RESPONSE OF HORIZONTAL FREE CONVECTION BOUNDARY LAYER TO PLATE TEMPERATURE OSCILLATIONS

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The free convection oscillatory heat transfer on a horizontal plate whose temperature differs from that of an ambient fluid is analysed. The basic flow is purely buoyancy induced and steady, while the oscillations in the plate temperature cause a time dependent boundary layer flow and heat transfer. Two separate solutions for high and low frequency ranges are developed. For very high frequencies, the oscillatory flow pattern is of 'shear wave' type, unaffected by mean flow and predicts a phase lead of $\pi/4$ in the local rate of heat transfer fluctuations, while the skin friction has a phase lead of $\pi/2$.

1. INTRODUCTION

The main aim of the present investigation is to study the effect of plate temperature oscillations on the free convection flow and heat transfer along a semi-infinite horizontal plate. We consider a thin flat plate extending from 0 to ∞ in the $\bar{x}$-direction, where $\bar{x}$ measures the distance along the plate. The plate is heated to a temperature $\bar{T}_w(\bar{x})$ and is placed in an ambient fluid at temperature $\bar{T}_\infty$. Thus, the basic flow is entirely due to buoyancy forces over a horizontal plate, the temperature of which differs from that of the free stream. The effect of buoyancy force is to induce a longitudinal pressure gradient which causes the flow. It is an interesting flow in its own right, yielding a steady outer flow for the boundary layer equations as a result of free convection alone. Moreover, this problem is amenable to experimentation in a laboratory. It was investigated by Krishnamurty *et al.* (1973). The steady free convection flow over the plate is perturbed due to a superimposed time varying plate temperature oscillation $\epsilon \bar{T}_w(\bar{x}) \cos \bar{\omega}\bar{t}$. The magnitude of the oscillation is assumed to be small (of order ϵ) compared with the mean velocity induced by the convection. This enables us to employ the technique of linearization for perturbation due to oscillation.

The basic steady flow is considered using Karman Pohlhausen method and an approximate solution to be used in the subsequent study of unsteady flow is obtained. Recently, Gill *et al.* (1965) considered the similarity solution of the free convection boundary layer equations for steady flow over a semi-infinite horizontal plate. The approximate result differs from the numerical exact result by less than 2%. In the analysis of unsteady flow, we have considered two different solutions : one for the low frequency range and the other for the high frequency range. The method of solving the problem is essentially the same as developed by Lighthill (1954). Here, of course, we have extended Lighthill's technique to obtain a series solution for high frequencies. For very high frequencies, the oscillatory component of the temperature field is of the simple shear-wave type unaffected by the mean flow predicting a phase lead of $\pi/4$ in the Nusselt number at the plate. The fluctuating skin friction in the shear wave flow is found to have phase lead of $\pi/2$ over the plate temperature oscillation.

2. Basic Equations

The non-dimensional boundary layer equations for two-dimensional free convection incompressible unsteady flow past a semi-infinite horizontal plate are (Muhuri and Maiti 1967) :

$$\frac{\partial}{\partial y}\left[\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y}\right] = \frac{\partial^3 u}{\partial y^3} - \frac{\partial G}{\partial x} \quad ...(1)$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad ...(2)$$

$$\frac{\partial G}{\partial t} + u\frac{\partial G}{\partial x} + v\frac{\partial G}{\partial y} = \frac{1}{\sigma}\frac{\partial^2 G}{\partial y^2} \quad ...(3)$$

where σ is the Prandtl number.

We shall consider the case when the plate temperature oscillates harmonically in time about a non-zero mean. The corresponding boundary conditions are :

$$y = 0 : u = v = 0,\ G = G_w(x) = G_{w0}(x)\,(1 + \epsilon e^{i\omega t}),\ \epsilon \ll 1 \quad ...(4)$$

$$y \to \infty : u \to 0,\ G \to 0 \quad ...(5)$$

where ω is the dimensionless frequency, and $G_{w0}(x)$ is yet an unspecified function of x. The plate temperature $[G_{w0}(x) + \epsilon\, G_{w0}(x)\, e^{i\omega t}]$ consists of a basic steady part $G_{w0}(x)$ with superimposed weak time varying distribution $\epsilon\, G_{w0}(x)\, e^{i\omega t}$. We now assume u, v, G as the sum of steady and small oscillating components.

