \frac{\sigma_{22}}{r^2} - p \\ \sigma^1_{33} &= \sigma_{33} - p. \end{aligned}\right\} \qquad \ldots(4.2)$$

Using (4.2) in (3.12), (3.13) and (3.14), we have

$$2\sigma^1_{11} - \frac{\sigma^1_{22}}{r^2} - \sigma^1_{33} = 3A\chi$$

$$\sigma^1_{11} - \frac{2\sigma^1_{22}}{r^2} + \sigma^1_{33} = -3A\chi$$

$$\sigma^1_{11} + \frac{\sigma^1_{22}}{r^2} - 2\sigma_{33} = 6A\chi.$$

From these three equations, we find that

$$\sigma^1_{11} + \frac{\sigma^1_{22}}{r^2}$$

and $$\sigma^1_{11} - \sigma^1_{33} = 3A\chi.$$

As $3A\chi > 0$, we have $\sigma^1_{11} > \sigma^1_{33}$. Equation (3.20) now becomes

$$\frac{d\,\sigma^1_{11}}{dr} + \rho\omega^2 r = 0.$$

On solving this equation and using the boundary condition $\sigma^1_{11} = -p$ at $r=a$, we obtain

$$\sigma^1_{11} = \rho\frac{\omega^2}{2}(a^2 - r^2) - p. \qquad \ldots(4.3)$$

The principal stresses are given by the determinantal equation

$$|\,\sigma^1_{ij} - sg_{ij}\,| = 0. \qquad \ldots(4.4)$$

Equation (4.4) has the roots

$$s_1 = \sigma^1_{11},\ s_2 = \frac{\sigma^1_{22}}{r^2},\ s_3 = \sigma_{33}$$

where

$$s_1 = s_2 > s_3.$$

On substituting for $s_1$ and $s_3$ in eqn. (1.2), we get

$$(\sqrt{1+k^2} + k)\,\sigma^1_{11} - (\sqrt{1+k^2} - k)\,\sigma^1_{33} = 2K - 2\alpha\,(N-1)\,\Omega\,(\tau). \qquad \ldots(4.5)$$

Substituting for $\Omega\,(\tau)$ and $\sigma^1_{11}$ from equations (3.24) and (4.3) in (4.5) and solving for $\sigma^1_{33}$ we obtain

$$\sigma^1_{33} = \frac{\sqrt{1+k^2}+k}{\sqrt{1+k^2}-k}\left[\rho\frac{\omega^2}{2}(a^2 - r^2) - p\right]$$

$$- \frac{2}{\sqrt{1+k^2}-k}\left[K - \alpha\,(N-1)\left(\rho\frac{\omega^2 a^2}{4} - 2\varpi\right)\right]. \qquad \ldots(4.6)$$

The end condition in this case is of the form

$$\int_0^a 2\pi r\,\sigma^1_{33}\,dr = -\pi a^2 p$$

Substituting the value of $\sigma_{33}^1$ from (4 6) in the above and integrating, we get

$$\omega^2 = \frac{8}{\rho a^2}\left[\frac{(K+kp)+2\alpha\omega\ (N-1)}{(\sqrt{1+k^2}+k)+2\alpha\ (N-1)}\right]. \qquad ...(4.7)$$

Or, we have

$$\left(\rho\frac{Ma^2}{4}-2\varpi\right)\left[(\sqrt{1+k^2}+k)+2\alpha\ (N-1)\right]$$

$$= 2kp + 2\left[K - \varpi\ (\sqrt{1+k^2}+k)\right] \qquad ...(4.8)$$

where $M = \omega^2$. The second factor in the left side of (4.8) is always positive. Also the right-hand member is positive because of (1.4). Hence

$$\rho\frac{Ma^2}{4} - 2\varpi > 0.$$

Therefore the endurance limit $L_1$ has the value

$$L_1 = M^{1/2} = \frac{2}{a}\left(\frac{2\varpi}{\rho}\right)^{\frac{1}{2}}.$$

As before, on setting $N = 1$, in equation (4.7), we get

$$\omega^2 = \frac{8\ (K+kp)}{\rho\ a^2(\sqrt{1+k^2}+k)}$$

which gives the angular speed $\omega$ necessary to cause fracture in a single loading operation under hydrostatic pressure.

