A TWO-UNIT WARM STANDBY SYSTEM WITH REPAIR AND PREVENTIVE MAINTENANCE

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A reliability model has been considered with a view to make a theoretical study of the effect of warm-standby redundancy on the time to failure of the system, the units of which are subject to repair and preventive maintenance. It is assumed that the failure, repair, inspection and preventive maintenance (preventive repair) time distributions of each unit for the system are all arbitrary. The Laplace Steiltjes Transform (LST) of the first system down is derived. Three particular cases have been discussed.

Introduction

Recently Osaki and Asakura (1970) carried out reliability analysis of a two unit standby system with repair and preventive maintenance. The units are assumed to be identical subject to general failure, repair, inspection and preventive maintenance time distributions. The main purpose of this paper is to investigate the case of two unit system with warm stand-by.

A warm stand-by unit can fail while the primary or the basic unit is still operating. Failure time distribution of a standby unit is different from an operating unit, thus a standby unit assumes the failure time distribution of an operating unit once the basic unit fails and the standby unit is put in place, its life time is counted afresh from the instant of switching over.

More recently some work was carried out in this direction by Kapur and Kapoor (1974) taking into account also the demand pattern of the system in the sense of Gaver (1964).

Description of the Model

Consider a system composed of two dissimilar units (the two units have equal status though their statistical properties are different, i.e. only a unit performs system's function perfectly). Appropriately labelling the units, we may call units 1 and 2. The failure time distribution of unit i(i= 1, 2),F_i (t), and the repair time distribution $G_i(t)$ are arbitrary functions. We assume that after repair a unit recovers its function perfectly. We also assume that the switching over times from the repair to the standby state, and from the standby states to the operative state are assumed to be instantaneous. The failure time of a standby unit i(i = 1, 2) has an arbitrary distribution $U_i(t)$, (t>0). Next we shall consider the preventive maintenance policy.

When an operative unit goes to a specified time t and it is free from failure in that interval, the unit undergoes inspection as the preventive maintenance policy. We assume that $A_i(t)$ the time distribution to the inspection of unit $i (i = 1, 2)$ and $B_i(t)$, the time distribution from the inspection to the inspection completion (or the preventive repair completion) are arbitrary functions. The time distribution to inspecton of a standby unit i has an arbitrary distribution $V_i(t)$. We also assume that $G_i(t) \leqslant B_i(t), \forall t$ and $i = 1, 2$, so as to make the preventive maintenance policy effective. We shall further consider a special situation, when an operative unit goes to the inspection time before the repair completion of the other failed unit (or the inspection completion of the other unit under inspection), we make no inspection for this operative unit since such an inspection leads to the system down. That is, the inspection of an operative unit is only made if the other is in stand-by or vice-versa. We assume that the switchover times occuring in the inspection are all instantaneous. We also assume that the random variables are mutually independent and non-negative. We should also assume that the failure time distribution of an operative unit has IFR. (see Barlow and Prochan 1965, p. 12) so as to make the preventive maintenance policy effective.

Results have also been obtained alongside for exponential failure and inspection of both operative and standby units.
These are given by

$$F_i(t) = 1 - \exp(-t_i t)$$
$$A_i(t) = 1 - \exp(-\alpha_i t)$$
$$U_i(t) = 1 - \exp(-\gamma_i t), \text{ and}$$
$$V_i(t) = 1 - \exp(-\nu_i t).$$

Analysis

We note that, at any instant of time, the system is found in one of the states listed in Table I.

Table I

The states of the system

State variable $X(t)$	State of the system
0	unit 1 begins to be operative and unit 2 begins to be standby
1	unit 2 is operative and unit 1 begins to get reparied.
2	unit 2 is operative and inspection time of unit 1 begins.
3	unit 1 is operative and unit 2 begins to get repaired.
4	unit 1 is operative and inspection time of unit 2 begins.
5	absorbing state, i.e. both the units are down.

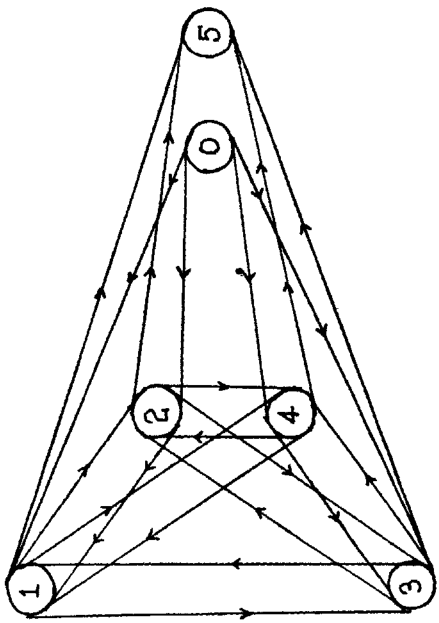

FIG. 1. Flow Chart

Our concern is to find the first time to system down starting from the state 0 at $t = 0$.