$$u = u_s + \epsilon u_1\, e^{i\omega t},\ v = v_s + \epsilon v_1\, e^{i\omega t},\ G = G_s + \epsilon G_1\, e^{i\omega t} \quad ...(6)$$

where u_s, v_s and G_s are the steady mean flow and satisfy equations

$$\frac{\partial}{\partial y}\left(u_s\frac{\partial u_s}{\partial x} + v_s\frac{\partial u_s}{\partial y}\right) = \frac{\partial^3 u_s}{\partial y^3} - \frac{\partial G_s}{\partial x} \quad ...(7)$$

$$\frac{\partial u_s}{\partial x} + \frac{\partial v_s}{\partial y} = 0 \quad ...(8)$$

$$u_s \frac{\partial G_s}{\partial x} + v_s \frac{\partial G_s}{\partial y} = \frac{1}{\sigma} \frac{\partial^2 G_s}{\partial y^2} \qquad \text{...(9)}$$

with boundary conditions

$$y = 0 : u_s = v_s = 0, \; G_s = G_{w0}; \quad y \to \infty : u_s, \; G_s \to 0 \qquad \text{...(10)}$$

while u_1, v_1 and G_1 satisfy the equations

$$\frac{\partial}{\partial y}\left(i\omega u_1 + u_1 \frac{\partial u_s}{\partial x} + u_s \frac{\partial u_1}{\partial x} + v_s \frac{\partial u_1}{\partial y} + v_1 \frac{\partial u_s}{\partial y} \right) = \frac{\partial^3 u_1}{\partial y^3} - \frac{\partial G_1}{\partial x} \qquad \text{...(11)}$$

$$\frac{\partial u_1}{\partial x} + \frac{\partial v_1}{\partial y} = 0 \qquad \text{...(12)}$$

$$i\omega G_1 + u_1 \frac{\partial G_s}{\partial x} + u_s \frac{\partial G_1}{\partial x} + v_1 \frac{\partial G_1}{\partial y} + v_s \frac{\partial G_1}{\partial y} = \frac{1}{\sigma} \frac{\partial^2 G_1}{\partial y^2} \qquad \text{...(13)}$$

with boundary conditions

$$y = 0 : u_1 = v_1 = 0, \; G_1 = G_{w0}; \quad y \to \infty : u_1 \; G_1 \to 0. \qquad \text{...(14)}$$

3. Steady State Solution

We consider the set of eqns. (7) - (10), which describe the steady free convection boundary layer flow and heat transfer on a horizontal plate. Integrating eqns. (7) and (9) from $y = 0$ to $y = \delta$, we get

$$\left(\frac{\partial^2 u_s}{\partial y^2}\right)_{y=0} + \frac{\partial}{\partial x} \int_0^\delta G_s dy = 0 \qquad \text{...(15)}$$

$$\frac{1}{\sigma} \left(\frac{\partial G_s}{\partial y}\right)_{y=0} + \frac{\partial}{\partial x} \int_0^\delta (G_s u_s) \, dy = 0 \qquad \text{...(16)}$$

where $\delta(x)$ denotes the dimensionless boundary layer thickness. Consistent with the boundary conditions, we take expressions for u_s and G_s as

$$u_s = \frac{B}{18} \frac{y}{\delta} \left(1 - \frac{y}{\delta} \right)^3 + A \frac{y}{\delta} \left(1 - \frac{y}{\delta} \right)^3 \left(1 + \frac{y}{\delta} \right) \qquad \text{...(17)}$$

$$G_s = G_{w0} \left(1 - \frac{y}{\delta} \right)^3 \left(1 + \frac{y}{\delta} \right) \qquad \text{...(18)}$$

where $B = \delta^3 \dfrac{d}{dx} G_{w0}$. A and δ are functions of x to be determined from eqns. (15) and (16).

