MAGNETOHYDRODYNAMIC AXIAL FLOW IN A TRIANGULAR PIPE UNDER TRANSVERSE MAGNETIC FIELD

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A numerical solution for steady magnetohydrodynamic flow in a triangular pipe of non-conducting walls in the presence of a transverse magnetic field perpendicular to a side has been obtained by the finite difference method. Tables and graphs are given illustrating the behaviour of induced magnetic field and velocity at various points of a section for different Hartmann numbers. Improved results have been obtained by extrapolation. An integration formula has been developed for finding out the flux through the section. The average velocity has been plotted against the Hartmann number.

1. INTRODUCTION

The magnetohydrodynamic flow problem in pipes under transverse magnetic field has become important, since it finds varied practical applications. The motion of a conducting fluid transverse to the magnetic field creates an electromotive force across the section of the pipe. The magnitude of the e.m.f. gives a measure of the flow and the system can be used as a flowmeter. A lot of experimental work is going on in this direction; magnetic field is used for measuring and controlling the flow of blood, which happens to be electrically conducting. The e.m.f. can also be used to generate electricity by connecting the ends of the section to an external load. In this form, it is called a magnetohydrodynamic generator using hot plasma as the conducting fluid. A lot of theoretical and experimental work is being done on the generation of electric energy from the kinetic energy of fluids.

Hartmann and Lazarus (1937) first made an experimental study of the flow of mercury as a conducting fluid in pipes of different cross-sections. Shercliff (1953, 1956) studied the flows from the flowmetry point of view. He solved the problem mathematically for a rectangular pipe. Gold (1962) gave an exact solution for flow in a circular pipe. Hunt and Stewartson (1965, 1967) studied similar problems for various types of walls and various sections. Gupta and Singh (1970, 1972, 1973) obtained exact solutions of some steady and unsteady flow problems in pipes of different sections.

In the present study, we have solved the corresponding problem for a pipe of triangular section with non-conducting walls. The triangle is taken to be equilateral and the magnetic field perpendicular to a side of the triangle. Numerical values of velocity and the induced field are obtained at various points of the section by the finite difference method. The results are obtained by fitting first a coarse and then a finer mesh. Improved values have been obtained by extrapolation. Numerical results are tabulated and are also presented graphically. Comparison has also been made with the numerical values obtained from the exact solution in hydrodynamic case. It is found that the magnetic field has a tendency to inhibit the flow rate. A numerical integration formula has been developed to obtain the velocity flux through a section. Average velocity values have been plotted against the Hartmann number.

2. Governing Equations

The coupled equations of magnetohydrodynamics governing the flow are

$$\nabla . \bar{v} = 0 \qquad \ldots(1)$$

$$\rho \frac{d\bar{v}}{dt} = \bar{J} \times \bar{B} - \nabla p + \eta \nabla^2 \bar{v} \qquad \ldots(2)$$

$$\nabla . \bar{B} = 0 \qquad \ldots(3)$$

$$\bar{J} = \sigma(\bar{E} + \bar{v} \times \bar{B}) \qquad \ldots(4)$$

$$\text{curl } \frac{\bar{B}}{\mu_0} = \bar{J} \qquad \ldots(5)$$

where $\bar{v}$, $\bar{B}$, $\bar{J}$, $\bar{E}$, ρ, p, η, σ are velocity, magnetic field, current density, electric field, density, pressure, coefficient of viscosity and conductivity and μ_0 is a constant having value $4\pi \times 10^{-7}$ in MKS system of units. The displacement and convection currents have been neglected. Substituting $\bar{J}$ from (5) in (2), we obtain

$$\rho \frac{d\bar{v}}{dt} = \frac{1}{\mu_0} (\text{curl } \bar{B}) \times \bar{B} - \nabla p + \eta \nabla^2 \bar{v}. \qquad \ldots(6)$$

Also, using Faraday's law

$$\text{Curl } \bar{E} = \bar{0}. \qquad \ldots(7)$$

Equations (4) and (5) give

$$\lambda \nabla^2 \bar{B} + \text{curl } (\bar{v} \times \bar{B}) = \bar{0} \qquad \ldots(8)$$

where

$$\lambda = (\mu_0 \sigma)^{-1} \qquad \ldots(9)$$

is the magnetic diffusivity.

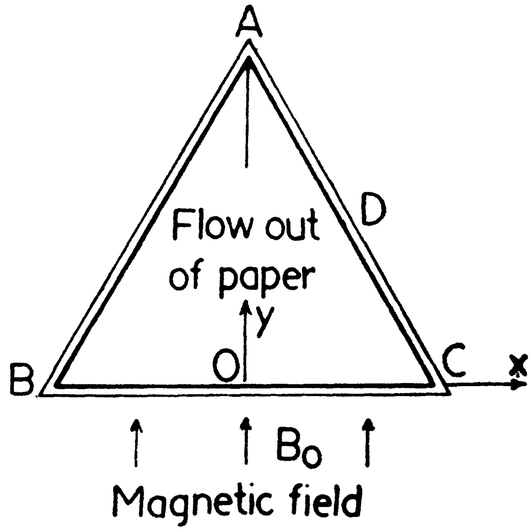

FIG. 1. Section of the channel.