Define,

$$P_{ij}(t) = \text{Prob. } \{X(t) = j \mid X(t_0) = i,\ 0 \leqslant t_0 < t\}$$
$$[i, j = 0, 1, 2, 3, 4, 5],$$

and

$$p_{ij}(s) = \int_0^\infty e^{-st}\, dp_{ij}(t). \qquad \text{Re}\,(s) > 0,$$

the Laplace Stieltjes transform (LST) of $P_{ij}(t)$ (the small letter function denotes the LST of corresponding capital one throughout this paper).

Now,

$$P_{01}(t) = \int_0^t \bar{A}_1(t).\ \bar{U}_2(t).\ \bar{V}_2(t)\ dF_1(t) \quad .. \quad .. \quad .. \qquad (1)$$

where $\bar{A}_i(t) = 1 - A_i(t)$ denotes the survival probability function. In general, the upper bar of the distribution denotes the survival probability function throughout the paper.

Taking LST of (1), we have

$$P_{01}(s) = \int_0^\infty e^{-st} A_1(t).\ \bar{U}_2(t).\ \bar{V}_2(t).\ dF_1(t) = \lambda_1/D \quad .. \quad (2)$$

where

$$D = [\, s + a_1 + \gamma_2 + \nu_2 + \lambda_1 \,] .$$

Similarly

$$P_{02}(s) = \int_0^\infty e^{-st} \ \bar{F}_1(t).\ \bar{U}_2(t).\ \bar{V}_2(t).\ dA_1(t) = a_1/D \quad .. \quad (3)$$

$$p_{03}(s) = \int_0^\infty e^{-st} \bar{F}_1(t).\ \bar{A}_1(t).\ \bar{V}_2(t).\ dU_2(t) = \gamma_2/D \quad .. \quad (4)$$

and

$$p_{04}(s) = \int_0^\infty e^{-st} \bar{F}_1(t).\ \bar{A}_1(t).\ \bar{U}_2(t).\ dV_2(t) = \nu_2/D. \quad .. \quad (5)$$

For the recurrence time distribution $P_{11}(t)$ for state 1 we have two exclusive and exhaustive cases :

(A) After the repair completion of unit 1, unit 1 is in standby and the unit 1 fails (before its inspection time begins) before unit 2 fails and before its inspection time starts. Then its distribution becomes

$$\int_0^t \bar{F}_2(x).\ \bar{A}_2(x).\ d\left[\int_0^x dG_1(u) \left\{\int_0^{x-u} \bar{V}_1(y)\, dU_1(y)\right\}\right].$$

$$= \int_0^t \bar{F}_2(x).\ \bar{A}_2(x) d\left[\ G_1(x) * \left\{\int_0^x V_1(y) dU_1(y)\right\}\right].$$

where *denotes the convolution operator.

(B) The inspection time of unit 2 comes before the repair completion of unit 1. In this case inspection is not made. Then the probability that the repair of unit 1 is completed during time x after the inspection time and before the inspection of standby unit 1 begins is

$$\int_0^x A_2(u)\ dG_1(u).\ \left\{\int_0^{x-u} \bar{V}_1(y)\ dU_1(y)\right\} .$$

Thus we have

$$P_{11}(t) = \int_0^t \bar{F}_2(x).\ \bar{A}_2(x).\ d\left[\ G_1(x)^* \left\{\int_0^x \bar{V}_1(y)\ dU_1(y)\right\}\right] +$$

$$\int_0^t \bar{F}_2(x)\ d\left[\int_0^x A_2(u)\ dG_1(u) \left\{\int_0^{x-u} \bar{V}_1(y)\ dU_1(y)\right\}\right]. \quad .. \qquad (6)$$

Therefore,

$$p_{11}(s) = \int_0 e^{-st}\ \bar{F}_2(t).\ \bar{A}_2(t)\ d\left[\ G_1(t)^* \left\{\int_0^t \bar{V}_1(y)\ dU_1(y)\right\}\right]$$

$$+\int_0^\infty e^{-st}\ \bar{F}_2(t)\ d\left[\int_0^t A_2(u)\ dG_1(u) \left\{\int_0^{t-u} \bar{V}_1(y)\ dU_1(y)\right\}\right].$$

$$= \gamma_1 \left[\frac{g_1(s+a_2+\frac{\lambda}{2})}{s+a_2+\frac{\lambda}{2}+\gamma_1+\nu_1} + \frac{g_1(s+\frac{\lambda}{2})}{s+\frac{\lambda}{2}+\gamma_1+\nu_2} - \frac{(s+a_2+\frac{\lambda}{2})\ g_1(s+a_2+\frac{\lambda}{2})}{(s+\frac{\lambda}{2})\ (s+\frac{\lambda}{2}+\gamma_1+\nu_1)}\right]$$

