ON THE RECURRENCY OF PRODUCT OF TWO SURFACES

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The product of two surfaces is considered by many authors and its gravitational properties are discussed. Here, it is shown that if the product space is 1-recurrent, the constituent surfaces reduce to be of constant scalar curvature, which is a necessary condition for it to represent a non-null electromagnetic field. It is also found that 2-recurrent product of two surfaces is a flat space-time devoid of gravitational significance.

The line-element is given by

$$ds^2 = -A(dx^2 + dy^2) - B(dz^2 - dt^2) \qquad \ldots(1)$$

where $A = A(x, y)$, $B = B(z, t)$. The variables x, y, z and t correspond to x^1, x^2, x^3 and x^4 respectively. The only two non-vanishing components of Riemann curvature tensor are

$$R_{\dot{1}\dot{2}}{}^{12} = \frac{A_{11} + A_{22}}{2A^2} - \frac{A_1^2 + A_2^2}{2A^3} = \alpha \text{ (say)}$$

$$R_{\dot{3}\dot{4}}{}^{34} = \frac{B_{33} - B_{44}}{2B^2} + \frac{B_4^2 - B_3^2}{2B^3} = \beta \text{ (say)}.$$

The lower suffixes 1, 2, 3 and 4 after an unknown function denote partial differentiation. The condition for a space-time to be recurrent (1-recurrent) is given by

$$R_{\dot{p}\dot{q};i}{}^{rs} = K_i R_{\dot{p}\dot{q}}{}^{rs},\ i = (1, 2, 3, 4) \qquad \ldots(2)$$

where K_i is the recurrence vector, semi-colon (;) denotes the covariant differentiation and $R_{\dot{p}\dot{q}}{}^{rs}$ is the Riemann curvature tensor of the second kind. The non-vanishing components of $R_{\dot{p}\dot{q};i}{}^{rs}$ are

$$R_{\dot{1}\dot{2};a}{}^{12} = \alpha_a,\ R_{\dot{3}\dot{4};b}{}^{34} = \beta_b. \qquad \ldots(3)$$

The lower suffixes a and b take the values (1, 2) and (3, 4) respectively. Using (3) in (2), we get the following equations

$$R_{\dot{3}\dot{4};a}{}^{34} = 0 = K_a R_{\dot{3}\dot{4}}{}^{34} \qquad \ldots(4)$$

$$R_{\dot{3}\dot{4};\,b}^{\;\;34} = \beta_b = K_b R_{\dot{3}\dot{4}}^{\;\;34} \qquad \ldots(5)$$

$$R_{\dot{1}\dot{2};\,b}^{\;\;12} = 0 = K_b R_{\dot{1}\dot{2}}^{\;\;12} \qquad \ldots(6)$$

$$R_{\dot{1}\dot{2};\,a}^{\;\;12} = \alpha_a = K_a R_{\dot{1}\dot{2}}^{\;\;12}. \qquad \ldots(7)$$

From (4) and (6), we have

$$K_1 = K_2 = K_3 = K_4 = 0. \qquad \ldots(8)$$

Now, we get

$$R_{pq\,;\,i}^{\cdot\cdot\;rs} = 0$$

i.e., $\alpha_a = \beta_b = 0$, which reduce α and β to constants. Thus,

$$R_{\dot{1}\dot{2}}^{\;\;12} = -R_1^1 = -R_2^2 = \frac{A_{11} + A_{22}}{2A^2} - \frac{A_1^2 + A_2^2}{2A^3} = c_1$$

and

$$R_{\dot{3}\dot{4}}^{\;\;34} = -R_3^3 = -R_4^4 = \frac{B_{33} - B_{44}}{2B^2} + \frac{B_4^2 - B_3^2}{2B^3} = c_2$$

where c_1 and c_2 are constants. From the above, it follows that R_i^j are constants and consequently the surfaces corresponding to the space-time (1) are of constant scalar curvatures. We may choose $c_1 = -c_2$, so that $R = 0$, which is the necessary and sufficient condition for the product space to represent a non-null electromagnetic field (Singh and Sharan 1965). Hence, the condition for 1-recurrency is that the two surfaces constituting the four-fold (1) be of constant scalar curvature, which is a necessary condition for the line-element to represent a non-null electromagnetic field. It is also found that the 2-recurrent product of two surfaces is a flat space-time.

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