Substituting u_s and G_s into (15) and (16) and solving the resulting equations with $G_{w0} = x^n$, we get

$$\left.\begin{aligned} \delta &= \delta_1\, x^{(2-n)/5} \\ A &= A_1\, x^{(1+2n)/5} \end{aligned}\right\} \qquad \ldots(19)$$

where

$$\delta_1 = \left[\frac{\dfrac{2}{\sigma}}{\left\{\dfrac{2n^2+n}{15120} + \dfrac{(2n+1)(9-7n)}{315000}\right\}}\right]^{1/5} \qquad \ldots(20)$$

$$A_1 = \frac{9-7n}{300}\,\delta_1^3$$

and $B = n\,\delta_1^3\, x^{(1+2n)/5}$. It should be noted that the mean horizontal velocity induced by steady convection is zero at the leading edge of the plate. The analysis is, therefore, invalid for small values of x. However, the mean velocity grows as $x^{(1+2n)/5}$ and so the results are valid far downstream. We find from the above expressions that the boundary layer thickness varies as $x^{(2-n)/5}$ and decreases as the Prandtl number increases. It is also clear that $\frac{y}{x^{(2-n)/5}}$ is the similarity variable for free convection from a horizontal plate. The values of δ_1 and A_1 have been calculated for various values of σ and n and are given in Table I, which shows that the boundary layer thickness decreases as n and σ increase.

TABLE I

n	0	0.5	1	0	0.5	1	0	0.5	1
	$\sigma = 0.72$			$\sigma = 1$			$\sigma = 2.4$		
δ_1	5.642	4.619	4.064	5.284	4.325	3.805	4.435	3.631	3.194
A_1	3.389	1.807	0.447	4.425	1.484	0.367	2.617	0.877	0.217

The results of practical interest are the rate of heat transfer and skin friction characteristics of the problem. The non-dimensional skin friction τ_s at the plate is obtained as

$$\tau_s = x^{(3n-1)/5}\left(n\,\frac{\delta_1^2}{18} + \frac{A_1}{\delta_1}\right). \qquad \ldots(21)$$

The rate of heat transfer at the plate in non-dimensional form, i.e. Nusselt number N_s, is determined as

$$N_s = x^{(6n-2)/5}\,\frac{2}{\delta_1}. \qquad \ldots(22)$$

The values of Nusselt number and dimensionless skin friction are given in Table II. The data in Table II show that both the skin friction and the Nusselt number increase as n increases. It is also seen that the Nusselt number at the plate increases, while the skin friction decreases as σ increases.

TABLE II

n	0	0.5	1	0	0.5	1	0	0.5	1
	$\sigma = 0.72$			$\sigma = 1$			$\sigma = 2.4$		
$\frac{\tau_s}{x^{(3n-1)/5}}$	0.955	0.984	1.038	0.837	0.862	0.901	0.590	0.608	0.635
$\frac{N_s}{x^{(6n-2)/5}}$	0.355	0.433	0.492	0.379	0.462	0.526	0.451	0.551	0.626

4. Low Frequency Solution

Equations (11) - (14) are next considered. It is convenient to write u_1, v_1, G_1 as the sum of in-phase and out-of-phase components :

$$u_1 = u_r + iu_2, \quad v_1 = v_r + iv_2, \quad G_1 = G_r + iG_2. \qquad \ldots(23)$$

Substituting (23) into (11) - (14) and separating real and imaginary parts, we get

$$\frac{\partial}{\partial y}\left[-\omega u_2 + u_r \frac{\partial u_s}{\partial x} + u_s \frac{\partial u_r}{\partial x} + v_r \frac{\partial u_s}{\partial x} + v_s \frac{\partial u_r}{\partial y}\right] = -\frac{\partial G_r}{\partial x} + \frac{\partial^3 u_r}{\partial y^3} \qquad \ldots(24)$$

$$\frac{\partial u_r}{\partial x} + \frac{\partial v_r}{\partial y} = 0 \qquad \ldots(25)$$

$$-\omega G_2 + u_s \frac{\partial G_r}{\partial x} + u_r \frac{\partial G_s}{\partial x} + v_r \frac{\partial G_s}{\partial y} + v_s \frac{\partial G_r}{\partial y} = \frac{1}{\sigma}\frac{\partial^2 G_r}{\partial y^2} \qquad \ldots(26)$$