The section of the pipe is shown in Fig. 1, where the magnetic field B_0 is imposed parallel to the y-axis. The flow is taking place along the z-axis, which has been taken out of paper. Since the flow induces a magnetic field along the z-axis, keeping in mind (1) and (3), the velocity and the magnetic field will be of the form

$$\bar{v} = [0,\ 0,\ v_z(x, y)] \qquad \text{...(10)}$$

$$\bar{B} = [0,\ B_0,\ B_z(x, y)]. \qquad \text{...(11)}$$

Then eqns. (6) and (8) give

$$\frac{\partial p}{\partial x} = -\frac{1}{\mu_0} B_z \frac{\partial B_z}{\partial x} \qquad \text{...(12)}$$

$$\frac{\partial p}{\partial y} = -\frac{1}{\mu_0} B_z \frac{\partial B_z}{\partial y} \qquad \text{...(13)}$$

$$\frac{\partial p}{\partial z} = \frac{B_0}{\mu_0} \frac{\partial B_z}{\partial y} + \eta \nabla^2 v_z \qquad \text{...(14)}$$

$$\lambda \nabla^2 B_z + B_0 \frac{\partial v_z}{\partial y} = 0 \qquad \text{...(15)}$$

where it has been assumed that $\partial/\partial z \equiv 0$, except for the pressure p, which from (12), (13) and (14), satisfies

$$\frac{\partial p}{\partial z} = \text{constant} = -P, \text{ say.} \qquad \text{...(16)}$$

Hence, (14) reduces to

$$\eta \nabla^2 v_z + \frac{B_0}{\mu_0} \frac{\partial B_z}{\partial y} = -P. \qquad \text{...(17)}$$

The current density $\bar{J}$ and the electric field $\bar{E}$ are given by

$$\bar{J} = \frac{1}{\mu_0} \left(\frac{\partial B_z}{\partial y}, -\frac{\partial B_z}{\partial x}, 0 \right) = \sigma \left[\bar{E} - (B_0 V_z, 0, 0) \right]. \qquad \text{...(18)}$$

3. Boundary Conditions

Complete solution of the problem requires the solution in the wall region as well as outside the walls. Let the primed and doubly primed quantities be used for the wall region and in the region outside it. In the region outside the walls, $\bar{J}'' = \bar{0}$. Hence,

$$\frac{\partial B_z''}{\partial y} = \frac{\partial B_z''}{\partial x} = 0. \qquad \text{...(19)}$$

Therefore,

$$B_z'' \equiv \text{constant}. \qquad \text{...(20)}$$

But B_z'' tends to zero, as the distance of the point from the channel increases to infinity. So,

$$B_z'' \equiv 0. \qquad \text{...(21)}$$

Similarly, in the wall region, $\bar{J}' = \bar{0}$, if the walls are made of non-conducting material. Proceeding as above, we find

$$B_z' = \text{constant}. \qquad \text{...(22)}$$

Applying the continuity of $\bar{B}$ at the outer boundary of the walls, we get

$$B_z' \equiv 0. \qquad \text{...(23)}$$

Again, using the continuity of the magnetic field at the inner boundary, we finally obtain

$$B_z = 0 \text{ on the inner boundary}. \qquad \text{...(24)}$$

Further, the nonslip condition at the walls gives

$$v_z = 0 \text{ on the inner boundary}. \qquad \text{...(25)}$$

4. Transformation of Equations

Introducing the nondimensional variables and parameters

$$B = \frac{B_z}{B_0},\ V = \frac{v_z}{V_0},\ V_0 = \frac{Pa^2}{\eta} \qquad ...(26)$$

$$X = \frac{x}{a},\ Y = \frac{y}{a} \qquad ...(27)$$

$$M^2 = B_0^2 a^2 \frac{\sigma}{\eta} \qquad ...(28)$$

$$R_m = V_0 \frac{a}{\lambda} \qquad ...(29)$$

where a is the side of the triangle, eqns. (15) and (17) in velocity and magnetic field are reduced to

$$\nabla^2 V + \frac{M^2}{R_m} \frac{\partial B}{\partial Y} = -1 \qquad ...(30)$$

and

$$\nabla^2 B + R_m \frac{\partial V}{\partial Y} = 0 \qquad ...(31)$$

with boundary conditions

$V = 0$ on the inner boundary ...(32)

$B = 0$ on the inner boundary. ...(33)

Note that in the nondimensional form, the side of the triangle becomes unity.

