ON THE MAGNUS INSTABILITY OF A SPINNING SHELL

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Investigation on the instability effects of a negative Magnus torque over the gyroscopic stability of a spinning artillery shell has been carried out. It has been observed that the gyroscopic stability is completely lost when a strong negative dissipative effect due to magnus torque is introduced.

1. INTRODUCTION

The aim of the present investigation is to study the effect of cross Magnus torque on the dynamic stability of a rotating artillery shell. For aerodynamically clean bodies of revolution (viz. the shell in question) the Magnus force and moment are linear with regard to the angle of attack (Platous 1965) for motions of the projectile confined to the ranges of low angles of attack. Such motions are technically known in ballistics as small (and slowly varying) yawing motion. In order to overcome resistance of air, all artillery projectiles are so designed that they execute small yawing motion initially near the muzzle of the gun. Stability demands that the fast precession and the nutational oscillations of the projectile must die quickly and the projectile must settle down to a position of equilibrium yaw. For accuracy purposes and to maintain an efficient tractability, the equilibrium yaw should not exceed two to three degrees at the most. This is the case with normally stable projectiles fired upto a quadrant elevation of 30°. Beyond this elevation the angle of attack (or yaw) of the projectile have been observed to be large depending on the muzzle velocity.

The cross Magnus force is essentially a spin force and measured to date is less than 5% of the normal force. Naturally the effect of this weak force on the stability of the projectile has not been taken into consideration by earlier ballisticians. More recent observations, however, have shown that the Magnus force and its associated moment about the body centre of gravity can have very important effect on the dynamic behaviour of the projectile.

We have analysed the initial motion and with the help of Liapunov's second method proved that a negative Magnus moment makes the projectile dynamically unstable inspite of the fact that the associated Magnus force remains weak. The phenomenon of Magnus instability has also been proved by using the stability criterion of Nielson and Synge (1946).

2. Equation of Intial Motion of the Projectile Due to over Turning and Magnus Moments

To study the initial oscillation of the projectile let us assume that its initial oscillatory motion is due to the over-turning and Magnus moments only, and the specifications of these two torques are as follows :

Over turning moment

$$\vec{M} = \mu \, (\vec{x} \times \vec{A}) \tag{2.1}$$

Magnus moment

$$\vec{J} = AN\Upsilon \, (\vec{A} \cos \delta - \vec{x}) \tag{2.2}$$

strictly as in Fowler's formalism (1920),

The direction of the axis of the shell $0A$ is represented by the unit vector $\vec{A}$ and the direction of motion of the centre of gravity '0' of the shell by the unit vector $\vec{X}$. The moment parameters μ and Υ are defined by

$$\mu = M/\sin \delta = \rho \, v^2 r^2 f_M \tag{2.3}$$

$$\Upsilon = J/AN \sin \delta = \rho \, A^{-1} \, r^4 \, v \, f_J \tag{2.4}$$

In the above relations δ is the angle of attack, A is the axial moment of interia, N the axial spin, v the velocity and $2r$ is the calibre of the shell. f_M and f_J are the dimensionless aerodynamic coefficients.

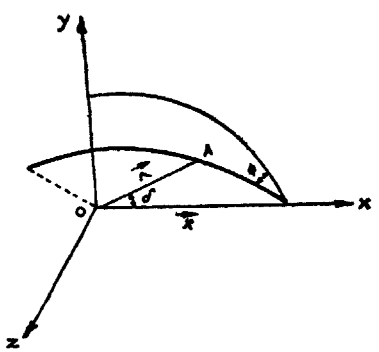

Fig. 1

As usual we assume that the centre of gravity of the shell moves uniformly along the line OX so that we can choose an orthogonal Newtonian frame of reference $0-XYZ$ as shown in the Fig. 1. whence the angular motion of the shell can be written out vectorially straight away relative to this frame of reference as

$$B\,(\vec{A} \times d^2\vec{A}/dt^2) + AN\; d\vec{A}/dt = \mu\,(\vec{X} \times \vec{A}) + AN\Upsilon\,(\vec{A}\cos\delta - \vec{X}).$$

In the above equation B is the transverse moment of inertia of the shell.

If l, m, n be the direction cosines of the axix OA we have

$$\vec{X} = (1, 0, 0)$$

$$\vec{A} = (l, m, n)$$

$$\frac{dl}{dt^2} = (dl/dt,\ dm/dt,\ dn/dt)$$

$$\frac{d^2l}{dt^2} = (d^2l/dt^2,\ d^2m/dt^2,\ d^2n/dt^2).$$

If ϕ denotes the angle between the plane AOX and YOX we can write

$$l = \cos\delta,\quad m = \sin\delta\cos\phi,\ n = \sin\delta\sin\phi. \tag{2.7}$$

Now projecting (2. 5) along the vectors X and $\vec{A} \times d\vec{A}/dt$ and making use of (2.6) and (2.7) we have the following scalar equations :

$$\frac{d}{dt}\Big[AN\cos\delta + \beta\,(d\phi/dt)\sin^2\delta\Big] + AN\Upsilon\sin^2\delta = 0 \tag{2.8}$$

$$\frac{1}{2}B\frac{d}{dt}\Big[\Big(\frac{d\delta}{dt}\Big)^2 + \Big(\frac{d\phi}{dt}\Big)^2\sin^2\delta\Big] + \\ +\mu\frac{d}{dt}(\cos\delta) + AN\Upsilon\sin^2\delta\,\frac{d\phi}{dt} = 0. \tag{2.9}$$

Defining

$$\Omega = AN/B \tag{2.10}$$

$$s = A^2N^2/4B\mu \tag{2.11}$$

$$\epsilon = \Upsilon/\Omega \tag{2.12}$$

$$\tau = \Omega\, t \tag{2.13}$$

the above two equations may be written as

$$(\cos\delta + \phi'\sin^2\delta)' + \epsilon\sin^2\delta = 0 \tag{2.8a}$$

$$(\delta'^2 + \phi'^2\sin^2\delta)' + \Big(\frac{\cos\delta}{2s}\Big)' + 2\epsilon\phi'\sin^2\delta = 0. \tag{2.9a}$$

Here the prime indicates differentiation with respect to.