with boundary conditions

$$y = 0 : u_r = v_r = 0, \ G_r = G_{w0}; \quad y \to \infty : u_r \to 0, \ G_r \to 0 \qquad \ldots(27)$$

and

$$\frac{\partial}{\partial y}\left[\omega u_r + u_2 \frac{\partial u_s}{\partial x} + u_s \frac{\partial u_2}{\partial y} + v_2 \frac{\partial u_s}{\partial y} + v_s \frac{\partial u_2}{\partial y}\right] = -\frac{\partial G_2}{\partial x} + \frac{\partial^3 u_2}{\partial y^3} \qquad \ldots(28)$$

$$\frac{\partial u_2}{\partial x} + \frac{\partial v_2}{\partial y} = 0 \qquad \ldots(29)$$

$$\omega G_r + u_s \frac{\partial G_2}{\partial x} + u_2 \frac{\partial G_s}{\partial x} + v_2 \frac{\partial G_s}{\partial y} + v_s \frac{\partial G_2}{\partial y} = \frac{1}{\sigma}\frac{\partial^2 G_2}{\partial y^2} \qquad \ldots(30)$$

with boundary conditions

$$y = 0 : u_2 = v_2 = 0, \ G_2 = 0 : y \to \infty : u_2 \to 0, \ G_2 \to 0. \quad \text{...(31)}$$

It is obvious that when $\omega = 0$, u_r, v_r, G_r give the quasi-steady solution of the problem. This can be seen from the fact that the same equations can be obtained by substituting $u = u_s + \epsilon u_r$, $v = v_s + \epsilon v_r$ and $G = G_s + \epsilon G_r$ in steady boundary layer equations. Following Lighthill (1954), u_r, v_r and G_r can be easily obtained as

$$\left.\begin{aligned} u_r &= \tfrac{2}{5} u_s + \tfrac{1}{5} y \frac{\partial u_s}{\partial y} \\ v_r &= \tfrac{1}{5} v_s + \tfrac{1}{5} y \frac{\partial v_s}{\partial y} \\ G_r &= G_s + \tfrac{1}{5} y \frac{\partial G_s}{\partial y}. \end{aligned}\right\} \quad \text{...(32)}$$

With the help of (17) and (18), u_r and G_r become

$$u_r = \tfrac{1}{5} \frac{y}{\delta} \left[\frac{B}{16} \left(1 - \frac{y}{\delta} \right)^2 \left(1 - 2 \frac{y}{\delta} \right) + A \left(1 - \frac{y}{\delta} \right)^2 \left(3 - 2 \frac{y}{\delta} - 7 \frac{y^2}{\delta^2} \right) \right] \quad \text{...(33)}$$

$$G_r = \tfrac{1}{5} G_{w0} \left(1 - \frac{y}{\delta} \right)^2 \left(5 - 2 \frac{y}{\delta} - 9 \frac{y^2}{\delta^2} \right). \quad \text{...(34)}$$

We shall again employ Karman Pohlhausen method to solve eqns. (28) - (31). Integrating eqns. (28) and (30) from $y = 0$ to $y = \delta$, we obtain the momentum and energy integral equations as

$$\frac{\partial}{\partial x} \int_0^\delta G_2 \, dy + \left(\frac{\partial^2 u_2}{\partial y^2} \right)_{y=0} = 0 \quad \text{...(35)}$$

$$\omega \int_0^\delta G_r \, dy + \frac{\partial}{\partial x} \int_0^\delta (u_s G_2 + u_2 G_s) \, dy + \frac{1}{\sigma} \left(\frac{\partial G_2}{\partial y} \right)_{y=0} = 0. \quad \text{...(36)}$$

We assume the following polynomials for u_2 and G_2 :

$$U_2 = \frac{y}{\delta} \left(1 - \frac{y}{\delta} \right)^3 \left\{ \frac{\omega \delta^2}{90} \left(\frac{B}{6} + 3A \right) - A_5 \left(1 + \frac{y}{\delta} \right) \right\} \quad \text{...(37)}$$

$$G_2 = -\tfrac{1}{3} \frac{y}{\delta} \left(1 - \frac{y}{\delta} \right)^3 \left\{ \tfrac{1}{2} \sigma \omega G_{w0} \delta^2 + B_5 \left(1 + 3 \frac{y}{\delta} \right) \right\} \quad \text{...(38)}$$

where A_5 and B_5 are functions of x to be determined.