The nature of the graphs of the hyperbolas (3.28) and (4.8) for $N \geqslant 1$, are shown in Fig. 2, for the convenience of comparison.

It is clear from Fig. 2, that, for a given number of cycles, the load aequired to cause fracture under hydrostatic pressure must be larger than that required under atmospheric pressure.

## 5. Alternating Rotation Cycles

The alternating rotation cycle is equivalent to a system of simple cycles of peak load $-2\omega$ under a fixed load $\omega$ or simple cycles of peak load $2\omega$ under a fixed load $-\omega$. In either case, the relation (3.24) now becomes

$$\Omega\ (\tau) = \rho\omega^2a^2 - 2\varpi. \qquad ...(5.1)$$

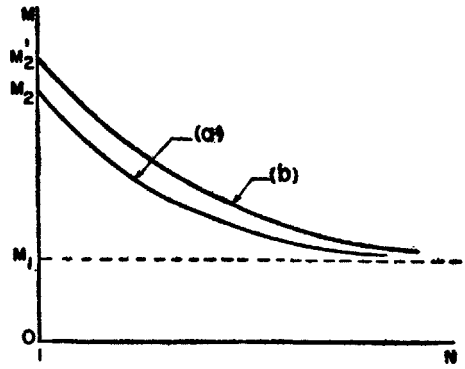


FIG. 2. (a) Graph of simple rotation cycles under atmospheric pressure.
(b) Graph of simple rotation cycles under hydrostatic pressure.

$M_1$, $M_2$ have the same values and significance as in Fig. 1.

$M_2' = \dfrac{8\,(k+kp)}{\rho a^2(\sqrt{1+k^2}+k)}$ = Square of the magnitude of peak load when fracture is produced by a single loading under hydrostatic pressure.

On substituting the value of $Q(\tau)$ given by the above equation and the value of $\sigma_{11}$ from (3.18) in eqn. (3.20), we get

$$\sigma_{33} = \frac{\sqrt{1+k^2}+k}{\sqrt{1+k^2}-k} \cdot \rho \frac{\omega^2}{2}(a^2 - r^2) - \frac{2}{\sqrt{1+k^2}-k}\left[K - \alpha(N-1) \quad (\rho\omega^2 a^2 - 2\varpi\right].$$

Using this value of $\sigma_{33}$ in the end condition (2.5) and integrating, the value of the angular speed prior to fracture is given by

$$\omega^2 = \frac{8}{\rho a^2}\left[\frac{K + 2\alpha(N-1)\varpi}{(\sqrt{1+k^2}+k) + 8\alpha(N-1)}\right]$$

in the case of alternating rotation cycles under atmospheric pressure. This can be written as

$$\left(\rho\frac{Ma^2}{4} - \frac{\varpi}{2}\right)\left[(\sqrt{1+k^2}+k) + 8\alpha(N-1)\right] = \frac{1}{2}\left[4K - \varpi(\sqrt{1+k^2}+k)\right] \qquad \ldots(5.2)$$

where $M = \omega^2$. As the second factor on the left side of the above equation is always positive and the right-hand member is positive because of (1.4), we have

$$\rho \frac{Ma^2}{2} - \frac{\varpi}{2} > 0$$

Hence the endurance limit $\mathfrak{L}_1$ has the value

$$\mathfrak{L}_1 = M^{1/2} = \frac{1}{a}\left(\frac{2\varpi}{\rho}\right)^{\frac{1}{2}}$$

The nature of the graph of equation (5.2) for $N \geqslant 1$ is shown in Fig. 3 together with the graph of simple rotation cycles, for convenience of comparison. The endurance limit $\mathfrak{L}_1$ is given by the asymptote $M = \frac{2\varpi}{\rho a^2}$ of the hyperbola. It is seen from Fig. 3 that cycles with $M \leqslant \frac{2\varpi}{\rho a^2}$ will not cause fracture in the material.

