ON THE LAMINAR FLOW ALONG A POROUS VERTICAL WALL*

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A method based on the momentum integral technique, assuming a fourth degree velocity profile, is presented for the case of a porous vertical wall to obtain the values of boundary layer and film thickness for different values of suction/injection parameter. A comparative study is also made with the Haugen's solution in which only second degree velocity profile is taken for the case of a non-porous vertical wall.

NOMENCLATURE

g = acceleration due to gravity

$h(x)$ = film thickness

h_0 = initial film thickness

$h^*(x)$ = dimensionless film thickess

u = velocity component in x-direction

U_0 = uniform entrance velocity

u_s = velocity outside the boundary layer

v = velocity component in y-direction

v_0 = uniform suction/injection velocity

x = co-ordinate axis along the direction of motion

x^* = x/h_0, dimensionless x-coordinate

y = coordinate axis normal to the wall

$\delta(x)$ = boundary layer thickness

δ^* = δ/h_0, dimensionless boundary layer thickness

η = y/δ, dimensionless y-coordinate

ν = μ/ρ, kinematic coefficient of viscosity

λ = $v_0\delta/\nu$, suction/injection parameter

ϕ = $3\nu U_0/gh_0^2$, dimensionless parameter

$\xi^2 = 1 + \dfrac{2gh_0x^*}{U_0^2}$, dimensionless variable

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Introduction

The flow of a film falling along a vertical wall is of considerable importance in engineering and technology. Such films occur frequently in wetted wall towers, evaporation and gas absorption experiments and trickling filters. It is observed that due to viscous effects, the boundary layer is formed along the wall.

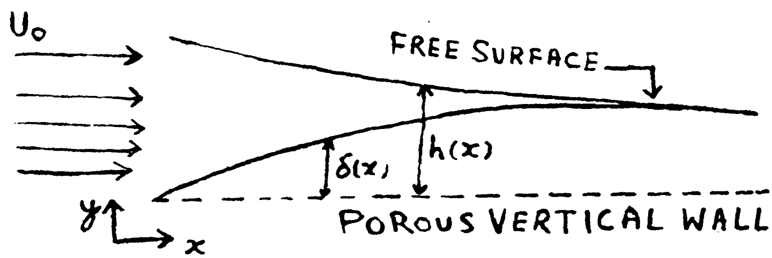
Bruley (1965), Hassan (1967) and Haugen (1968) have discussed the laminar flow along a vertical wall and obtained the boundary layer thickness, film thickness and entrance length. Haugen (1968) presented a pertinent analysis of the laminar accelerating flow of a thin film for which the surface tension effects were neglected.

In the present investigation, we have studied the same problem for porous wall by assuming the fourth degree velocity profile by the Karman Pohlohagan technique.

The laminar flow along a solid wall and the results of Haugen (1968) follow as particular cases. A comparison with Haugen's solution for the second degree velocity profile has also been made. Boundary layer and film thickness have been presented graphically.

Equations of Motion

We consider a steady laminar flow of a viscous incompressible fluid along a vertical porous wall. It is assumed that the fluid enters with a uniform velocity U_0 and initial film thickness h_0. The velocity U_s outside the boundary layer increases along the length because of the gravitational force considered.



Schematic diagram of flow considered.

Assuming the axis of x along the wall and y-axis perpendicular to it, the boundary layer equations governing the motion can be written as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \text{...(1)}$$

and

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = g + \nu\frac{\partial^2 u}{\partial y^2} \qquad \text{...(2)}$$

with the boundary conditions :

$$\left.\begin{aligned} y = 0 &: u = 0, \quad v = v_0 \\ y = \delta &: u = u_s, \frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} = \frac{\partial^3 y}{\partial y^3} = 0. \end{aligned}\right\} \qquad \ldots(3)$$

Analysis

Integrating eqns. (1) and (2), we have

$$\int_0^\delta u\, dy + U_s(h - \delta) = \text{constant} \qquad \ldots(4)$$

and

$$\frac{d}{dx}\int_0^\delta u^2\, dy + v_0 U_s - U_s \frac{d}{dx}\int_0^\delta u\, dy = g\delta - \nu\left(\frac{\partial u}{\partial y}\right)_{y=0}. \qquad \ldots(5)$$

For the solution of eqns. (5), let us assume a polynomial of degree four as

$$\frac{u}{U_s} = a_0 + a_1\eta + a_2\eta^2 + a_3\eta^3 + a_4\eta^4 \qquad \ldots(6)$$

where $\eta = \frac{y}{\delta}$ (δ being the boundary layer thickness).