Substituting u_s, u_2, G_s, G_r and G_2 into (35) and (36) and solving the resulting equations, we get

$$A_5 = \omega F x, \quad B_5 = \omega E x^{(3n+4)/5}$$

where E and F are given by

$$\frac{120F}{\delta_1^2} - \frac{n\delta_1^3}{3} - 6A_1 - \frac{3+n}{10}\sigma\delta_1^3 - \frac{2n+6}{5}E\delta_1 = 0 \quad \ldots(39)$$

$$\frac{6}{25}\delta_1 + \frac{7+4n}{5}\left[\frac{11}{45360}\left(\tfrac{1}{6}n\delta_1^6 + 3A_1\delta_1^3\right) - \frac{17}{630}F\delta_1 - \frac{317}{54422}\sigma\delta_1^6 n - \frac{13}{15120}\sigma\delta_1^3 A_1 - \frac{19}{136080}En\delta_1^4 - \frac{139}{41580}BA_1\delta_1\right] - \tfrac{1}{6}\delta_1 - \frac{E}{3\sigma\delta_1} = 0. \quad \ldots(40)$$

These algebraic equations are solved for various values of σ and n and the corresponding values of E and F are given in Table III.

TABLE III

n	0	0.5	1	0	0.5	1	0	0.5	1
	$\sigma = 0.72$			$\sigma = 1$			$\sigma = 2.4$		
E	−5.701	−81.619	−99.114	−6.470	−99.035	−120.401	−9.740	−166.531	−202.814
F	8.631	−84.577	−82.580	6.927	−85.587	−83.321	4.371	−87.104	−84.432

When the frequency of oscillation is small, the longitudinal component of velocity and temperature field may be written as

$$u = u_s + \epsilon R_1 \cos(\omega t + \alpha_1) \quad \ldots(41)$$

$$G = G_s + \epsilon R_2 \cos(\omega t + \alpha_2) \quad \ldots(42)$$

where

$$R_1 = (u_r^2 + u_2^2)^{1/2}, \quad R_2 = (G_r^2 + G_2^2)^{1/2}$$

$$\alpha_1 = \tan^{-1}\frac{u_2}{u_r}, \quad \alpha_2 = \tan^{-1}\frac{G_2}{G_r}.$$

The non-dimensional skin friction at the plate τ is obtained as

$$\tau = \tau_s + \epsilon R_3 \cos(\omega t + \alpha_3) \quad \ldots(43)$$

where

$$R_3 = \left[\frac{9}{25}\left\{\frac{\delta_1^2 n\, x^{(3n-1)/5}}{18} + \frac{A_1}{\delta_1} x^{(3n-1)/5}\right\}^2 + \left\{\frac{\omega\delta_1\, x^{(3+n)/5}}{90}\left(\frac{n\delta_1^3}{6} + 3A_1\right) - \frac{\omega F}{\delta_1} x^{(3+n)/5}\right\}^2\right]^{1/2} \quad \ldots(44)$$

$$\alpha_3 = \tan^{-1}\left[\frac{5\omega\left\{\frac{\delta_1^2}{90}\left(\frac{n\delta_1^3}{6} + 3A\right) - F\right\}}{3\left\{\frac{\delta_1^3 n}{18} + A_1\right\}} x^{(4-2n)/5}\right]. \quad \ldots(45)$$