Introducing new variables ϕ and U defined by

$$\phi = V + \frac{M}{R_m} B \qquad ...(34)$$

$$U = V - \frac{M}{R_m} B \qquad ...(35)$$

Equations (30) and (31) are simplified to the noncoupled system

$$\nabla^2 \phi + M \frac{\partial \phi}{\partial Y} = -1 \qquad ...(36)$$

$$\nabla^2 U - M \frac{\partial U}{\partial Y} = -1 \qquad ...(37)$$

where

and $\left.\begin{array}{l} \phi = 0 \quad ...(38) \\ U = 0 \quad ...(39) \end{array}\right\}$ on the inner boundary.

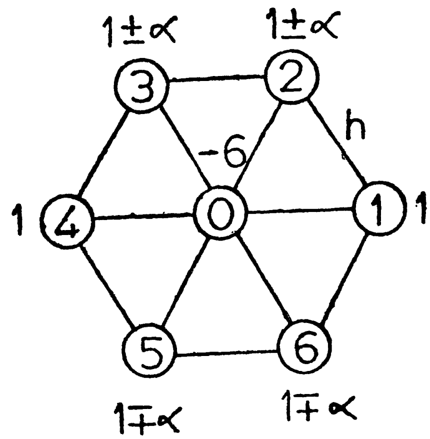

FIG. 2. Hexagonal element. Upper sign for ϕ and lower sign for U.

5. Discretization of Equations

Let us divide the section of the pipe into regular hexagonal elements of side h (nondimensional). One such element is shown in Fig. 2.

Using Taylor's expansion, we can easily obtain

$$[\nabla^2\phi + M\phi_y]_0 = \frac{2}{3h^2}(\phi_1 + \phi_2 + \phi_3 + \phi_4 + \phi_5 + \phi_6 - 6\phi_0)$$

$$+ \frac{M}{2h\sqrt{3}}(\phi_2 + \phi_3 + \phi_5 + \phi_6)$$

$$- \frac{h^2}{16}\left(\frac{\partial^4\phi}{\partial x^4} + 2\frac{\partial^4\phi}{\partial x^2\partial y^2} + \frac{\partial^4\phi}{\partial y^4} + 2M\frac{\partial^3\phi}{\partial x^2\partial y} + 2M\frac{\partial^3\phi}{\partial y^3}\right)_0$$

$+$ higher order terms in h.

So, eqn. (36) gets discretized to

$$-6\phi_0 + \phi_1 + (1+\alpha)\phi_2 + (1+\alpha)\phi_3 + \phi_4 + (1-\alpha)\phi_5 + (1-\alpha)\phi_6 = -\beta \qquad \text{...(40)}$$

where

$$\alpha = \frac{Mh\sqrt{3}}{4} \qquad \text{...(41)}$$

and

$$\beta = \frac{3h^2}{2}. \qquad \text{...(42)}$$

Note that eqn. (40) has discretization error of the order of h^2.

Similarly, eqn. (37) gets discretized to :

$$-6U_0 + U_1 + (1-\alpha)U_2 + (1-\alpha)U_3 + U_4 + (1+\alpha)U_5 + (1+\alpha)U_6 = -\beta \quad \ldots(43)$$

with discretization error again of the order of h^2.

In the present problem, there is symmetry in velocity and magnetic field about the line OA (Fig. 1). So, ϕ and U will also be symmetric about this line. Taking advantage of this symmetry, the total number of equations is sufficiently reduced. We have written down these equations for points on and to the left of OA.

If there are n such points, eqns. (40) and (43) give rise to a system of n linear simultaneous equations each for ϕ and U. In matrix form, the equations can be written as

$$[A]\{\phi\} = \{\beta\} \quad \ldots(44)$$

$$[A^*]\{U\} = \{\beta\} \quad \ldots(45)$$

where A is an $n \times n$ matrix and A^* is obtained from it by replacing α and $-\alpha$ and

$$\{\phi\} = (\phi_1, \phi_2, \ldots, \phi_n)^T \quad \ldots(46)$$

$$\{U\} = (U_1, U_2, \ldots, U_n)^T \quad \ldots(47)$$

and

$$\{\beta\} = -\beta(1, 1, \ldots, 1)^T. \quad \ldots(48)$$

Now let

$$\{V\} = (V_1, V_2, \ldots, V_n)^T \quad \ldots(49)$$

$$\{B\} = (B_1, B_2, \ldots, B_n)^T. \quad \ldots(50)$$

Using (34) and (35), we have

$$\{\phi\} = \{V\} + \frac{M}{R_m}\{B\}$$

$$\{U\} = \{V\} - \frac{M}{R_m}\{B\}.$$

Therefore,

$$2\{V\} = \{\phi\} + \{U\} = ([A]^{-1} + [A^*]^{-1})\{\beta\} \quad \ldots(51)$$

$$\frac{2M}{R_m}\{B\} = \{\phi\} - \{U\} = ([A]^{-1} - [A^*]^{-1})\{\beta\}. \quad \ldots(52)$$