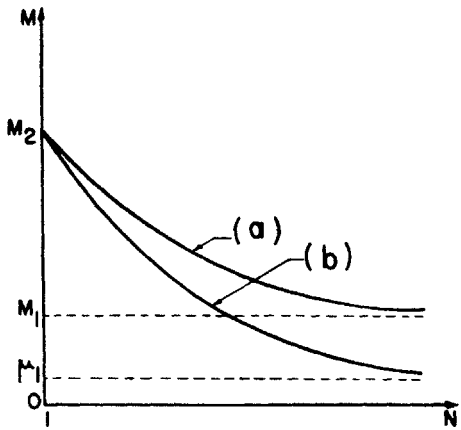


Fig. 3. (a) Graph of simple rotation cycles under atmospheric pressure.
(b) Graph of alternating rotation cycles under atmospheric pressure.

$\mu_1 = \frac{2\varpi}{\rho a^2}$ = Square of the endurance limit $\mathfrak{L}_1$.

$M_1$ and $M_1$ have the same values and significance as in Fig. 1.

Substituting the values of $\Omega(\tau)$ and $\sigma_{11}^1$ from the equations (5.1) and (4.3) respectively in (4.5), we obtain

$$\tau_{33}^1 = \frac{\sqrt{1+k^2}+k}{\sqrt{1+k^2}-k} \cdot \left[\rho \frac{\omega^2}{2}(a^2 - r^2) - p\right]$$

$$- \frac{2}{\sqrt{1+k^2}-k} \left[K - \alpha (N-1)\ (\rho\omega^2 a^2 - 2\varpi)\right].$$

Using this value of $\sigma_{33}^{1}$ in the end condition

$$\int_0^a 2\pi r\, \sigma_{33}^{1}\, dr = -\pi a^2 p$$

and integrating we get the value of the critical angular speed in the case of alternating rotation cycles under hydrostatic pressure in the form

$$\omega^2 = \frac{8}{\rho a^2}\left[\frac{(K + kp) + 2\alpha\varpi\,(N-1)}{(\sqrt{1+k^2}+k) + 8\alpha\,(N-1)}\right]$$

which can also be put in the form

$$\left(\rho\frac{Ma^2}{4} - \frac{\varpi}{2}\right)\left[(\sqrt{1+k^2}+k) + 2\alpha\,(N-1)\right]$$

$$= 2kp + \frac{1}{2}\left[4K - \omega\,(\sqrt{1+k^2}+k)\right] \qquad \ldots.(5.3)$$

from which we easily infer that the endurance limit in this case is given by

$$f_1 = M^{1/2} = \frac{1}{a}\left(\frac{2\varpi}{\rho}\right)^{\frac{1}{2}}.$$

The nature of the graph of (5.3) for $N \geq 1$ is shown in Fig. 4 together with the graph of simple rotation cycles under hydrostatic pressure.

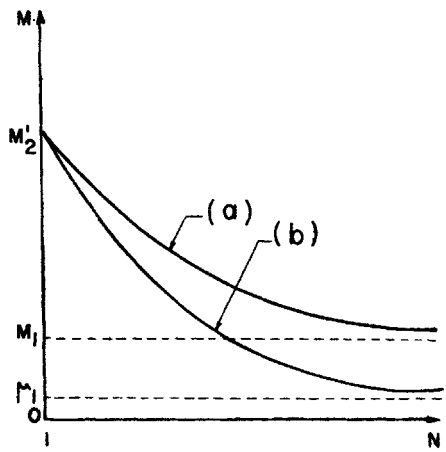


FIG. 4. (a) Graph of simple rotation cycles under hydrostatic pressure.
(b) Graph of alternating rotation cycles under hydrostatic pressure.
$\mu_1$ and $M_1$ have the same values and significance as in Fig. 3.
$M_2'$ has the same value and significance as in Fig. 2.

## ACKNOWLEDGEMENT

The authors would like to thank their Principal, Prof. P. S. Mani Sundaram, for his help and encouragement.

## REFERENCES

Hoffman, O., and Sach, G. (1953). Introduction to the Theory of Plasticity for Engineers. McGraw-Hill Book Co., Inc., New York.

Nadai, A. L. (1950). Theory of Flow and Fracture of Solids, Vol. I. McGraw-Hill Book Co., Inc., New York.

Thomas, T. Y. (1961). Plastic Flow and Fracture in Solids. Academic Press, New York.

Thomas, T. Y. (1970). Fatigue of brittle solids. *Int J. Engng Sci.*, 8, 113-36.