Determining coefficients of different powers of η with the boundary conditions in (3), we get

$$\frac{u}{U_s} = 4\eta - 6\eta^2 + 4\eta^3 - \eta^4. \qquad \ldots(7)$$

Using eqn. (7), eqn. (5) becomes

$$\frac{d}{dx}\left(\frac{32\delta}{45} U_s^2\right) + v_0 U_s - U_s \frac{d}{dx}\left(\frac{4\delta}{5} U_s\right) = g\delta - \frac{4U_s\nu}{\delta} \qquad \ldots(8)$$

or

$$\frac{4}{45} U_s^2 \frac{d\delta}{dx} - \frac{19}{45}\delta U_s \frac{dU_s}{dx} - v_0 U_s + g\delta = \frac{4U_s\nu}{\delta}. \qquad \ldots(9)$$

For the case of zero pressure gradient, Euler's equation may be written as

$$\frac{d}{dx}\left(\tfrac{1}{2} U_s^2\right) = g \qquad \ldots(10)$$

i.e., $U_s^2 = U_0^2 + 2gx.$ $\qquad \ldots(11)$

Substituting the value of U_s from eqn. (11) into eqn. (9), we get

$$\frac{4}{45}(U_0^2 + 2gx)\frac{d\delta}{dx} + \frac{26}{45}g\delta - V_0(U_0^2 + 2gx)^{1/2} = \frac{4\nu}{\delta}(U_0^2 + 2gx)^{1/2} \quad \ldots(12)$$

or

$$\frac{d}{d\xi}(\delta^{*2}) + 13\frac{\delta^{*2}}{\xi} = \frac{15}{2}\lambda\phi^2 + 30\phi^2 \quad \ldots(13)$$

where

$$\left.\begin{aligned} \delta^* &= \frac{\delta}{h_0}, \quad \xi^2 = 1 + \frac{2gh_0\bar{x}}{U_0^2} \\ x^* &= \frac{x}{h_0}, \quad \phi^2 = \frac{3\nu U_0}{gh_0^2} \text{ and } \lambda = \frac{V_0\delta}{\nu}\,. \end{aligned}\right\} \quad \ldots(14)$$