If

$$[A]^{-1} = [a_{ij}], [A^*]^{-1} = [a^*_{ij}] \qquad \text{...(53)}$$

then

$$2V_i = -\frac{3h^2}{2}\sum_{j=1}^{n}(a_{ij} + a^*_{ij}) \qquad \text{...(54)}$$

$$\frac{2M}{R_m} B_i = -\frac{3h^2}{2}\sum_{j=1}^{n}(a_{ij} - a^*_{ij}). \qquad \text{...(55)}$$

6. Numerical Results

Equations (44) and (45) have been solved for $\{\phi\}$ and $\{U\}$ on IBM 1620 using a special program for band matrices. The velocity and magnetic field are then computed from (51) and (52). First the results are obtained for $h = 1/9$ and then for $h = 1/18$. All computations have been carried out for the Hartmann numbers 0, 2, 5 and 10. For $h = 1/9$, the total number of internal nodes is 28 and for $h = 1/18$, their number is increased to 136. Using symmetry, we have considered 16 nodes in the first case and 72 in the second case. These are shown in Fig. 3.

The numerical values of velocity and magnetic field are given in Tables I and II. Since the discretization error is of the order of h^2, the values at the 16 common points have been further improved using Richardson extrapolation. These are given in Table III. The behaviour of velocity and induced magnetic field on OA and BD for different values of M is shown in Fig. 5(a) – (d).

The errors in the computed values will be made of two parts : (i) the discretization error, and (ii) errors introduced while solving the finite difference equations. The former have been reduced through extrapolation. To see the effect of rounding errors while solving the equations, we have carried out calculations first with single precision and then with double precision arithmetic. It is found that the results are not affected appreciably.

The case $M = 0$ corresponds to the hydrodynamic case for which the exact solution is easily available. In this case, eqn. (30) for velocity is reduced to

$$\nabla^2 V = -1. \qquad \text{...(56)}$$

The exact solution of eqn. (56) satisfying the boundary condition is found to be

$$V = \frac{y}{2\sqrt{3}}(y^2 - 3x^2 - \sqrt{3}y + \tfrac{3}{4}). \qquad \text{...(57)}$$

The numerical values obtained are found to be in close agreement with the ones found by finite difference approximations. In fact, in this case, the approximation becomes exact. It also confirms further the negligible effect of rounding errors introduced while solving the set of linear equations.