3. Damped Nutational Oscillations of The Shell Due to a Weak Magnus Torque

If one neglects the relatively small term $\epsilon \sin^2\delta$ in (2.8a) and eliminates ϕ' between (2.8a) and (2.9b) the resulting differential equation will be in δ only. Then a substitution

$$z = \sin \delta/2 \tag{3.1}$$

gives

$$\left(\frac{4z'^2}{1-z^2}\right)' + \left(\frac{z^2}{1-z^2}\right)' - \frac{2zz'}{s} -+ 4\epsilon z^2 = 0. \tag{3.2}$$

Noting that the yawing motion is small and slowly varying, i.e. (z, and $z' << 1$) we carry out the implied differentiation in (3.2) and retaining terms upto order three in z and z' obtain the following quasi-harmonic equation :

$$z'' + \sigma^2 \left(\frac{z}{4}\right) + \epsilon \left(\frac{z^2}{2z'}\right) = 0 \tag{3.3}$$

where

$$\sigma^2 = \left(1 - \frac{1}{s}\right).^*$$

The equivalent linear equation of (3.3) by the method of equivalent linearization (Bogolinbov and Mitropolsky 1961) is

$$z'' + \lambda_c\,(a)\, z' + k_e\,(a)\, z = 0 \tag{3.4}$$

where

$$\lambda_c\,(a) = \frac{2\epsilon}{a\sigma} \int_0^{2\pi} \frac{a^2 \cos^2 \psi}{a\sigma \sin \psi} \sin \psi \, d\psi = \frac{2\epsilon}{\sigma^2} \tag{3.5}$$

and

$$K_e\,(a) = \frac{\sigma^2}{4} - \frac{\epsilon}{a} \int_0^{2\pi} \frac{a^2 \cos^2 \psi}{a\sigma \sin \psi} \cos \psi \, d\psi = \frac{\sigma^2}{4} \tag{3.6}$$

as the principal value of the improper integral $\int_0^{2\pi} \cos^3 \psi \operatorname{cose} \psi \, d\psi$ vanishes. It may be noted that a and $\sigma/2$ are the amplitudes and frequency of the nutational oscillations of the shell, ψ is the phase angle where the motion is characterized by the reduced harmonic equation

$$z'' + \left(\frac{\sigma^2}{4}\right) z = 0.$$

The motion represented by (3.4) is now oscillatory and stable if

$$0 < \epsilon < \frac{\sigma^3}{2}. \tag{3.7}$$

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*σ^2 is the cranz stability parameter of the shell and must be positive for stable shells.

Further it is interesting to observe that eqn. (3. 3) written as the system

$$z' = y$$

$$y' = -\left(\frac{\sigma^2}{4}\right)z - \left(\frac{\epsilon}{2}\right)\left(\frac{z^2}{y}\right) \tag{3.8}$$

with $z(0)/y(0) \underline{\Delta} 0$ (which is a physically correct assumption) is such that its trivial solution $y=z=0$ is asymptotically, stable, stable and unstable in the sense of Liapunov according as

$$\epsilon \gtreqless 0. \tag{3.9}$$

For this one needs to choose Liapunov function

$$V(y,z) = y^2 + \frac{\sigma^2}{4} z^2$$

which is positive definite and

$$v' = -\epsilon z^2$$

along the system (3.8).

Now the assertion (3.9) follows from the basic theorems of Liapunov (Lasalle and Lefschetz 1961).

The same result is also true for (3.2). This can be seen by choosing the Liapunov function

$$v = \frac{4z'^2 + z^4}{1 - z^2} + \sigma^2 z^2 \tag{3.11}$$

which is positive definite for $|z| < 1$ and $\sigma^2 > 0$.

Now due to (3.2) we have

$$\frac{dv}{d} + 4\epsilon z^2 = 0 \tag{3.12}$$

which proves the assertion.

One of the most interesting features of the system (3.3) is that it not only preserves the structural character of asymptotic stability of (3.2) but the nutational oscillation characterized by this system is quantitatively the same (upto a very high order of accuracy) as that given by (3.2). This may be observed from the graphs shown below. The graphs give the values of $\sin \delta/2\ (=z)$ against the dimensionless time τ. z has been estimated by solving (3.2) numerically with the help of a digital computer. These solution for 7 cycles of oscillations are compared with those of (3.4) which are the equivalent linear solutions of (3.3).