The Nusselt number at the plate can be written as

$$N = N_s + \epsilon R_4 \cos(\omega t + \alpha_4) \quad \ldots(46)$$

where

$$R_4 = \left[\left(\frac{12\, x^{(6n-2)/5}}{5\delta_1}\right)^2 + x^{4(2n+1)/5}\left(\frac{\sigma\omega\delta_1}{6} + \frac{\omega E}{3\delta_1}\right)^2\right]^{1/2} \quad \ldots(47)$$

$$\alpha_4 = \tan^{-1}\left[\frac{5\omega\, x^{(4-2n)/5}}{36}\left(\frac{\sigma\delta_1^2}{2} + E\right)\right]. \quad \ldots(48)$$

Equations (45) and (48) indicate that both $\tan\alpha_3$ and $\tan\alpha_4$ vary as $x^{(4-2n)/5}$. For $n = 0$, this result is the same as obtained by Muhuri and Maiti (1967). In Figs. 1 and 2 $\tan\alpha_3$ and $\tan\alpha_4$ have been plotted against n for different values of σ. It is found that the skin friction has a phase lag for $n = 0$, but as n increases, the phase angle becomes positive, showing a maximum phase lead between $n = 0$ and $n = 1$ and then it decreases as n increases. It is found that α_4 is positive for $n = 0$ and then it decreases to a minimum value between $n = 1$ and $n = 2$, after which it increases with n. This shows that the oscillating component of Nusselt number at the plate has a phase lead for $n = 0$ and then as n increases, it lags behind the plate temperature oscillations. The amplitudes R_3 and R_4 increase as ω increases. The minimum values of R_3 and R_4 corresponding to $\omega = 0$ are given in Table IV for different values of σ and n.

TABLE IV

n	0	0.5	1	0	0.5	1	0	0.5	1
	$\sigma = 0.72$			$\sigma = 1$			$\sigma = 2.4$		
$\frac{R_3}{x^{(3n-1)/5}}$	0.573	0.590	0.623	0.502	0.518	0.541	0.354	0.365	0.381
$\frac{R_4}{x^{(6n-2)/5}}$	0.425	0.520	0.591	0.454	0.555	0.631	0.541	0.661	0.751

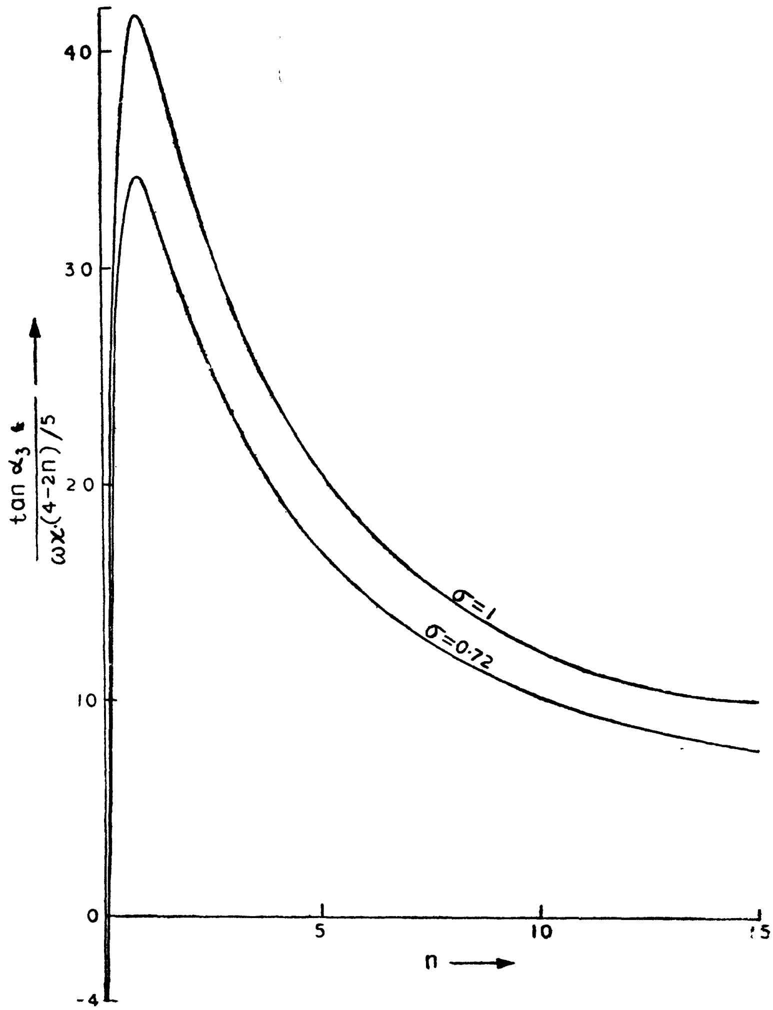

FIG. 1. Phase angle of low frequency oscillating skin friction $\frac{\tan \alpha_3}{x^{(4-2n)/5}}$ versus n for $\sigma = 0.72, 1.0$.