$$.. \quad .. \quad .. \qquad (7)$$

Similarly,

$$P_{12}(s) = \int_0^\infty e^{-st}\ \ \bar{F}_2(t)\ \ \bar{A}_2(t)\ d\left[\ G_1(t) * \left\{\int_0^t \bar{U}_1(y)\ dV_1(y)\right\}\right]$$

$$+ \int_0^\infty e^{-st}\ \bar{F}_2(t)\ d\left[\int_0^t A_2(u)\ dG_1(u) \left\{\int_0^{t-u} \bar{U}_1(y)\ dV_1(y)\right\}\right].$$

$$= \nu_1 \left[\frac{g_1(s+a_2+\frac{\lambda}{2})}{s+a_2+\frac{\lambda}{2}+\gamma_1+\nu_1} + \frac{g_1(s+\frac{\lambda}{2})}{s+\frac{\lambda}{2}+\gamma^2+\nu_1} - \frac{(s+a_2+\frac{\lambda}{2})\ g_1(s+a_2+\frac{\lambda}{2})}{(s+\frac{\lambda}{2})\ (s+\frac{\lambda}{2}+\gamma_1+\nu_1)}\right]$$

$$.. \quad .. \qquad (8)$$

$$P_{13}(s) = \int_0^\infty e^{-st}\ \bar{A}_2(t)\ \left[\ G_1(t) * (\bar{U}_1(t).\ \bar{V}_1(t))\ \right] dF_2(t)$$

$$+ \int_0^\infty e^{-st} \left[\int_0^t A_2(x) d\left[\ G_1(x) * (\bar{U}_1(x)\bar{V}_1(x))\ \right] dF_2(t)\right.$$

$$= \lambda_2 \quad \frac{g_1(s+a_2+\frac{\lambda}{2})}{s+a_2+\frac{\lambda}{2}+\gamma_1+\nu_1} + \frac{g_1(s+\frac{\lambda}{2})}{s+\frac{\lambda}{2}+\gamma_1+\nu_1} - \frac{(s+a_2+\frac{\lambda}{2})g_1(s+a_2+\frac{\lambda}{2})}{(s+\frac{\lambda}{2})\ (s+a_2+\frac{\lambda}{2}+\gamma_1+\nu_1)}\Bigg]$$

$$.. \quad .. \qquad (9)$$

$$p_{14}(s) = \int_0^\infty e^{-st}\,\bar{F}_2(t)\Big[\; G_1(t) \; * \;\Big]\,(\bar{U}_1(t).\,\bar{V}_1(t))\,\Big]\,dA_2(t)$$

$$= a_2.\, g_1\,(s+\alpha_2+\tfrac{\lambda}{2})/(s+\alpha_2+\tfrac{\lambda}{2}+\gamma_1+\nu_1) \qquad \ldots \qquad \ldots \qquad (10)$$

$$p_{15}(s) = \int_0^\infty e^{-st}\,\bar{G}_1(t)\,dF_2(t) = \tfrac{\lambda}{2}\Big[\; 1-g_1(s+\tfrac{\lambda}{2}) \;\Big]\Big/\,(s+\tfrac{\lambda}{2}) \qquad \ldots \qquad (11)$$

$$p_{21}(s) = \int e^{-st}\,\bar{F}_2(t).\,\bar{A}_2(t)d\Big[\; B_1(t) \; * \;\Big\{\int_0 \bar{V}_1(y)\,dU_1(y)\Big\}\Big]$$

$$+ \int_0^\infty e^{-st}\,\bar{F}_2(t)\;d\Big[\int_0^t A_2(u)\,dB_1(u)\;\Big\{\int_0^{t-u} \bar{V}_1(y)\,dU_1(y)\Big\}\Big]$$

$$= \gamma_1\Big[\frac{b_1(s+\alpha_2+\frac{\lambda}{2})}{s+\alpha_2+\frac{\lambda}{2}+\gamma_1+\nu_1} + \frac{b_1(s+\frac{\lambda}{2})}{s+\frac{\lambda}{2}+\gamma_1+\nu_1} - \frac{(s+\alpha_2+\frac{\lambda}{2}).\;b_1(s+\alpha_2+\frac{\lambda}{2})}{(s+\frac{\lambda}{2})\;(s+\frac{\lambda}{2}+\gamma_1+\nu_1)}\Big]$$

$$\ldots \qquad \ldots \qquad (12)$$

$$p_{22}(s) = \int_0^\infty e^{-st}\,\bar{F}_2(t)\;\bar{A}_2(t)\;d\Big[B_1(t) \; * \;\Big\{\int_0^t \bar{U}_1(y)\,dV_1(y)\Big\}\Big]$$