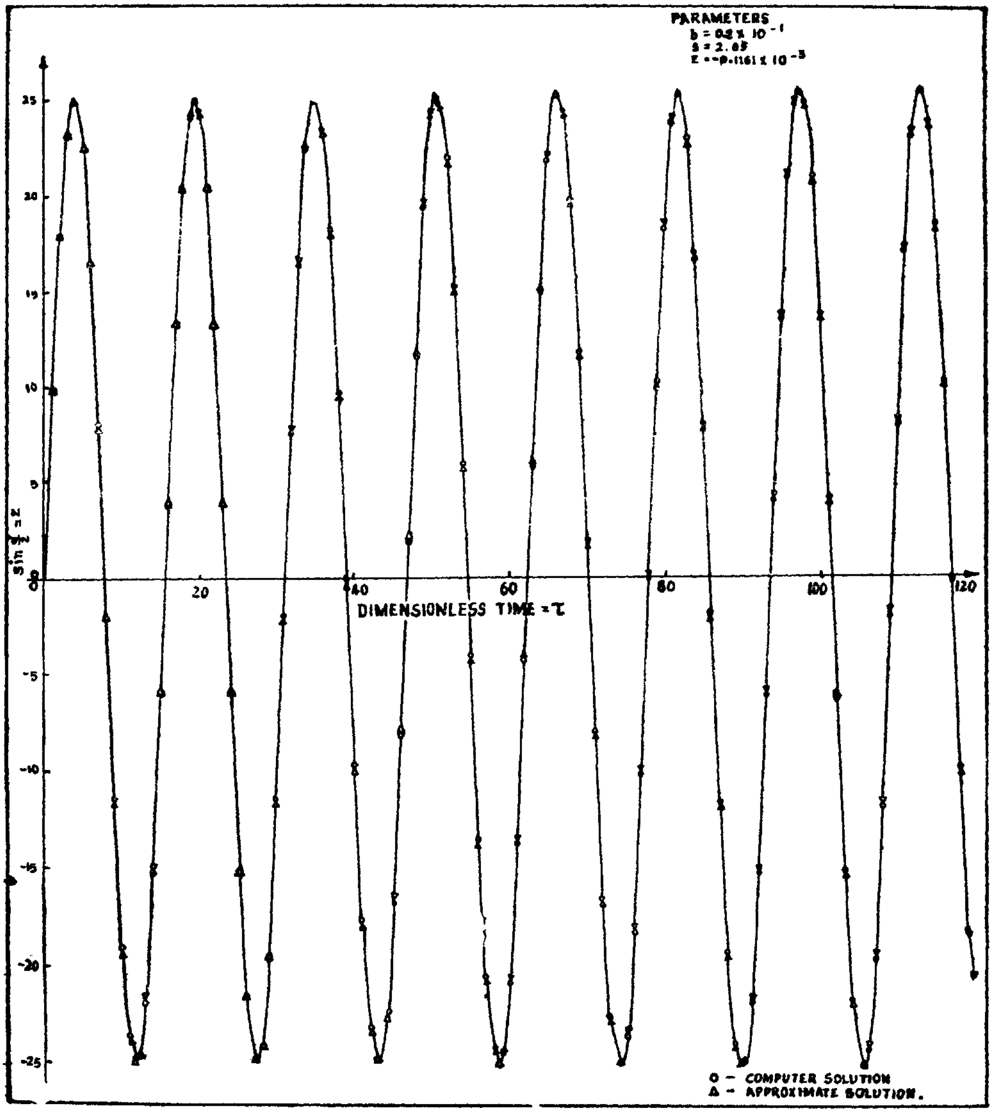

PARAMETERS
b = 0.2 x 10^-1
E = -0.1161 x 10^-3
0
20
40
60
80
100
120
DIMENSIONLESS TIME = τ
O – COMPUTER SOLUTION
Δ – APPROXIMATE SOLUTION.

FIG. 2

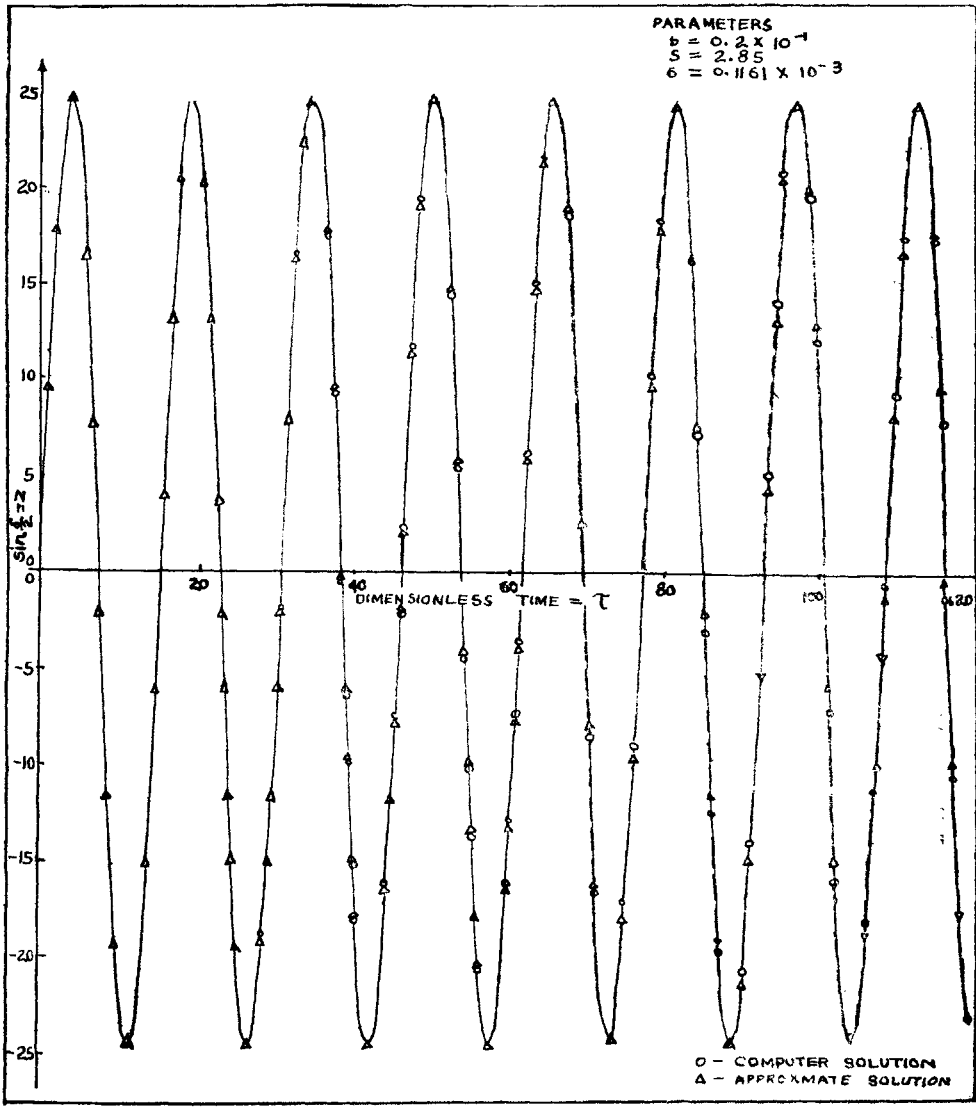

PARAMETERS
b = 0.2 X 10⁻¹
S = 2.85
σ = 0.1161 X 10⁻³
25
20
15
10
5
0
-5
-10
-15
-20
-25
20
40
60
80
100
120
DIMENSIONLESS TIME = τ
O – COMPUTER SOLUTION
Δ – APPROXMATE SOLUTION