The data in Table IV indicate that R_3 decreases, while R_4 increases as σ increases. Both R_3 and R_4 increase with n. It is interesting to note that the ratios R_3/τ_s $(= \frac{3}{5})$ and R_4/N_s $(= \frac{6}{5})$ remain constant for all values of σ and n. For $\epsilon = 0.1$ and for all values of σ and n, the quasisteady skin friction is 6% of the steady mean value, while the quasisteady Nusselt number is 12% of the steady mean value.

5. HIGH FREQUENCY

Introducing the variable $z = \sqrt{\omega} y$ in eqns. (11) – (13), we get

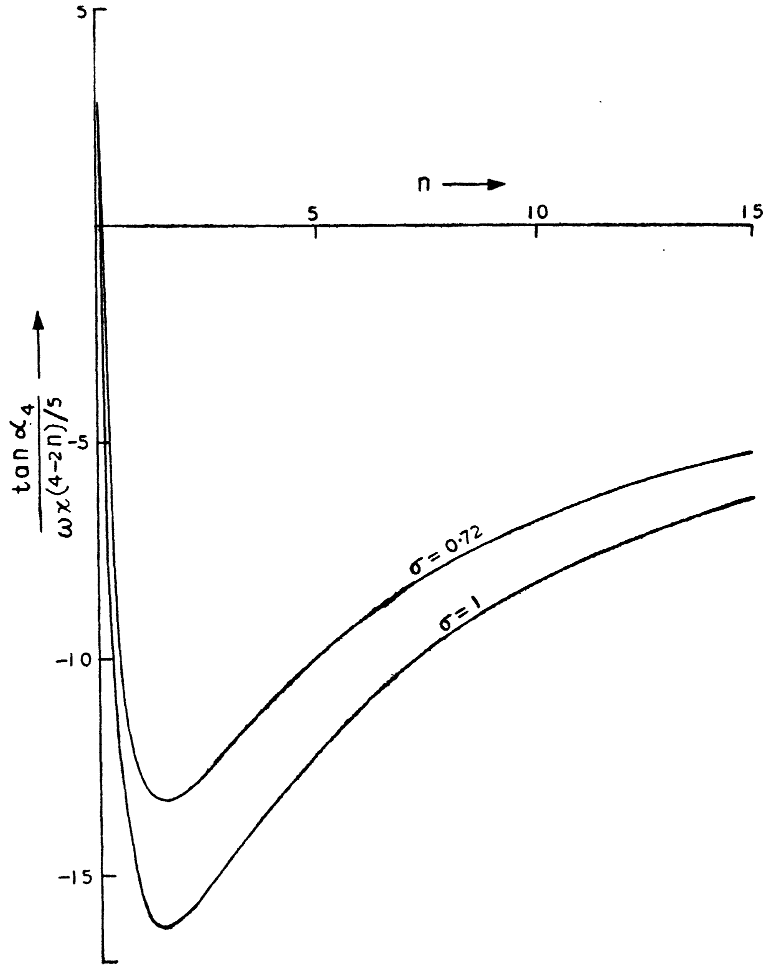

FIG. 2. Phase angle of low frequency oscillating Nusselt number $\frac{\tan \alpha_4}{x^{(4-2n)/5}}$ versus n for $\sigma = 0.72, 1.0$.

