TRANSITION THEORY OF CREEP IN ROTATING DISKS OF VARIABLE THICKNESS

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The transition theory of creep has been employed to find the creep stresses in rotating disks with variable thickness. No assumption has been made about the form of the creep laws, which is a characteristic feature of the transition theory developed by Seth (1962). However, the results obtained correspond to the creep law with Tresca rule and agree with those derived by Wahl (1957).

1. INTRODUCTION

The transition theory of creep proposed by Seth (1962) has been applied to the creep phenomenon in rotating disks of variable thickness. The essence of the transition theory lies in recognizing the fact that creep is a transition phenomenon and should be reflected in the asymptotic nature of the solution of the differential equation governing the deformations at the critical points. The role of the generalized strain measure in this connection becomes very important. Seth (1966) proposed a very general strain measure, which can take into account not only the creep phenomenon, but also those of relaxation, failure and fatigue. Here, we shall confine ourselves to a generalized strain measure with only one exponent. A discussion with a more generalized strain measure for rotating cylinders has been given by Seth (1974). It is needless to emphasize that we do not assume any creep law, since any such assumption is superfluous in the transition theory treatment of the creep phenomenon. However, the results obtained here correspond to the creep law with Tresca rule and agree with those derived by Wahl (1957).

2. TRANSITION POINTS

We consider a circular disk of variable thickness with a central hole rotating with an angular velocity ω. Let the internal and external radii of the disk be a and b respectively. We treat the problem as one of plane strain and steady state creep. Due to the symmetry of the problem, we can take the displacements in cylindrical coordinates (r, θ, z) as

$$u = r(1 - \beta), \quad v = 0, \quad w = 0 \qquad \text{...(2.1)}$$

where β is a function of r only.

The components of the generalized strain measure with only one exponent are given by

$$\left.\begin{aligned} e_{rr} &= \frac{1}{n}\{1 - (r\beta' + \beta)^n\} \\ e_{\theta\theta} &= \frac{1}{n}(1 - \beta^n) \\ e_{zz} &= e_{r\theta} = e_{\theta z} = e_{rz} = 0. \end{aligned}\right\} \quad \ldots(2.2)$$

The stress-strain relations

$$T_{ij} = \lambda \triangle \delta_{ij} + 2\mu e_{ij} \quad \ldots(2.3)$$

with strains given by eqns. (2.2) give

$$\left.\begin{aligned} T_{rr} &= \lambda \triangle + \frac{2\mu}{n}\{1 - (r\beta' + \beta)^n\} \\ T_{\theta\theta} &= \lambda \triangle + \frac{2\mu}{n}(1 - \beta^n) \\ T_{zz} &= \lambda \triangle \\ T_{r\theta} &= T_{\theta z} = T_{rz} = 0 \end{aligned}\right\} \quad \ldots(2.4)$$

where

$$\triangle = \frac{1}{n}\{2 - \beta^n - (r\beta' + \beta)^n\}.$$

The equation of equilibrium to be satisfied by the stresses is

$$\frac{d}{dr}(rhT_{rr}) - hT_{\theta\theta} + \rho\omega^2 r^2 h = 0 \quad \ldots(2.5)$$

where $h = h(r)$ represents the variable thickness of the disk.

Substituting the stresses from eqns. (2.4) in eqn. (2.5), we get

$$(r\beta' + \beta)^{n-1}(r\beta'' + 2\beta') + (1 - c)\beta^{n-1}\beta' + \frac{c}{nr}\{(r\beta' + \beta)^n - \beta^n\}$$
$$- \frac{c\rho\omega^2 r}{2\mu} + \frac{1}{h}\frac{dh}{dr}\{(2 - c) - (1 - c)\beta^n - (r\beta' + \beta)^n\} = 0$$

which on integration yields

$$(r\beta' + \beta)^n + (1 - c)\beta^n + c\int\{(r\beta' + \beta)^n - \beta^n\}\frac{dr}{r} - \frac{nc\rho\omega^2 r^2}{4\mu}$$
$$+ n\int\frac{1}{h}\frac{dh}{dr}\{(2 - c) - (1 - c)\beta^n - (r\beta' + \beta)^n\}\,dr = A_0 \quad \ldots(2.6)$$

where $c = \dfrac{2\mu}{(\lambda + 2\mu)}$ and A_0 is a constant of integration.