FIG. 3

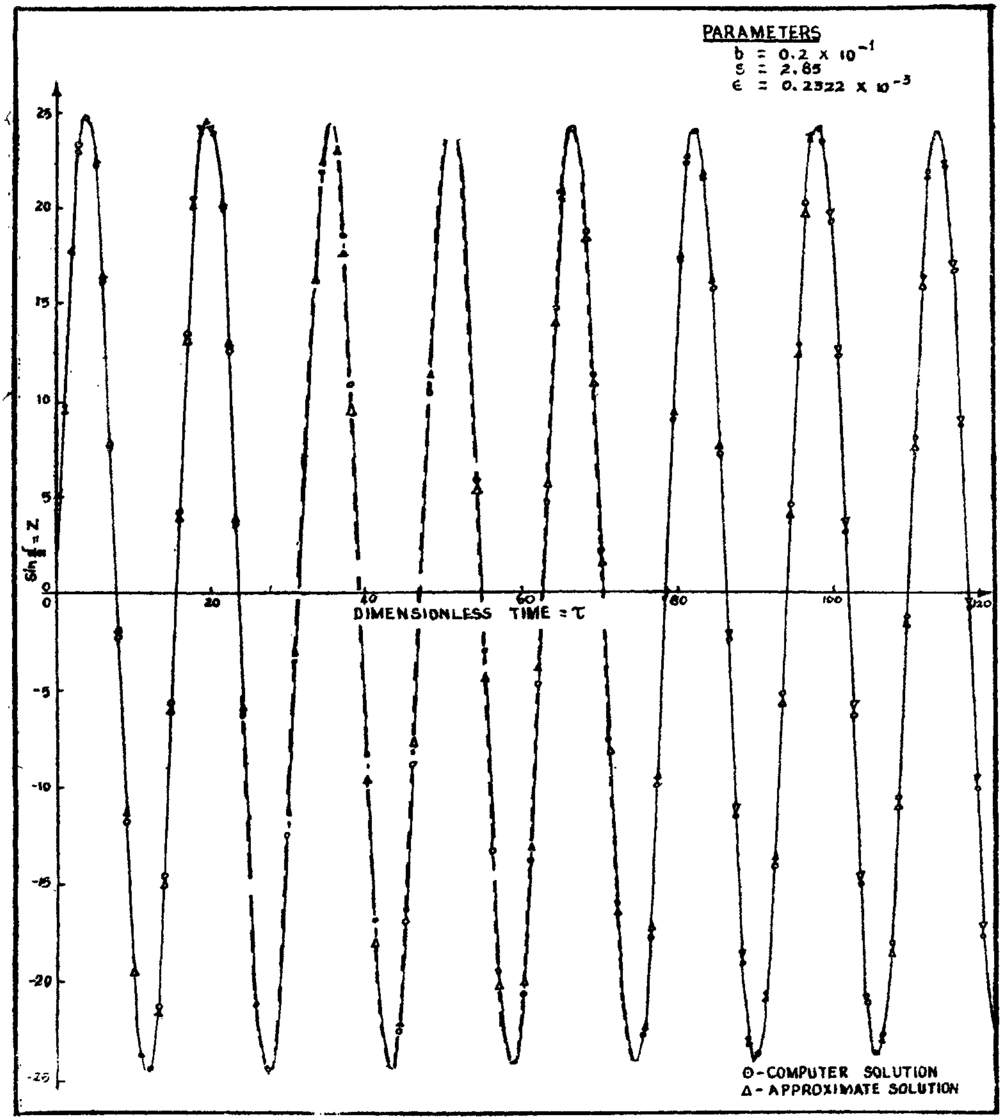

PARAMETERS
b = 0.2 x 10^-1
δ = 2.65
ε = 0.2322 x 10^-3
DIMENSIONLESS TIME = τ
0-COMPUTER SOLUTION
Δ-APPROXIMATE SOLUTION

FIG. 4

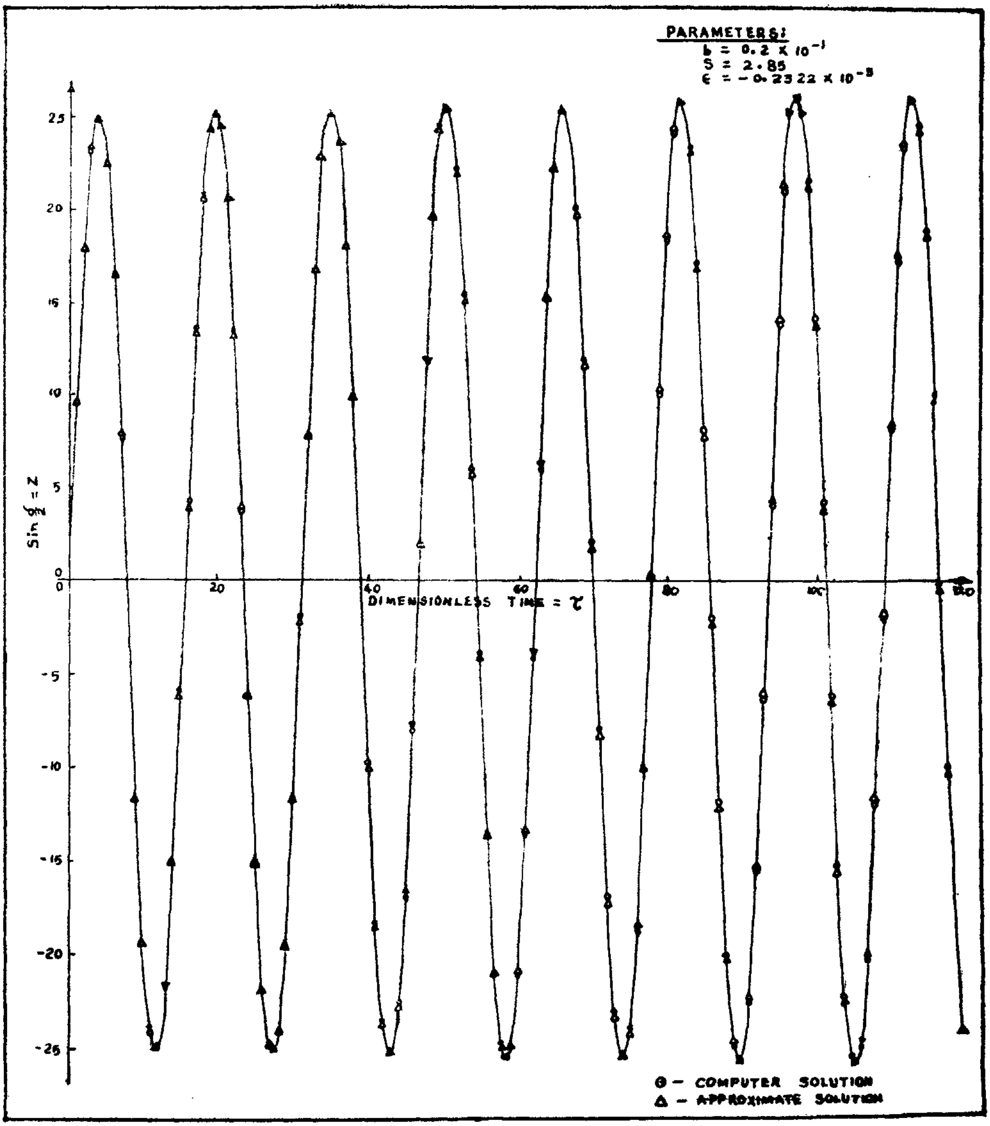

PARAMETERS:
b = 0.2 × 10⁻¹
S = 2.85
ε = −0.2322 × 10⁻⁵
Sin $\frac{\delta}{2}$ = Z
25
20
15
10
5
0
-5
-10
-15
-20
-25
0
20
40
60
80
100
DIMENSIONLESS TIME = τ
⊖ — COMPUTER SOLUTION
△ — APPROXIMATE SOLUTION