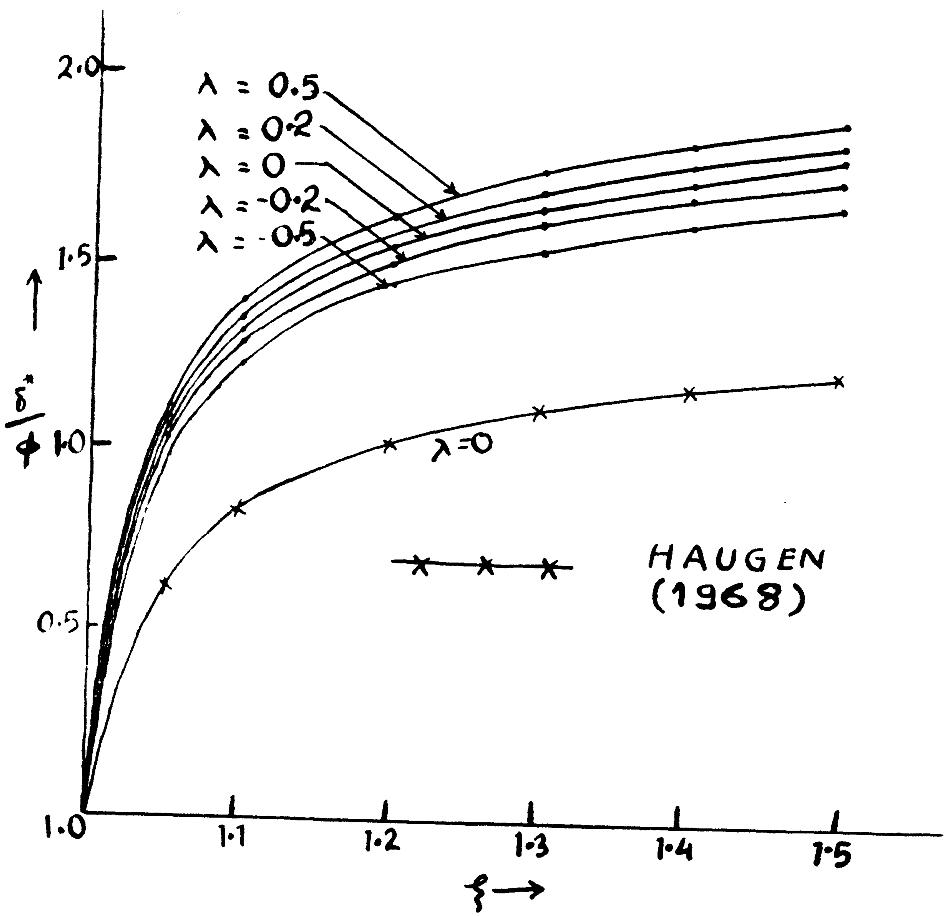


FIG. 1. Boundary layer thickness versus ξ.

Here, λ is suction/injection parameter according as $v_0 < 0$ or $v_0 > 0$ (v_0 is so adjusted as to give constant λ). Equation (13) is a linear differential equation whose solution is given by

$$\delta^{*2} = \frac{15}{28} \phi^2(\lambda + 4) (\xi - \xi^{-13}). \quad \ldots(15)$$

SOLUTION FOR FILM THICKNESS

Using eqn. (4), the film thickness $h(x)$ is given by

$$h(x) = \delta(x) + \frac{U_0 h_0}{U_s} - \int_0^\infty \frac{u}{U_s} dy \quad \ldots(16)$$

or

$$h^* = \frac{h}{h_0} = \delta^* + \frac{U_0}{U_s} - \frac{1}{h_0} \int_0^\infty \frac{u}{U_s} dy. \quad \ldots(17)$$

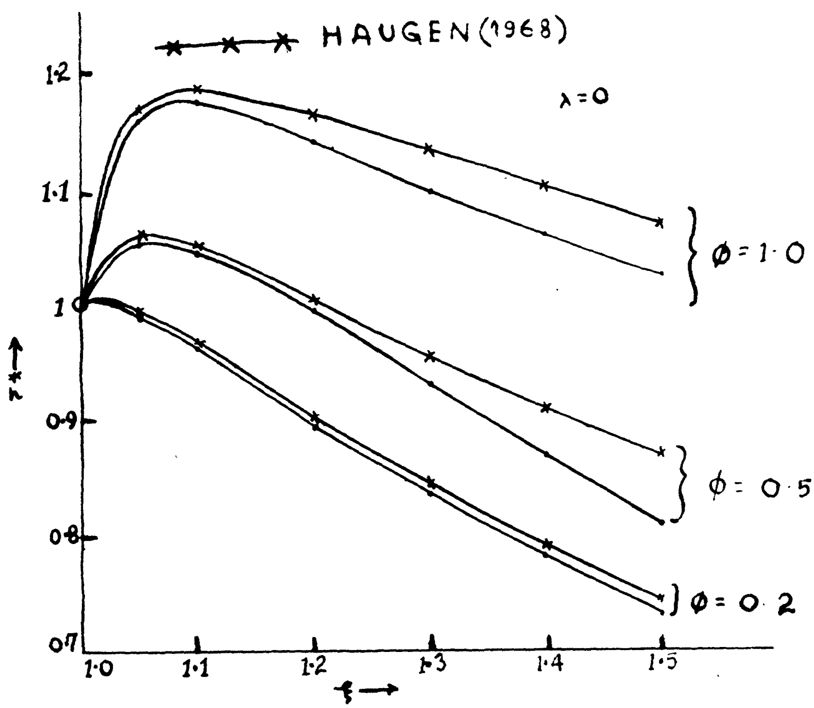


FIG. 2. Film thickness versus ξ.