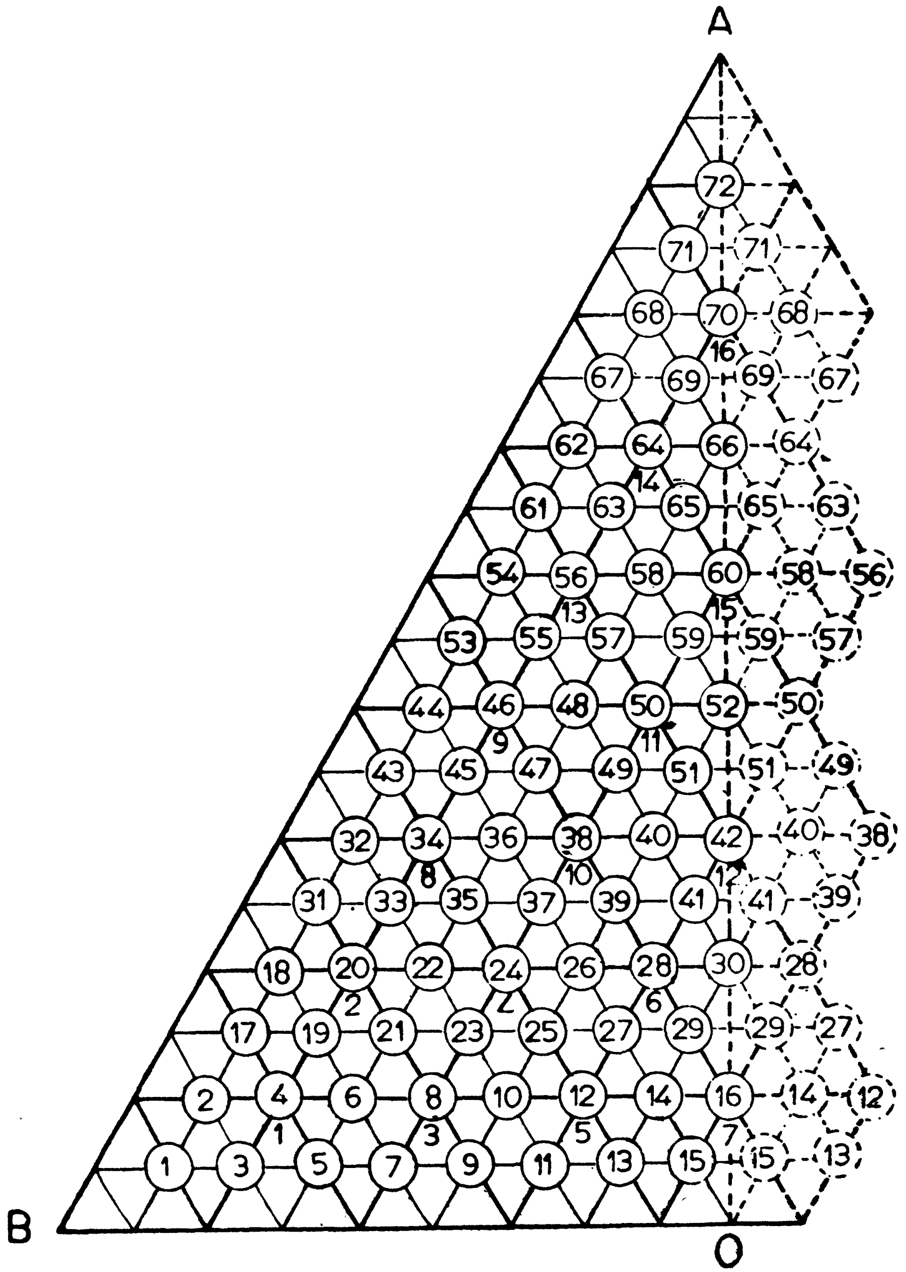

FIG. 3. Mesh points.

TABLE I

Velocity and magnetic field values
($h = 1/9$)

Point No.	Velocity × 10^1				Magnetic field × 10^3		
	$M = 0$	$M = 2$	$M = 5$	$M = 10$	$M = 2$	$M = 5$	$M = 10$
1	0.07202	0.07183	0.07093	0.06818	0.20476	0.19420	0.16542
2	0.12346	0.12254	0.11804	0.10539	0.13105	0.11675	0.07946
3	0.12346	0.12303	0.12093	0.11469	0.50795	0.48051	0.40602
4	0.20576	0.20389	0.19478	0.16957	0.39580	0.36349	0.27857
5	0.15432	0.15374	0.15089	0.14256	0.74564	0.70354	0.58980
6	0.24691	0.24449	0.23274	0.20058	0.57911	0.53437	0.41630
7	0.16461	0.16398	0.16088	0.15192	0.83369	0.78590	0.65704
8	0.15432	0.15288	0.14593	0.12731	– 0.12022	– 0.11670	– 0.10596
9	0.16461	0.16328	0.15684	0.13946	– 0.41562	– 0.38603	– 0.30822
10	0.24691	0.24420	0.23115	0.19633	– 0.04757	– 0.04395	– 0.03364
11	0.24691	0.24461	0.23345	0.20308	– 0.53027	– 0.48320	– 0.36100
12	0.27778	0.27459	0.25923	0.21839	0.00061	0.00353	0.01124
13	0.15432	0.15367	0.15042	0.14069	– 0.62454	– 0.58181	– 0.46850
14	0.12346	0.12360	0.12420	0.12468	– 0.64097	– 0.60840	– 0.51848
15	0.20576	0.20475	0.19972	0.18449	– 0.79265	– 0.73300	– 0.57507
16	0.07202	0.07248	0.07470	0.08074	– 0.41189	– 0.40199	– 0.37278

7. VELOCITY FLUX AND AVERAGE VELOCITY

To obtain the velocity flux across a section of the pipe, we have divided the section into nine similar equilateral triangles of side $3h$, each with $h = 1/9$. One such triangle is shown in Fig. 4.

After some lengthy calculations, the details of which are not given here, it has been possible to arrive at the following integration formula :

$$\iint_R V(x, y)\, dx\, dy = \frac{\Delta}{120}[54V_0 + 9(V_1 + V_2 + V_3 + V_4 + V_5 + V_6) + 4(V_7 + V_8 + V_9)] + E \qquad \ldots(58)$$

where R is the region bounded by one such elementary triangle and Δ is its area. The leading term in the error is found to be :

$$\frac{-81h^6\sqrt{3}}{5120}\left(\frac{\partial^4 V}{\partial X^4} + 2\frac{\partial^4 V}{\partial X^2 \partial Y^2} + \frac{\partial^4 V}{\partial Y^4}\right)_0 \qquad \ldots(59)$$

which is of the order of h^6.

TABLE II

Velocity and magnetic field values
($h = 1/18$)