FIG. 5

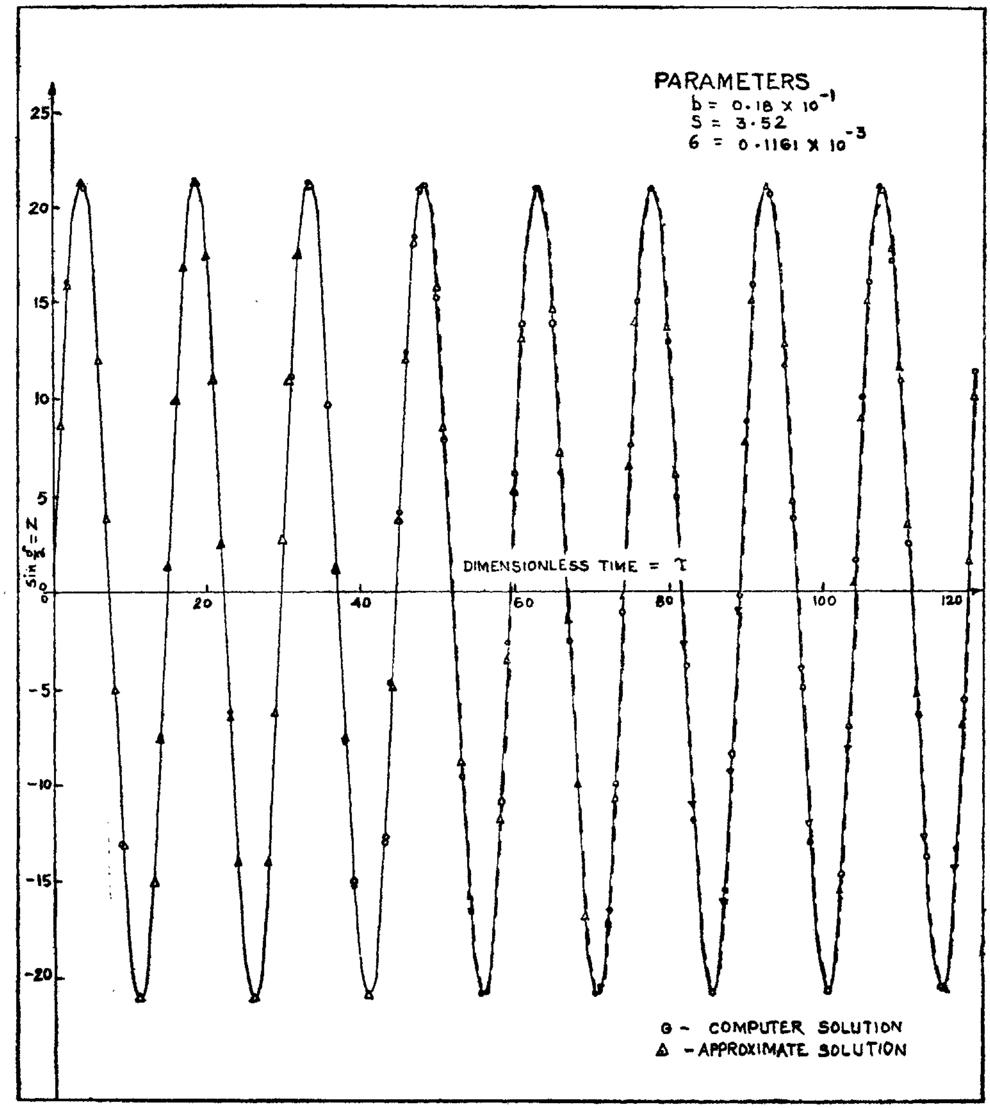

PARAMETERS
b = 0.18 X 10^-1
S = 3.52
6 = 0.1161 X 10^-3
DIMENSIONLESS TIME = τ
0
20
40
60
80
100
120
25
20
15
10
5
-5
-10
-15
-20
⊖ - COMPUTER SOLUTION
Δ - APPROXIMATE SOLUTION

FIG. 6

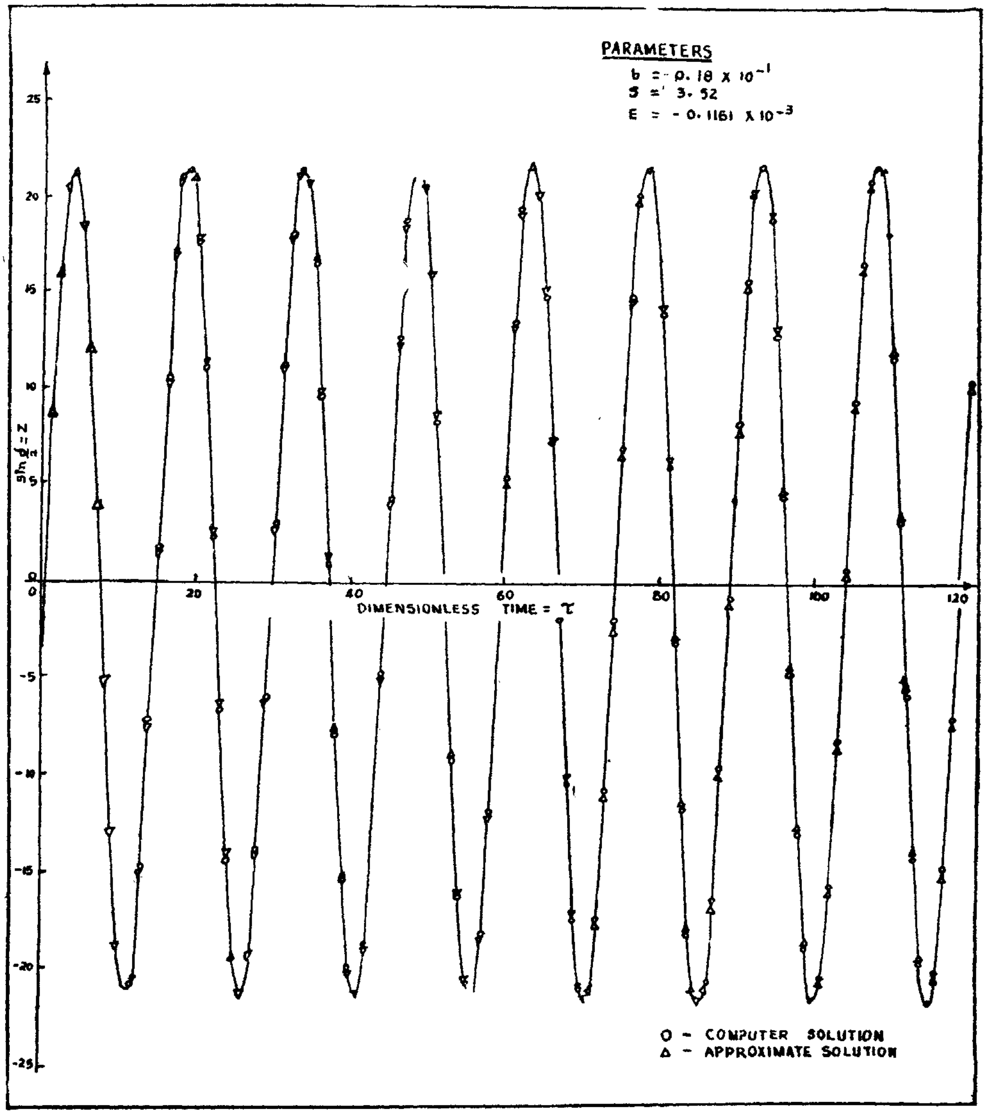

PARAMETERS
b = -0.18 X 10^-1
S = 3.52
E = -0.1161 X 10^-3
DIMENSIONLESS TIME = τ
O - COMPUTER SOLUTION
Δ - APPROXIMATE SOLUTION