The substitutions given by

$$\frac{c\rho\omega^2}{2\mu} = \omega_0^2, \quad s = \frac{r\omega_0}{A_0}, \quad y = \frac{\beta}{A_0^{1/n}}, \quad H = \frac{h}{A_0} \qquad \ldots(2.7)$$

transform eqn. (2.6) into

$$\left(s\frac{dy}{ds} + y\right)^n + (1-c)\,y^n + c\int\left\{\left(s\frac{dy}{ds} + y\right)^n - y^n\right\}\frac{ds}{s} - \frac{n}{2}s^2$$
$$+ n\int \frac{1}{H}\frac{dH}{ds}\left\{\frac{2-c}{A_0} - (1-c)\,y^n - \left(s\frac{dy}{ds} + y\right)^n\right\} ds = 1. \qquad \ldots(2.8)$$

The substitution $s = \exp(\alpha)$ followed by differentiation with respect to α and further substitutions (Seth 1963)

$$y = s^{2/n}P, \quad P' + \frac{2}{n}P = Q, \quad Q = FP, \quad P^{-n} = T \qquad \ldots(2.9)$$

in eqn. (2.8) leads to

$$\left[F(1+F)^n + (1-c)\,F + \frac{c}{n}\{(1+F)^n - 1\} - T\right.$$
$$\left. + \frac{1}{H}\frac{dH}{d\alpha}\{Te^{-2\alpha} - (1-c) - (1+F)^n\}\right]\frac{d\beta}{dF} + \beta F(1+F)^{n-1} = 0. \qquad \ldots(2.10)$$

This shows that the transition points of β are

$$F = -1, \quad F = \pm\infty.$$

Here, we will consider only the transition point $F = -1$. It can be easily shown that T is given by

$$(1-c) + \frac{c}{n} + T + \frac{1}{H}\frac{dH}{d\alpha}\{Te^{-2\alpha} - 1 - c\} = 0 \qquad \ldots(2.11)$$

at the transition point $F = -1$.

3. Creep Stresses

We define the transition function R as

$$R = \frac{nT_{\theta\theta}}{2\mu} \qquad \ldots(3.1)$$

and consider its asymptotic value at the transition point $F = -1$. The general discussion on the role of transition functions in conjunction with the transition points is fully demonstrated in the case of a spherical shell under internal pressure (Hulsurkar 1966).

From eqns. (2.4), (2.7), (2.9) and (3.1), we get

$$R = \frac{A_0 y^n}{c}\left\{\frac{2-c}{A_0}\frac{T}{s^2} - 1 - (1-c)(1+F)^n\right\}. \qquad \ldots(3.2)$$

Taking logarithmic differentiation of eqn. (3.2) and using eqn. (2.10), we arrive at

$$\frac{d\log R}{d\log s} = nF + \left[\frac{2-c}{A_0} s \frac{d}{ds}\left(\frac{T}{s^2}\right) - n(1-c)\left\{F(1+F)^n + (1-c)F + \frac{c}{n}\{(1+F)^n - 1\} - T - \frac{1}{H}\frac{dH}{d\alpha} \times \{Te^{-2\alpha} - (1-c) - (1+F)^n\}\right\}\right] \Big/ \left[\frac{2-c}{A_0}\frac{T}{s^2} - 1 - (1-c)(1+F)^n\right]. \qquad \ldots(3.3)$$

Letting $F \to -1$ in eqn. (3.3), using eqn. (2.11), allowing $A_0 \to 0$ and noting that $(A_0 s)$ is finite as $A_0 \to 0$, we get

$$\frac{d\log R}{d\log s} = -n$$

which gives

$$T_{\theta\theta} = Ar^{-n} \qquad \ldots(3.4)$$

where A is a constant, after recalling eqns. (2.7) and (3.1).

