PHOTON EMISSION FROM CYLINDRICAL BODIES

by K. D. Krori and Sumita Chaudhury, *Mathematical Physics Forum, Cotton College, Gauhati*

(*Received* 5 *March* 1977; *after revision* 11 *May* 1977)

The results of a general investigation of the problem of photon emission from a cylindrical body are presented.

1. Introduction

Synge (1966) has shown that for gravitationally intense stars, only those photons which are emitted within a slender cylindrical cone can escape to infinitely. But in the limit when the surface of the star approaches the Schwarzchild radius, only the radially moving photons can escape. Banerjee (1968) found in a corresponding problem involving a cylindrically symmetric mass distribution that for photons moving in a plane normal to the cylindrical axis, there exists a critical value for the mass per unit length of the cylinder below which the photons escape to infinity for all angles of emergence. In other cases, they have to turn back somewhere in their courses, the only exception being those moving radially. Here, we present a general investigation of the problem of photon emission from a cylindrical body. We find for emission in a plane intersecting the axis of the cylinder that for all values of the mass per unit length of the cylinder, photons will turn back after traversing some distance, the only exception being for photons moving radially, which will escape to infinity. We also notice that for photons moving in a plane passing through the cylindrical axis, there exists a critical value for the mass per unit length of the cylinder above which the photons escape to infinity for all angles of emergence. In other cases, they have to turn back, only the radially moving photons being the exceptions.

2. Detailed Calculations

We consider the cylindrically symmetric metric obtained by Marder (1958).

$$ds^2 = r^{2c}\, dt^2 - r^{2(1-c)}\, d\phi^2 - A^2 r^{2c(c-1)}\, (dr^2 + dz^2) \qquad \ldots(1)$$

where A is a constant and $\frac{1}{2}c$ is the mass per unit length of the cylinder.

The equations for the null geodesics are,

$$r^{2c}\dot{t}^2 - r^{2(1-c)}\, \dot{\phi}^2 - A^2 r^{2c(c-1)}\, (\dot{r}^2 + \dot{z}^2) = 0 \qquad \ldots(2)$$

$$\ddot{z} + \frac{2c(c-1)}{r}\dot{z}\dot{r} = 0 \qquad ...(3)$$

$$\ddot{\phi} + \frac{2(1-c)}{r}\dot{r}\dot{\phi} = 0 \qquad ...(4)$$

$$\ddot{t} + \frac{2c}{r}\dot{r}\dot{t} = 0 \qquad ...(5)$$

where dots denote differentiation with respect to some affine parameter.

Equations (3), (4) and (5), after integration, become

$$\dot{z} = \frac{B}{r^{2c(c-1)}} \qquad ...(6)$$

$$\dot{\phi} = \frac{F}{r^{2(1-c)}} \qquad ...(7)$$

$$\dot{t} = \frac{D}{r^{2c}}\,. \qquad ...(8)$$

Using (6), (7) and (8) in (2), we get

$$\dot{r}^2 = \frac{D^2 r^{2c^2-4c+2}\, r^{4c(c-1)} - F^2 r^{2c^2}\, r^{4c(c-1)} - A^2 B^2 r^{2c^2}\, r^{2c^2-4c+2}}{A^2 r^{2c^2}\, r^{2c^2-4c+2}\, r^{4c(c-1)}}\,. \qquad ...(9)$$

Dividing eqn. (6) by (9),

$$\left(\frac{dz}{dr}\right)^2 = \frac{A^2 B^2 r^2}{D^2 r^{2c^2-4c+2} - F^2 r^{2c^2} - A^2 B^2 r^2}\,. \qquad ...(10)$$

Using eqns. (1), (7) and (9),

$$\frac{g_{33}}{g_{11}}\left(\frac{d\phi}{dr}\right)^2 = \frac{F^2 r^{2c^2}}{D^2 r^{2c^2-4c+2} - F^2 r^{2c^2}}\,. \qquad ...(11)$$

Now, if ψ is the angle of inclination made by the light ray with the radial direction, then

$$\cos\psi = \left[1 + \frac{g_{22}}{g_{11}}\left(\frac{dz}{dr}\right)^2 + \frac{g_{33}}{g_{11}}\left(\frac{d\phi}{dr}\right)^2\right]^{-1/2}. \qquad ...(12)$$

Using (10) and (11) in (12), we get

$$\cos\psi = \left[1 + \frac{F^2 r^{2c^2}}{D^2 r^{2c^2-4c+2} - F^2 r^{2c^2} - A^2 B^2 r^2} + \frac{A^2 B^2 r^2}{D^2 r^{2c^2-4c+2} - F^2 r^{2c^2} - A^2 B^2 r^2}\right]^{-1/2}$$

$$\operatorname{cosec}^2\psi = \frac{D^2 r^{2c^2-4c+2}}{F^2 r^{2c^2} + A^2 B^2 r^2}\,. \qquad ...(13)$$

3. Discussion

From (13),

$$D^2 = \operatorname{cosec}^2 \psi \, [F^2 r^{2(2c-1)} + A^2 B^2 r^{2c(2-c)}]. \qquad \ldots(14)$$

It is evident from eqn. (14) that for emission in a plane perpendicular to the cylindrical axis (with z constant, so that $B = 0$), we find that for $c = \frac{1}{2}$, photons may escape at all angles and these angles will remain constant during the journey. But for $c < \frac{1}{2}$, photons may escape with any angle of emergence, which will, however, vary during the journey. When $c > \frac{1}{2}$, only the radially moving photons will escape to infinity, provided D is infinite. Similar results were obtained by Banerjee (1968).

Also eqn. (14) shows that for photons moving in a plane intersecting the cylindrical axis ($F \neq 0$, $B \neq 0$), only the radially moving photons will escape to infinity. For any value of the mass per unit length of the cylinder, photons moving in other directions will turn back somewhere in their courses. We now consider the escape of photons in a plane passing through the cylindrical axis (with ϕ = constant, so that $F = 0$). In this case, for $c = 2$, photons may escape at all angles and these angles will remain constant throughout the journey. But for $c > 2$, photon may escape with any angle of emergence, which will, however, vary during the journey. When $c < 2$, photons will turn back after traversing a certain distance, and the only exception will occur for radially moving photons, when D is infinite in which case the photons will escape to infinity.

Acknowledgement

The authors express their gratitude to the Government of Assam for facilities provided at Cotton College, Gauhati, for carrying out this work.

References

Banerjee, A. (1968). Escape of photons from infinite cylinders. *J. Phys.*, A **1**, 495.

Marder, L. (1958). Gravitational waves in general relativity. *Proc. R. Soc.*, A **244**, 524.

Synge, J. L. (1966). The escape of photons from gravitationally intense stars. *Mon. Not. R. astr. Soc.*, **131**, 463.