FIG. 7

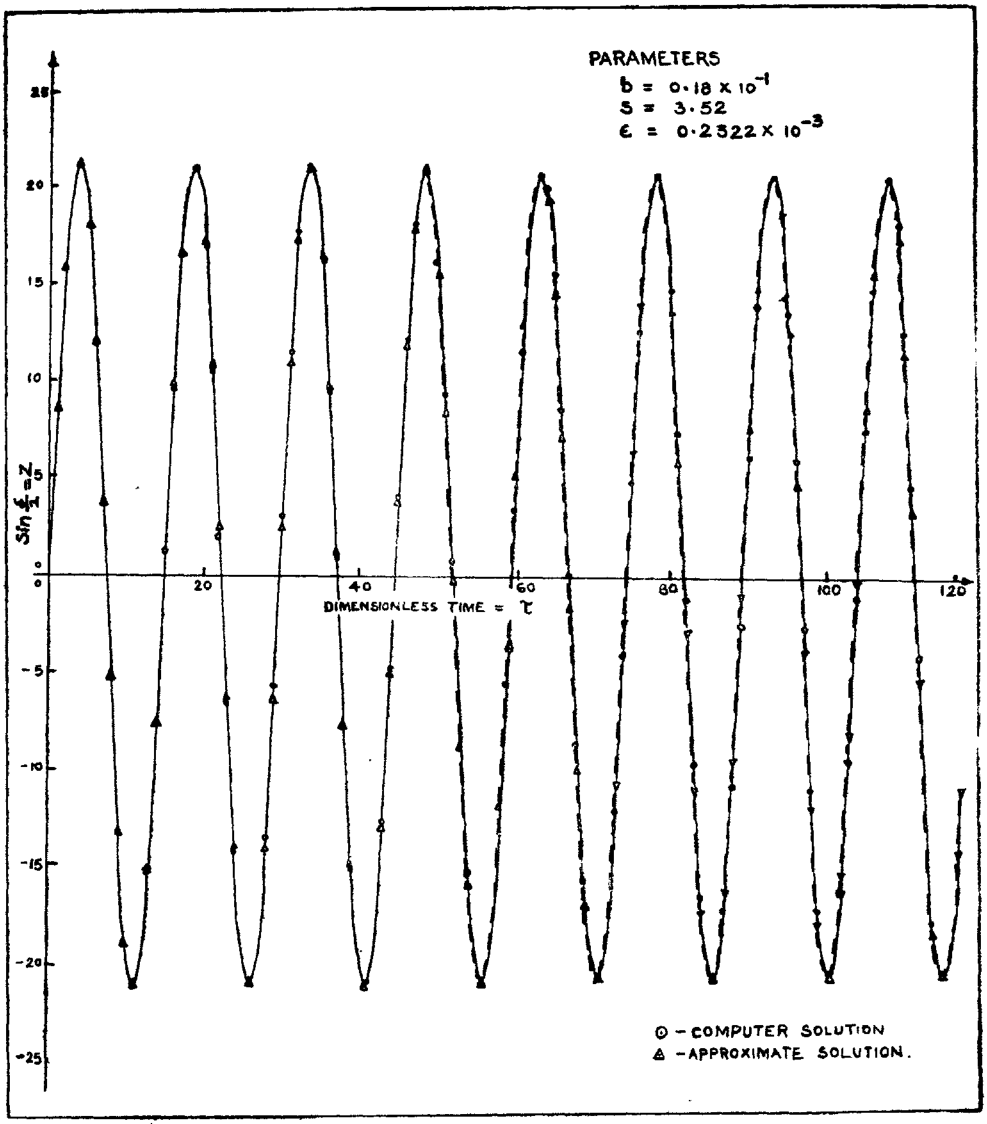
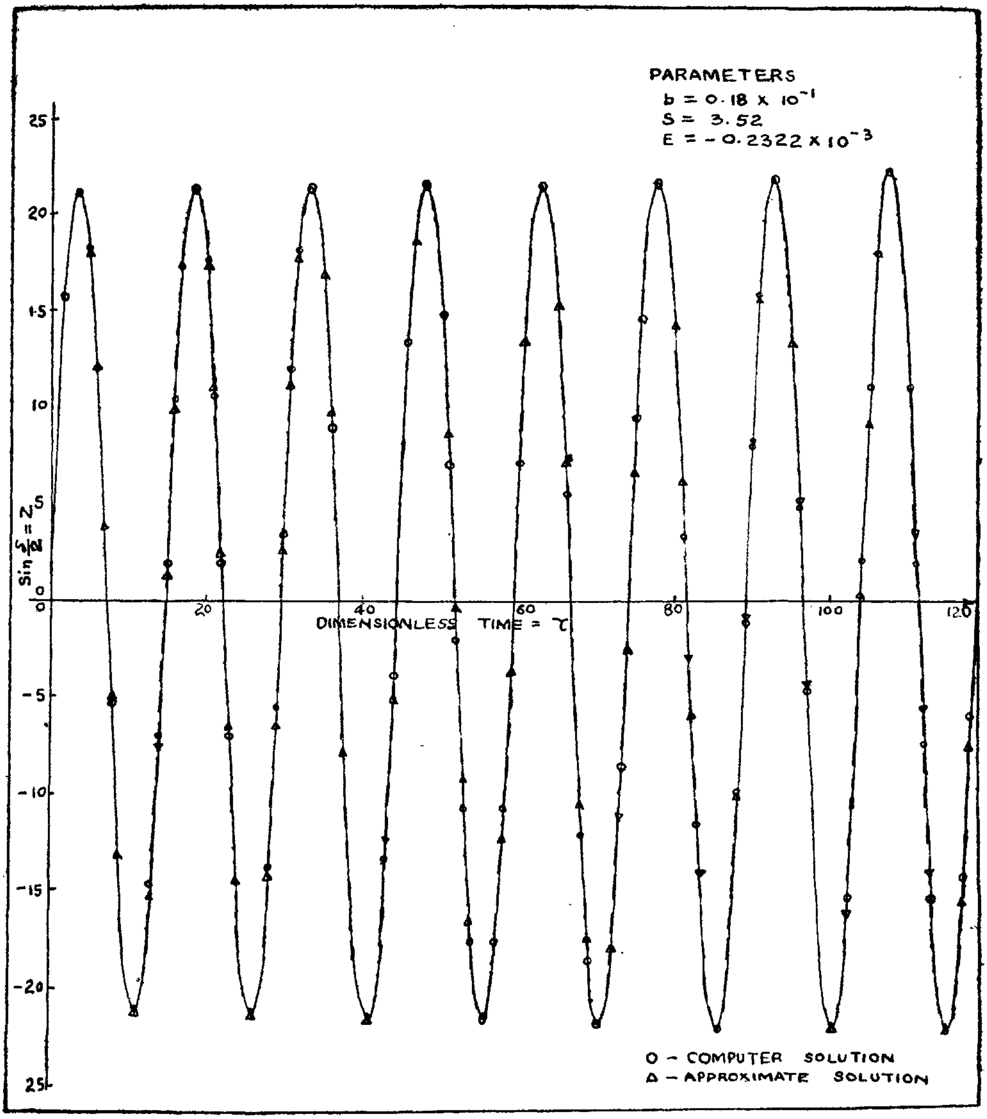