Point No.	Velocity $\times 10^1$				Magnetic field $\times 10^3$		
	$M = 0$	$M = 2$	$M = 5$	$M = 10$	$M = 2$	$M = 5$	$M = 10$
1	0.02058	0.02056	0.02047	0.02016	0.05113	0.04885	0.04270
2	0.03858	0.03845	0.03779	0.03578	0.07469	0.06914	0.05462
3	0.03858	0.03854	0.03833	0.03758	0.13960	0.13308	0.11551
4	0.07202	0.07171	0.07021	0.06567	0.20374	0.18968	0.15292
5	0.05401	0.05396	0.05366	0.05257	0.24300	0.23098	0.19875
6	0.10031	0.09984	0.09750	0.09055	0.35523	0.33090	0.26744
7	0.06687	0.06681	0.06647	0.06522	0.34678	0.32878	0.28081
8	0.12346	0.12284	0.11979	0.11082	0.50615	0.47118	0.38020
9	0.07716	0.07710	0.07677	0.07549	0.44059	0.41690	0.35406
10	0.14146	0.14073	0.13713	0.12664	0.63959	0.59489	0.47891
11	0.08488	0.08482	0.08453	0.08331	0.51700	0.48848	0.41319
12	0.15432	0.15351	0.14952	0.13799	0.74325	0.69080	0.55502
13	0.09002	0.08998	0.08972	0.08859	0.57074	0.53875	0.45454
14	0.16204	0.16118	0.15696	0.14483	0.80873	0.75130	0.60287
15	0.09259	0.09255	0.09232	0.09125	0.59843	0.56463	0.47579
16	0.16461	0.16373	0.15944	0.14712	0.83109	0.77195	0.61918
17	0.05401	0.05370	0.05217	0.04783	0.06667	0.05919	0.04044
18	0.06687	0.06636	0.06391	0.05724	0.03002	0.02338	0.00764
19	0.10031	0.09962	0.09626	0.08678	0.19693	0.17901	0.13382
20	0.12346	0.12236	0.11709	0.10290	0.13053	0.11484	0.07683
21	0.13889	0.13783	0.13266	0.11828	0.35407	0.32408	0.24820
22	0.16975	0.16809	0.16012	0.13883	0.26225	0.23620	0.17225
23	0.16975	0.16836	0.16163	0.14308	0.51027	0.46819	0.36157
24	0.20576	0.20361	0.19333	0.16612	0.39472	0.35853	0.26893
25	0.19290	0.19125	0.18329	0.16155	0.64470	0.59204	0.45859
26	0.23148	0.22897	0.21694	0.18535	0.50517	0.46051	0.34943
27	0.20833	0.20651	0.19771	0.17384	0.74239	0.68194	0.52874
28	0.24691	0.24417	0.23107	0.19681	0.57769	0.52745	0.40216
29	0.21605	0.21414	0.20492	0.17998	0.79363	0.72905	0.56542
30	0.25206	0.24924	0.23578	0.20061	0.60290	0.55071	0.42047
31	0.07716	0.07648	0.07324	0.06474	− 0.02917	− 0.03186	− 0.03718
32	0.08488	0.08409	0.08037	0.07078	− 0.10331	− 0.09956	− 0.08882
33	0.14146	0.14003	0.13322	0.11539	0.01945	0.01223	− 0.00377
34	0.15432	0.15270	0.14499	0.12509	− 0.11975	− 0.11382	− 0.09733
35	0.19290	0.19077	0.18061	0.15419	− 0.10524	0.09232	0.06197

(Table II contd.)

TABLE II—(*contd.*)

Point No.	Velocity $\times 10^1$				Magnetic field $\times 10^3$		
	$M = 0$	$M = 2$	$M = 5$	$M = 10$	$M = 2$	$M = 5$	$M = 10$
36	0.20833	0.20595	0.19462	0.16548	− 0.09043	− 0.08387	− 0.06644
37	0.23148	0.22877	0.21585	0.18247	0.19659	0.17794	0.13274
38	0.24691	0.24393	0.22976	0.19347	− 0.04701	− 0.04078	− 0.02517
39	0.25720	0.25407	0.23922	0.20100	0.27025	0.24700	0.18976
40	0.27006	0.26669	0.25073	0.20997	− 0.01199	− 0.00632	0.00704
41	0.27006	0.26672	0.25086	0.21019	0.31089	0.28509	0.22114
42	0.27778	0.27428	0.25771	0.21542	0.00117	0.00659	0.01900
43	0.09002	0.08922	0.08540	0.07559	− 0.18418	− 0.17270	− 0.14316
44	0.09259	0.09186	0.08837	0.07924	− 0.26340	− 0.24444	− 0.19669
45	0.16204	0.16039	0.15258	0.13240	− 0.26995	− 0.24885	− 0.19558
46	0.16461	0.16312	0.15602	0.13732	− 0.41428	− 0.37907	− 0.29141
47	0.21605	0.21367	0.20236	0.17315	− 0.29829	− 0.27013	− 0.20042
48	0.21605	0.21393	0.20381	0.17712	− 0.49291	− 0.44559	− 0.32906
49	0.25206	0.24914	0.23528	0.19955	− 0.30002	− 0.26737	− 0.18779
50	0.24691	0.24438	0.23228	0.20039	− 0.52835	− 0.47365	− 0.33994
51	0.27006	0.26686	0.25166	0.21253	− 0.29571	− 0.26091	− 0.17682
52	0.25720	0.25453	0.24174	0.20805	− 0.53820	− 0.48103	− 0.34165
53	0.09259	0.09201	0.08922	0.08163	− 0.33286	− 0.30814	− 0.24598
54	0.09002	0.08965	0.08784	0.08248	− 0.38501	− 0.35744	− 0.28733
55	0.16204	0.16086	0.15518	0.13958	− 0.53674	− 0.49132	− 0.37788
56	0.15432	0.15356	0.14979	0.13858	− 0.62280	− 0.57315	− 0.44737
57	0.20833	0.20669	0.19872	0.17682	− 0.65060	− 0.59055	− 0.44150
58	0.19290	0.19186	0.18666	0.17121	− 0.75036	− 0.68662	− 0.52567
59	0.23148	0.22958	0.22038	0.19505	− 0.70086	− 0.63329	− 0.46613
60	0.20576	0.20461	0.19891	0.18193	− 0.79034	− 0.72178	− 0.54885
61	0.08488	0.08475	0.08405	0.08139	− 0.41323	− 0.38631	− 0.31641
62	0.07716	0.07726	0.07765	0.07780	− 0.41222	− 0.38934	− 0.32811
63	0.14146	0.14115	0.13949	0.13342	− 0.66012	− 0.61306	− 0.49102
64	0.12346	0.12355	0.12383	0.12290	− 0.63941	− 0.60113	− 0.49859
65	0.16975	0.16932	0.16704	0.15881	− 0.77484	− 0.71710	− 0.56757
66	0.13889	0.13897	0.13916	0.13768	− 0.71173	− 0.66805	− 0.55104
67	0.06687	0.06714	0.06838	0.07100	− 0.37861	− 0.36228	− 0.31669
68	0.05401	0.05436	0.05603	0.06021	− 0.31195	− 0.30301	− 0.27653
69	0.10031	0.10068	0.10235	0.10558	− 0.55580	− 0.53029	− 0.45893
70	0.07202	0.07247	0.07462	0.07993	− 0.41088	− 0.39858	− 0.36205
71	0.03858	0.03890	0.04043	0.04471	− 0.21626	− 0.21349	− 0.20426
72	0.02058	0.02075	0.02162	0.02426	− 0.10328	− 0.10358	− 0.10394