$$+ \int_0^\infty e^{-st}\,\bar{F}_2(t)\;d\Big[\int_0^t A_2(u)\,dB_1(u)\;\Big\{\int_0^{t-u} \bar{U}_1(y)\,dV_1(y)\Big\}\Big]$$

$$= \nu_1\,\frac{b_1(s+\alpha_2+\frac{\lambda}{2})}{s+\alpha_2+\frac{\lambda}{2}+\gamma_1+\nu_1} + \frac{b_1(s+\frac{\lambda}{2})}{s+\frac{\lambda}{2}+\gamma_1+\nu_1} - \frac{(s+\alpha_2+\frac{\lambda}{2}).\;b_1(s+\alpha_2+\frac{\lambda}{2})}{(s+\frac{\lambda}{2})\;(s+\lambda_2+\gamma_1+\nu_1)}\Big]$$

$$\ldots \qquad \ldots \qquad (13)$$

$$p_{23}(s) = \int_0^\infty e^{-st}\bar{A}_2(t)\Big[B_1(t) \; * \;\Big(\bar{U}_1(t).\bar{V}_1(t)\Big)\Big]\,dF_2(t)$$

$$+ \int_0^\infty e^{-st}\Big[\int_0^t A_2(x)d\Big[\; B_1(x) \; * \;\Big(\bar{U}_1(x).\,\bar{V}_1(x)\Big)\Big]dF_2(t)$$

$$\lambda_2 = \Big[\frac{b_1(s+\alpha_2+\frac{\lambda}{2})}{s+\alpha_2+\frac{\lambda}{2}+\gamma_1+\nu_1} + \frac{b_1(s+\frac{\lambda}{2})}{s+\frac{\lambda}{2}+\gamma_1+\nu_1} - \frac{(s+\alpha_2+\frac{\lambda}{2})b_1(s+\alpha_2+\frac{\lambda}{2})}{(s+\alpha_2+\frac{\lambda}{2}+\gamma_1+\nu_1)\;(s+\frac{\lambda}{2})}\Big]$$

$$(14)$$

$$p_{24}(s) = \int_0^\infty e^{-st}\,\bar{F}_2(t)\Big[B_1(t) \;*\; \Big(\bar{U}_1(t).\bar{V}_1(t)\Big)\Big]\,dA_2(t)$$

$$= a_2.b_1(s+a_2+\lambda_2)\Big/(s+a_2+\lambda_2+\gamma_1+v_1) \quad \ldots \quad \ldots \quad \ldots \qquad (15)$$

$$p_{25}(s) = \int_0^\infty e^{-st}\bar{B}_1(t)dF_2(t). = \lambda_2\Big[1-b_1(s+\lambda_2)\Big]\Big/\;(s+\lambda_2) \quad \ldots \qquad (16)$$

$$p_{31}(s) = \int_0^\infty e^{-st}\bar{A}_1(t)\Big[G_2(t) \;*\; \Big(\bar{U}_2(t).\bar{V}_2(t)\Big)\Big]dF_1(t)$$

$$+ \int_0^\infty e^{-st}\Big[\int_0^t A_1(x)d\Big[G_2(x) \;*\; \Big(\bar{U}_2(x).\bar{V}_2(x)\Big)\Big]dF_1(t)$$

$$= \lambda_1\left[\frac{g_2(s+a_1+\lambda_1)}{s+a_1+\lambda_1+\nu_2+\gamma_2} + \frac{g_1(s+\lambda_1)}{s+\lambda_1+\gamma_2+\nu_2} - \frac{(s+a_1+\lambda_1)g_2(s+a_1+\lambda_1)}{(s+\lambda_1)(s+a_1+\lambda_1+\gamma_2+\nu_2)}\right] \quad (17)$$

$$p_{32}(s) = \int_0^\infty e^{-st}\bar{F}_1(t)\Big[G_2(t) \;*\; \Big(\bar{U}_2(t)\bar{V}_2(t)\Big)\Big]\,dA_1(t)$$

$$= a_1.g_2(s+a_1+\lambda_1)\Big/(s+a_1+\gamma_1+\gamma_2+\nu_2) \quad \ldots \quad \ldots \qquad (18)$$

$$p_{33}(s) = \int_0^\infty e^{-st}\,\bar{F}_1(t)\,\bar{A}_1(t)d\Big[G_2(t) \;*\; \Big\{\int_0^t \bar{V}_2(y)dU_2(y)\Big\}\Big]$$

$$+ \int_0^\infty e^{-st}F_1(t)d\Big[\int_0^t A_1(u)dG_2(u)\Big\{\int_0^{t-u}\bar{V}_2(y)dU_2(y)\Big\}\Big]$$