Substitution of eqn. (3.4) in eqn. (2.5) leads to

$$rhT_{rr} + \frac{2\mu}{n} A \int hr^{-n}\,dr + \rho\omega^2 \int hr^2\,dr = B \qquad \ldots(3.5)$$

where B is a constant of integration.

Now the boundary conditions in our problem are

$$\left.\begin{aligned} T_{rr} &= 0 \quad \text{at } r = a \\ T_{rr} &= T_0 \quad \text{at } r = b \end{aligned}\right\} \qquad \ldots(3.6)$$

where T_0 is the stress acting on the outer edge due to the externally applied load.

The stress T_{rr} given by eqns. (3.5) subjected to the boundary conditions given by eqn. (3.6) yields

$$rhT_{rr} + \frac{2\mu}{nc} A \int_a^r hr^{-n}\,dr + \rho\omega^2 \int_a^r hr^2\,dr = 0 \qquad \ldots(3.7a)$$

$$A = -\frac{nc\rho\omega^2}{2\mu}\left\{\frac{\int_a^b hr^2\,dr + T_0 bh}{\int_a^b hr^{-n}\,dr}\right\}. \quad \ldots(3.7\text{b})$$

From eqns. (3.4) and (3.7), we get

$$\text{(i)} \quad T_{\theta\theta} = \rho\omega^2\left\{\frac{\int_a^b hr^2 dr + T_0 bh}{\int_a^b hr^{-n}\,dr}\right\}\frac{1}{r^n}$$

$$\text{(ii)} \quad T_{rr} = \frac{\rho\omega^2}{rh}\left[\left\{\frac{\int_a^b hr^2\,dr + T_0 bh}{\int_a^b hr^{-n}\,dr}\right\}\int_a^r hr^{-n}\,dr - \int_a^r hr^2 dr\right]. \quad \ldots(3.8)$$

Also, T_{zz} can be found from $T_{zz} = \frac{1-c}{2-c}(T_{rr} + T_{\theta\theta})$.

These expressions are the same as those given by Wahl (1957), who obtained them by assuming the creep law with Tresca rule. The measure exponent here is inverse of the creep index.

4. A Special Case

We consider a special case, when the profile of the disk is given by a power function

$$h = kr^m \quad \ldots(4.1)$$

where k and m are constants.

Substituting the function h from eqns. (4.1) in eqns. (3.8), we get

$$\left.\begin{aligned} T_{\theta\theta} &= \frac{k_1 A (T_{\theta\theta})_{av}}{k(b^{k_1} - a^{k_1})}\frac{1}{r^n} \\ T_{rr} &= \frac{1}{kr^{m+1}}\left\{\frac{(T_{\theta\theta})_{av}\, A(r^{k_1} - a^{k_1})}{(b^{k_1} - a^{k_1})} - \rho\omega^2 I\right\} \end{aligned}\right\} \quad \ldots(4.2)$$

where

$$k_1 = m + 1 - n$$

$$(T_{\theta\theta})_{av} = \frac{kb^{m+1}T_0}{A} + \frac{\rho\omega^2 I_0}{A}$$

$$I = \int_a^r hr^2\, dr = \frac{k}{m+3}(r^{m+3} - a^{m+3})$$

$$I_0 = \int_a^b hr^2\, dr = \frac{k}{m+3}(b^{m+3} - a^{m+3})$$

$$A = \int_a^b h\, dr = \frac{k}{m+1}(b^{m+1} - a^{m+1}). \qquad \ldots(4.3)$$

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