TABLE III

Extrapolated values of velocity and magnetic field

Point No.	Velocity $\times 10^1$				Magnetic field $\times 10^8$		
	$M = 0$	$M = 2$	$M = 5$	$M = 10$	$M = 2$	$M = 5$	$M = 10$
1	0.07202	0.07167	0.06997	0.06483	0.20340	0.18817	0.14875
2	0.12346	0.12230	0.11677	0.10207	0.13036	0.11420	0.07594
3	0.12346	0.12278	0.11941	0.10953	0.50555	0.46807	0.37159
4	0.20576	0.20352	0.19285	0.16497	0.39436	0.35688	0.26572
5	0.15432	0.15343	0.14906	0.13647	0.74245	0.68655	0.54353
6	0.24691	0.24406	0.23051	0.19555	0.57722	0.52514	0.39745
7	0.16461	0.16365	0.15896	0.14552	0.83022	0.76730	0.60656
8	0.15432	0.15264	0.14468	0.12435	− 0.11959	− 0.11286	− 0.09445
9	0.16461	0.16307	0.15575	0.13661	− 0.41383	− 0.37675	− 0.28581
10	0.24691	0.24384	0.22930	0.19252	− 0.04682	− 0.03972	− 0.02235
11	0.24691	0.24430	0.23189	0.19949	− 0.52771	− 0.47047	− 0.33292
12	0.27778	0.27418	0.25720	0.21443	0.00136	0.00761	0.02159
13	0.15432	0.15352	0.14958	0.13788	− 0.62222	− 0.57026	− 0.44033
14	0.12346	0.12353	0.12371	0.18108	− 0.63889	− 0.59871	− 0.49196
15	0.20576	0.20456	0.19864	0.12231	− 0.78957	− 0.71804	− 0.54011
16	0.07202	0.07247	0.07459	0.07966	− 0.41054	− 0.39744	− 0.35847

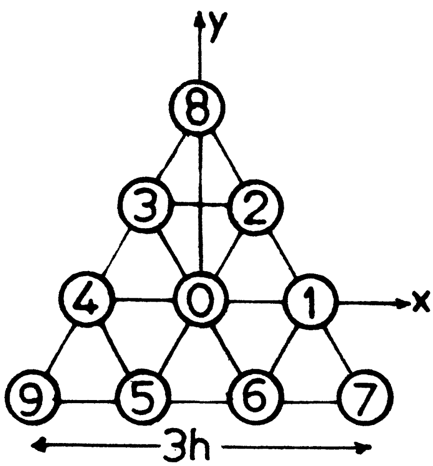

FIG. 4. Points for integration formula.

Using eqn. (58), we have found the flux through each of the nine triangles. Their sum gives the total flux. The average velocity is found by dividing the total flux by 9Δ. The results are given in Table IV. In Fig. 5(a), the average velocity is plotted against the Hartmann number.