$$= \gamma_2\left[\frac{g_2(s+a_1+\lambda_1)}{s+a_1+\lambda_1+\gamma_2+\nu_2} + \frac{g_2(s+\lambda_1)}{s+\lambda_1+\gamma_2+\nu_2} - \frac{(s+a_1+\lambda_1)g_2(s+a_1+\lambda_1)}{(s+\lambda_1)\;(s+\lambda_1+\gamma_2+\nu_2)}\right]$$

$$(19)$$

$$p_{34}(s) = \int_0^\infty e^{-st}\bar{F}_1(t)\,\bar{A}_1(t)d\Big[G_2(t) \;*\; \Big\{\int_0^t \bar{U}_2(v)dV_2(y)\Big\}\Big]$$

$$+ \int_0^\infty e^{-st}F_1(t)d\Big[\int_0^t A_1(u)dG_2(u)\Big\{\int_0^{t-u}\bar{U}_2(y)dV_2(y)\Big\}\Big]$$

$$= \nu_2\left[\frac{g_2(s+a_1+\lambda_1)}{s+a_1+\lambda_1+\gamma_2+\nu_2} + \frac{g_2(s+\lambda_1)}{s+\lambda_1+\gamma_2+\nu_2} - \frac{(s+a_1+\lambda_1)g_2(s+a_1+\lambda_1)}{(s+\lambda_1)\;(s+\lambda_1+\gamma_2+\nu_2)}\right] \qquad (20)$$

$$p_{3b}(s) = \int_0^\infty e^{-st}\bar{G}_2(t)dF_1(t). = \lambda_1\left[1-g_2(s+\lambda_1)\right] \Big/ (s+\lambda_1) \qquad .. \qquad (21)$$

$$p_{41}(s) = \int_0^\infty e^{-st}\bar{A}_1(t)\left[B_2(t) * \left(\bar{U}_2(t).\bar{V}_2(t)\right)\right]dF_1(t)$$

$$+ \int_0^\infty e^{-st}\left[\int_0^t A_1(x)\,d\left[B_2(x) * \left(\bar{U}_2(x).\bar{V}_2(x)\right)\right]\right] dF_1(t)$$

$$= \lambda_1\left[\frac{b_2(s+a_1+\lambda_1)}{s+a_1+\lambda_1+\gamma_2+\nu_2} + \frac{b_2(s+\lambda_1)}{s+\lambda_1+\gamma_2+\nu_2} - \frac{(s+a_1+\lambda_1)b_2(s+a_1+\lambda_1)}{(s+\lambda_1)\ (s+a_1+\lambda_1+\nu_2+\nu_2)}\right] \qquad (22)$$

$$p_{42}(s) = \int_0^\infty e^{-st}\bar{F}_1(t)\left[B_2(t) * \left(\bar{U}_2(t)\bar{V}_2(t)\right)\right] dA_1(t)$$

$$= a_1.b_2(s+a_1+\lambda_1)\Big/(s+a_1+\lambda_1+\gamma_2+\nu_2) \qquad .. \qquad .. \qquad (23)$$

$$p_{43}(s) = \int_0^\infty e^{-st}\bar{F}_1(t)\bar{A}_1(t)d\left[B_2(t) * \left\{\int_0^t \bar{V}_2(y)dU_2(y)\right\}\right]$$

$$+ \int_0^\infty e^{-st}\bar{F}_1(t)d\left[\int_0^t A_1(u)\,{}_2dB_2(u)\left\{\int_0^{t-u} \bar{V}_2(y)dU_2(y)\right\}\right]$$

$$= \gamma_2\left[\frac{b_2(s+a_1+\lambda_1)}{s+a_1+\lambda_1+\gamma_2+\nu_2} + \frac{b_2(s+\lambda_1)}{s+\lambda_1+\gamma_2+\nu_2} - \frac{(s+a_1+\lambda_1)b_2(s+a_1+\lambda_1)}{(s+\lambda_1)\ (s+\lambda_1+\gamma_2+\nu_2)}\right] \qquad (24)$$

$$p_{44}(s) = \int_0^\infty e^{-st}\bar{F}_1(t)\bar{A}_1(t)d\left[B_2(t) * \left\{\int_0^t \bar{U}_2(y)dV_2(y)\right\}\right]$$

$$+ \int_0^\infty e^{-st}\bar{F}_1(t)d\left[\int_0^t A_1(u)\,dB_2(u)\left\{\int_0^{t-u} \bar{U}_2(y)dV_2(y)\right\}\right]$$

$$= \nu_2\left[\frac{b_2(s+a_1+\lambda_1)}{s+a_1+\lambda_1+\gamma_2+\nu_2} + \frac{b_2(s+\lambda_1)}{s+\lambda_1+\gamma_2+\nu_2} - \frac{(s+a_1+\lambda_1).b_2(s+a_1+\lambda_1)}{(s+\lambda_1)\ (s+\lambda_1+\gamma_2+\nu_2)}\right] \qquad (25)$$

and

$$p_{45}(s) = \int_0^\infty e^{-st}\bar{B}_2(t)dF_1(t). = \lambda_1 \left[1 - b_2(s+\lambda_1) \right] / (s+\lambda_1) \quad .. \quad (26)$$