$$\frac{\partial^3 u_1}{\partial z^3} - i\,\frac{\partial u_1}{\partial z} = \frac{1}{\sqrt{\omega}}\left(v_s \frac{\partial u_1}{\partial z} + v_1 \frac{\partial u_s}{\partial z}\right) + \frac{1}{\omega}\left(u_1 \frac{\partial u_s}{\partial x} + u_s \frac{\partial u_1}{\partial x}\right) + \frac{1}{\omega^{3/2}} \frac{\partial G_1}{\partial x} \quad ...(49)$$

$$\frac{\partial v}{\partial z} + \frac{1}{\sqrt{\omega}} \frac{\partial u}{\partial x} = 0 \quad ...(50)$$

$$\frac{1}{\sigma} \frac{\partial^2 G_1}{\partial z^2} - iG_1 = \frac{1}{\sqrt{\omega}}\left(u_1 \frac{\partial G_s}{\partial z} + v_s \frac{\partial G_1}{\partial z}\right) + \frac{1}{\omega}\left(u_1 \frac{\partial G_s}{\partial x} + u_s \frac{\partial G_1}{\partial x}\right). \quad ...(51)$$

This suggests that for large ω, a solution may be developed in inverse power of $\sqrt{\omega}$. We write

$$u_1 = u_{10} + \frac{1}{\sqrt{\omega}} u_{11} + \frac{1}{\omega} u_{12} + \frac{1}{\omega^{3/2}} u_{13} + \dots \qquad \dots(52)$$

$$G_1 = G_{10} + \frac{1}{\sqrt{\omega}} G_{11} + \frac{1}{\omega} G_{12} + \frac{1}{\omega^{3/2}} G_{13} + \dots . \qquad \dots(53)$$

Substituting (52) and (53) into (49) and (51) and equating coefficients of like powers of ω, and solving these equations, we get

$$G_{10} = G_{w0}\, e^{-\sqrt{i\sigma}\, z} \qquad \dots(54)$$

$$u_{13} = \frac{nx^{n-1}}{i\sqrt{i\sigma}\,(1-\sigma)} \left\{ e^{-\sqrt{i\sigma}\, z} - e^{-\sqrt{i}\, z} \right\} \qquad \dots(55)$$

showing that these solutions are unaffected by the steady mean flow and the velocity u_{13} comprises two wave modes. Interaction terms, however, appear in the subsequent higher approximation. The longitudinal component of velocity and the temperature field in the shear wave flow are

$$u = u_s + \epsilon \frac{nx^{n-1}}{\sqrt{\sigma}\sqrt{1-\sigma}\,\omega^{3/2}} \left[e^{-\sqrt{(\omega/2)}\, y} \cos\left(\omega t + \frac{\pi}{4} - \sqrt{\frac{\omega}{2}}\, y\right) - e^{-\sqrt{(\sigma\omega/2)}\, y} \cos\left(\omega t + \frac{\pi}{4} - \sqrt{\frac{\sigma\omega}{2}}\, y\right)\right] \qquad \dots(56)$$

$$G = G_s + \epsilon G_{w0}\, e^{-\sqrt{(\sigma\omega/2)}\, y} \cos\left(\omega t - \sqrt{\frac{\sigma\omega}{2}}\, y\right). \qquad \dots(57)$$

The non-dimensional skin friction for the shear wave is now obtained as

$$\tau_h = \tau_s - \epsilon \frac{nx^{n-1}}{\sqrt{\sigma}\,(1+\sqrt{\sigma})\,\omega} \cos\left(\omega t + \frac{\pi}{2}\right). \qquad \dots(58)$$

Equation (58) shows that the amplitude of the fluctuating skin friction in the shear wave flow decreases with frequency, while it increases with n. The phase of the fluctuating skin friction is ahead of that of fluctuation of the surface temperature by $\pi/2$.

The Nusselt number at the plate for high frequency solution is given by

$$N_h = N_s + \epsilon \sqrt{\sigma\omega} \cos\left(\omega t + \frac{\pi}{4}\right). \qquad \dots(59)$$

This shows that Nusselt number at the plate for a very high frequency has a phase lead of $\pi/4$ over the plate temperature oscillation. It may also be remarked that its amplitude increases with frequency. This result is the same as obtained by Singh and Sinha (1971) for combined free and forced convection flow on a horizontal plate.

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