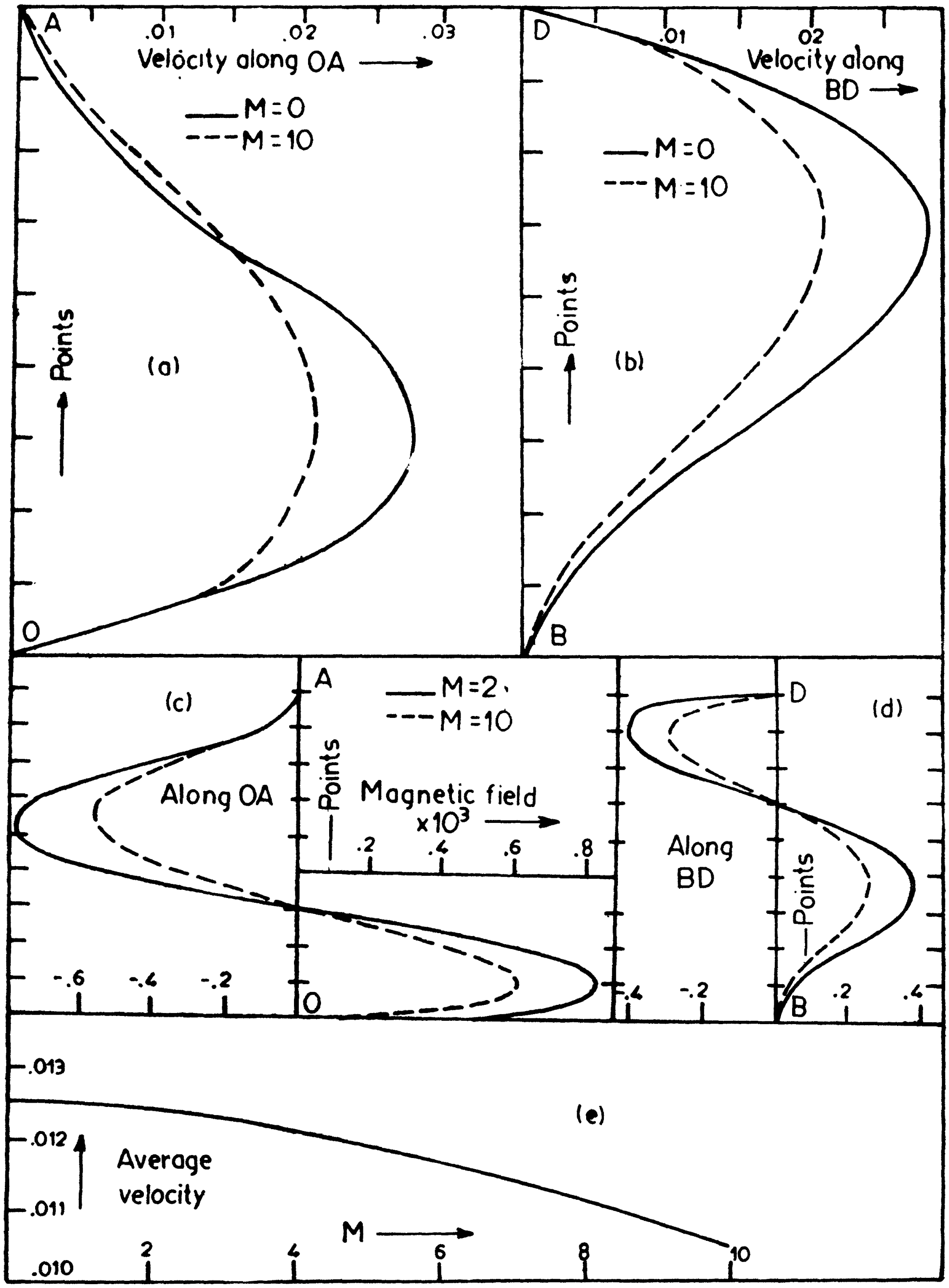

FIG. 5. Velocity, magnetic field and average velocity.

TABLE IV

Flux and average velocity values

M	0	2	5	10
Flux $\times 10^2$	0.54126	0.53678	0.51533	0.45617
Average velocity	0.01250	0.01240	0.01190	0.01053

8. DISCUSSION

From Figs. 5(a) and 5(b) it is clear that the velocity decreases with increasing values of *M*, i.e., the magnetic field has the tendency to inhibit the flow rate. The velocity profile gets flattened in the presence of magnetic field, thereby giving rise to small velocity in the middle of the channel and comparatively larger values near the boundary. This effect is depicted in Fig. 5(a). The magnetic lines of forces get stretched in the flow direction. This results in positive induced magnetic field in the lower part of the channel and negative in the upper part. This is clear from Figs. 5(c) and 5(d). The overall flow rate is reduced in the presence of magnetic field (Fig. 5e). These are all well-known characteristics of magnetohydrodynamic channel flows in the presence of traverse magnetic fields.

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