Define, $\Omega_K(s)$: $K = 0, 1, 2, 3, 4$, the Laplace Stieltjes transform of the first down-time distribution starting from state K. We have

$$\Omega_0(s) = \sum_{i=1}^{4} p_{0i}(s).\ \Omega_i(s) \quad .. \quad .. \quad .. \quad (27)$$

and

$$\Omega_j(s) = \sum_{i=1}^{4} p_{ji}(s).\ \Omega_i(s) + p_{j5}(s).(1 \leqslant j \leqslant 4). \quad .. \quad .. \quad (28)$$

The matrix representation of (27) — (28) is given by

$$\Delta(s).\ \Omega(s) = A(s) \quad .. \quad .. \quad .. \quad .. \quad (29)$$

where

$$\Delta(s) = \begin{bmatrix} -1 & p_{01}(s) & p_{02}(s) & p_{03}(s) & p_{04}(s) \\ 0 & p_{11}(s)-1 & p_{12}(s) & p_{13}(s) & p_{14}(s) \\ 0 & p_{21}(s) & p_{22}(s)-1 & p_{23}(s) & p_{24}(s) \\ 0 & p_{31}(s) & p_{32}(s) & p_{33}(s)-1 & p_{34}(s) \\ 0 & p_{41}(s) & p_{42}(s) & p_{43}(s) & p_{44}(s)-1 \end{bmatrix}$$

$$A(s) = \begin{bmatrix} 0 \\ p_{15}(s) \\ k_{25}(s) \\ p_{35}(s) \\ p_{45}(s) \end{bmatrix} \quad \text{and} \quad \Omega(s) = \begin{bmatrix} \Omega_0(s) \\ \Omega_1(s) \\ \Omega_2(s) \\ \Omega_3(s) \\ \Omega_4(s) \end{bmatrix}.$$

Thus,

$$\Omega_i(s) = [\text{ Det. } \Delta_i(s)] \ / \ [\text{Det. } \Delta(s) \qquad (1 \leqslant i \leqslant 4) \quad .. \quad (30)$$

where $\Delta_i(s)$ is the matrix obtained by replacing the ith column of the matirx $\Delta(s)$ by the coloumn matrix $A(s)$.

It could be verified that $\lim\limits_{s \to 0} \Omega_0(s) = 1$, which would lead us to remark that $\Omega_0(s)$ is a proper distribution. The mean time is given by the relation

$$\hat{T} = -\frac{d}{ds}\,\Omega_0(s)\Big|_{s=0} = -\frac{d}{ds}\left[\left\{\text{Det.}\,\Delta_i(s)\right\}\Big/\left\{\text{Det.}\,\Delta(s)\right\}\right]\Big|_{s=0} \tag{31}$$

PARTICULAR CASES

Case 1

Two-unit standby redundant system of similar units 'without standby failure' (i.e. 'with cold standby')

For this case we need not use the memoryless property of the failure and inspection time. In this case setting that $U_i(t) = V_i(t) = 0$, $(i = 1, 2)$

$F_i(t) = F(t) = 1 - \exp(-\lambda t)$, say

$A_i(t) = A(t) = 1 - \exp(-at)$ say

$G_i(t) = G(t)$, and $B_i(t) = B(t)$, say $(i = 1, 2)$

We will be left with four cases only (see Table II)

TABLE II

State variable $X(t)$	State of the system
0	One unit begins to be operative and the other is in standby
1	One unit is operative and the other failed unit undergoes repair.
2	One unit is operative while the inspection time of the other unit begins.
3	Absorbing state, i.e. both the units are down.

Making the necessary substitutions in relations for $P_{ij}(s)$ we have

$$p_{01}(s) = \int_0^\infty e^{-st}\,\bar{A}(t).dF(t). = \lambda/(s+a+\lambda) \quad .. \quad .. \quad .. \tag{1·1}$$

$$p_{02}(s) = \int_0^\infty e^{-st}\bar{F}(t).dA(t). = a/(s+a+\lambda) \quad .. \quad .. \quad .. \quad (1\cdot2)$$

$$p_{11}(s) = \int_0^\infty e^{-st}\bar{A}(t).G(t)dF(t) + \int_0^\infty e^{-st}\left[\int_0^t A(y)dG(y)\right] dF(t)$$

$$= \lambda\left[\frac{g(s+a+\lambda)}{s+a+\lambda} + \frac{g(s+\lambda)}{s+\lambda} - \frac{g(s+a+\lambda)}{s+\lambda}\right] \quad .. \quad .. \quad (1\cdot3)$$