Since $$\frac{U_s}{U_0} = \xi \text{ and } \int_0^\delta \frac{u}{U_s}\, dy = \frac{4\delta}{5} \qquad ...(18)$$

$$h^* = \delta^* + \frac{1}{\xi} - \frac{4\delta^*}{5}$$

or

$$h^* = \frac{1}{\xi} + \frac{\delta^*}{5}. \qquad ...(19)$$

Substituting the value of δ^* from eqn. (15), the final form of the dimensionless film thickness may be written as

$$h^* = \frac{1}{\xi} + \frac{1}{5}\left[\frac{15}{28}\phi^2(\lambda + 4)(\xi - \xi^{-13})\right]^{1/2} \qquad ...(20)$$

$$= \frac{1}{\xi} + \frac{\phi}{5}\left[\frac{15}{28}(\lambda + 4)(\xi - \xi^{-13})\right]^{1/2}. \qquad ...(21)$$

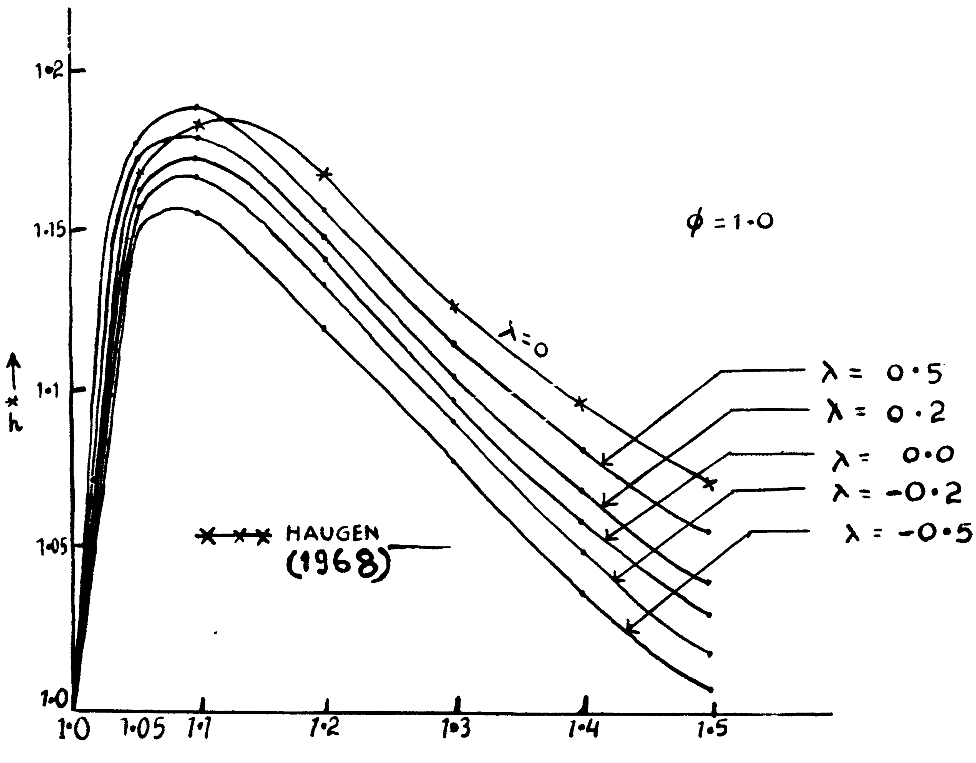


FIG. 3. Film thickness versus ξ.

Numerical Discussion

For different values of the suction/injection parameter λ, the graphs for the boundary layer thickness have been drawn against δ, the dimensionless parameter for the length along the wall (Fig. 1). Graphs for Haugen's solution (1968) have also been drawn for the solid wall and a comparison of the same with those of our solution for the solid wall has been made. Fig. 2 shows a comparison of the film thickness due to our method and that due to Haugen's method (1968) for the solid wall. It is interesting to note that in our case, the film thickness takes a steady shape for a smaller distance along the wall, which appears to be nearer the practical problems.

The effect of suction/injection parameter λ on film thickness is represented in Fig. 3. For every value of λ, h^* first increases rapidly and then decreases continuously. Due to suction, the film thickness decreases.

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