$$P_{12}(s) = \int_0^\infty e^{-st}\bar{F}(t)G(t)dA(t) = ag(s+a+\lambda)/(s+a+\lambda) \quad (1\cdot4)$$

$$p_{13}(s) = \int_0^\infty e^{-st}G(t)dF(t) = \lambda\ [1 - g(s+\lambda)\ /\ (s+\lambda)] \quad .. \quad (1\cdot5)$$

$$p_{21}(s) = \int_0^\infty e^{-st}\,A(t)B(t)dF(t) + \int_0^\infty e^{-st}\left[\int_0^t A(y)dB(y)\right] dF(t)$$

$$= \lambda\left[\frac{b(s+a+\lambda)}{s+a+\lambda} + \frac{b(s+\lambda)}{s+\lambda} - \frac{b(s+a+\lambda)}{s+\lambda}\right] \quad .. \quad (1\cdot6)$$

$$p_{22}(s) = \int_0^\infty e^{-st}\bar{F}(t)B(t)dA(t) = ab(s+a+\lambda)/(s+a+\lambda) \quad .. \quad (1\cdot7)$$

$$p_{23}(s) = \int_0^\infty e^{-st}\bar{B}(t)dF(t) = \lambda\ [1 - b(s+\lambda)]/(s+\lambda) \quad .. \quad (1\cdot8)$$

Let $\Omega_k(s)$: $k = 0, 1, 2$ be the LST of the first system down-time distribution starting from state k at t=0.

Therefore,

$$\Omega_0(s) = p_{01}(s).\ \Omega_1(s) + p_{02}(s)\ \Omega_2(s) \quad .. \quad .. \quad .. \quad (1\cdot9)$$

$$\Omega_0(s) = p_{i3}(s) + p_{i1}(s).\ \Omega_1(s) + p_{i2}(s)\ \Omega_2(s) \qquad (i = 1, 2). \quad .. \quad (1\cdot10)$$

Now one could see that

$$\Omega_0(s) = [p_{01}(s).p_{13}(s)\ (1-p_{22}(s) + p_{01}(s).p_{12}(s).p_{23}(s)$$
$$+p_{02}(s).p_{23}(s).\ 1-p_{11}(s) + p_{02}(s).p_{21}(s).p_{13}(s)]/\ [1-p_{11}(s)(1-p_{22}(s)$$
$$- p_{12}(s).\ p_{21}(s)] \quad .. \quad .. \quad .. \quad (1\cdot11)$$

and the mean time $\hat{T}$ is given by

$$\hat{T} = \Big[-p'_{01} - p'_{02} - \Big\{ (p_{01} \cdot p_{23} + p_{21}) \Big(\sum_{j=1}^{3} p_{ij} \Big) + (1 - p_{11} - p_{01} p_{13}) \cdot \Big(\sum_{j=1}^{3} p_{2j} \Big) \Big\} \Big/ \Big\{ (1 - p_{11})\, p_{23} + p_{21} p_{13} \Big\} \Big] \quad \ldots \quad (1{\cdot}12)$$

where

$$p'_{ij} = p_{ij}(s)\Big|_{s=} \text{ and } p_{ij} = \frac{d}{ds}\, p_{ij}(s)\Big|_{s=0}$$

These results have been given by Osaki and Asakura (1970).

Case 2

In this case we will csnsider a two-unit warm standby redundant system with repair maintenance only. For such a case we shall have

$$A_i(t) = B_i(t) = 0. \qquad (i = 1, 2)$$

In such a case we would have the following states (see Table III).

TABLE III

State variable $X(t)$	State of the system
0	Unit 1 begins to be operative and unit 2 begins to be standby.
1	Unit 2 is operative and unit 1 undergoes repair.
2	Unit 1 is operative and unit 2 undergoes repair.
3	Absorbing state.

For such a situation we have

$$p_{01}(s) = \int_0^{\infty} e^{-st}\, \bar{U}_2(t) dF_1(t). = \lambda_1/(s+\lambda_1+\gamma_2) \quad \ldots \quad (2.1)$$

$$p_{02}(s) = \int e^{-st}\, \bar{F}_1(t)\, dU_2(t) = \gamma_2/(s+\lambda_1+\gamma_2) \quad \ldots \quad (2{\cdot}2)$$

$$p_{11}(s) = \int_0^{\infty} e^{-st}\, \bar{F}_2(t)\, d\Big[G_1(t) * \Big] U_1(t) = \gamma_1 g_1(s+\lambda_2)/(s+\lambda_1+\gamma_1) \quad (2{\cdot}3)$$

$$p_{12}(s) = \int_0 e^{-st} \left[G_1(t) * \bar{U}_1(t) \right] dF_2(t) = {}^{\lambda}_{2} g_1 \, (s+{}^{\lambda}_{2})/(s+\gamma_1+{}^{\lambda}_{2}) \qquad (2{\cdot}4)$$

$$p_{21}(s) = \int_0^\infty e^{-st} \, \bar{G}_1(t) \, dF_2(t). = {}_2\left[1-g_1 \, (s+{}^{\lambda}_{2}) \right] /(S+{}^{\lambda}_{2}) \qquad (2{\cdot}5)$$

$$p_{21}(s) = \int_0^\infty e^{-st} \left[G_2(t) * \bar{U}_2(t) \right] dF_1(t) = {}^{\lambda}_{1} \, g_2 \, (s+{}^{\lambda}_{1})/(s+{}^{\lambda}_{1}+\gamma_2) \qquad (2{\cdot}6)$$

$$p_{22}(s) = \int_0^\infty e^{-st} \, \bar{F}_1 \, (t) d \left[G_2(t) * U_2(t) \right] = \gamma_2 g_2 \, (s+{}^{\lambda}_{1})/((+{}^{\lambda}_{1}+\gamma_2) \qquad (2{\cdot}7)$$

$$p_{23}(s) = \int_0^\infty e^{-st} \, \bar{G}_2(t) \, dF_1(t) = {}^{\lambda}_{1} \left[1-g_2 \, (s+{}^{\lambda}_{1}) \right] /(s+{}^{\lambda}_{1}). \quad .. \qquad (2{\cdot}8)$$

Therefore,

$$\Omega_0(s) = \Big[p_{01}(s) p_{13}(s) \Big((1-p_{22}(s) \Big) + p_{01}(s). \, p_{12}(s). \, p_{23}(s)$$

$$+ p_{02}(s). \, p_{23}(s) \Big(1-p_{11}(s) \Big) + p_{02}(s). \; p_{21}(s) \Big] \Big/ \Big[\Big(1-p_{11}(s) \Big)$$

$$\Big(1-p_{22}(s) \Big) - p_{12}(s). \, p_{21}(s) \Big] \quad .. \qquad (2{\cdot}9)$$

and

$$\hat{T} = \Big[-p'_{01} - p'_{02} - (p_{01} \, p_{23} + p_{21}) \Big(\sum_{j=1}^{3} p_{1j} \Big) + (1 - p_{11} - p_{01} \, p_{13})$$

$$\times \Big(\sum_{j=1}^{3} p_{2j} \Big) \Big\} \Big/ \Big\{ (1-p_{11}) \, p_{23} + p_{21} \, p_{13} \Big\} \Big]. \quad .. \quad ... \qquad (2{\cdot}10)$$

Case 3

Here we consider a two unit cold standby redundant system with identical units. For such a case we have

$A_i \, (t) = B_i \, (t) = U_i \, (t) = V_i \, (t) = 0$

$i = 1, 2$

$F_i \, (t) = F \, (t)$ and $G_i \, (t) = G \, (t)$, say

The states are described in Table IV.

TABLE IV

State variable	State of the system
0	a unit begins to be operative and the other unit begins to be standby
1	a unit is operative and the other unit undergoes repair
2	absorbing state

LST of various probability distributions are given as follows:

$$p_{01}(s) = \int_0^\infty e^{-st}\, dF(t) = \lambda/(s+\lambda) \qquad (3\cdot1)$$

$$p_{11}(s) = \int_0^\infty e^{-st}\, G(t)\, dF(t) = \lambda\, g\, (s+\lambda)/(s+\lambda) \qquad (3\cdot2)$$

and

$$p_{12}(s) = \int_0^\infty e^{-st}\, \bar{G}(t)\, dF(t) = \lambda\, [\, 1-g\, (s+\lambda)]/(s+\lambda). \qquad (3\cdot3)$$

Let $\Omega_0(s)$ and $\Omega_1(s)$ be the LST's of the first down-time distribution starting from states 0 and 1.

Than

$$\Omega_0(s) = p_{01}(s).\, \Omega_1(s) \qquad (3\cdot4)$$

$$\Omega_1(s) = p_{12}(s) + p_{11}(s).\, \Omega_1(s) \qquad (3\cdot5)$$

Therefore

$$\Omega_0(s) = p_{01}(s).\, p_{22}(s)/1 - [p_{11}(s)]$$

$$= \lambda^2\, [1 - g(s+\lambda)]/[(s+\lambda)^2\, \{1 - \lambda g(s+\lambda)/(s+\lambda)\}] \qquad (3\cdot6)$$

Clearly,

$$\mathrm{Lim}_{s\to 0}\, \Omega_0(s) = 1, \text{ and}$$

$$\hat{T} = g(\lambda)/[1 - g(\lambda)]\, .$$

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