PARAMETERS
b = 0·18 x 10⁻¹
S = 3·52
ε = 0·2322 x 10⁻³
Sin φ/2 = Z
DIMENSIONLESS TIME = τ
O - COMPUTER SOLUTION
Δ - APPROXIMATE SOLUTION.

FIG. 8


PARAMETERS
b = 0·18 x 10⁻¹
S = 3.52
E = -0.2322 x 10⁻³
Sin ξ/2 = Z
DIMENSIONLESS TIME = τ
O - COMPUTER SOLUTION
Δ - APPROXIMATE SOLUTION

Fig, 9

4. Qualitative Analysis of the Nutational Oscillation in the Presence of Magnus Effects

In the present section we shall study all the solutions of eqns. (3.8) geometrically, particularly the behaviour of the trajectories of (3.8) near the singular point, i.e. the origin.

Transforming the system (3.8) into the polar coordinates by the substitution

$$z = \rho \cos \theta, \qquad y = \rho \sin \theta \tag{4.1}$$

we have,

$$\rho' = \rho\, z(\theta) \qquad \theta' = N(\theta) \tag{4.2}$$

where

$$z(\theta) = (1-\sigma^2/4) \sin\theta \cos\theta - (\epsilon/2) \cos^2\theta \tag{4.3}$$

$$N(\theta) = -\left[(\sigma^2/4) \cos^2\theta + \sin^2\theta + (\epsilon/2) \cos^2\theta \cot\theta \right]. \tag{4.4}$$

Since the pole in the present case is an isolated singular point for (4.2) we have

$$z^2(\theta) + N^2(\theta) > 0. \tag{4.5}$$

For any point P distinct from the pole of argument θ one indicates with α $(-\pi < \alpha \leq \pi)$ the angle that the radius vector OP forms with the positive tangent at P to the trajectory of (4.2) from P. The positive tangent means oriented according to the positive direction on the trajectoroy, i.e. in accordance with the way the trajectory has been described for increasing τ and we have

$$\sin\alpha = N(\theta) \operatorname{cosec}\theta \left[1 + \frac{\sigma^4}{16} \cot^2\theta + \frac{\epsilon^2}{4} \cot^4\theta + \frac{\sigma^2\epsilon}{4} \cot^3\theta \right]^{-\frac{1}{2}}. \tag{4.6}$$

Hence the rays from O (with O delected) are the isoclines for the family of trajectories of (4.2). If for one such ray we have $\alpha = 0$, the ray itself is the trajectory known as invariant ray (Sansone and Conti 1964).

The ray in the opposite direction is also an invariant ray. Obviously from (4.6) we have

$$N(\theta) = 0 \tag{4.7}$$

for an invariant ray. It may now be seen that for the present system we have an isolated invariant ray given by the solution of

$$\frac{\sigma^2}{4} \cos^2\theta + \sin^2\theta + \frac{\epsilon}{2} \cos^2\theta \cot\theta = 0 \tag{4.8}$$

since $\theta = \frac{\pi}{2}$ is not a solution of (4.8) the equivalent condition may be written as

$$\tan^3 \theta + \frac{\sigma^2}{4} \tan \theta + \frac{\epsilon}{2} = 0. \tag{4.8a}$$

This equation has only one real root given by

$$\tan \theta_0 = \left\{ \left[\frac{\epsilon}{4} + \sqrt{\frac{\epsilon^2}{16} + \frac{\sigma^6}{27 \times 64}} \right\}^{\frac{1}{3}} + \left\{ \frac{\epsilon}{4} - \sqrt{\frac{\epsilon^2}{16} + \frac{\sigma^6}{27 \times 64}} \right\}^{\frac{1}{3}}. \tag{4.9}$$

This root is positive or negative according as

$$\epsilon \lessgtr 0.$$

Now θ_0 is an isolated zero of the equation (4.8*a*) and it can be seen that

$$z(\theta_0) = \tan \theta_0. \tag{4.10}$$

Now the pole (the singular point) is stable or unstable according as the radius (i.e. the trajectory) is traversed towards or away the equilibrium point for increasing τ. Due to (4.9) and (4.10), $z(\theta_0)$ is positive or negative according as $\epsilon \lessgtr 0$. It can be seen from the equation for ρ' in (4.2) that the pole is (a nodal point) stable or unstable according as $\epsilon \gtrless 0$.

Further we discuss the behaviour of the trajectory in the neighbourhood of the invariant ray $\theta = \theta_0$. Since $z(\theta_0) = \tan \theta_0 = 0$, due to (4.10), there exists an angle $|\theta - \theta_0| < \beta$, of vertex 0 called the normal angle.

For definiteness one can assume

$$0 < \beta < \frac{\pi}{2}.$$

It can be easily verified that θ_0 is a simple zero of the function $N(\theta)$ and

$$\left[\frac{dN(\theta)}{d\theta}\right]_{\theta=\theta_0} = \left[-2\left(1 - \frac{\sigma^2}{4}\right) \cos\theta \sin\theta + \frac{\epsilon}{2} \cot^2\theta + \epsilon \cos^2\theta\right]_{\theta=\theta_0}$$

and using (4.8*a*) we have.

$$\left[\frac{dN(\theta)}{d\theta}\right]_{\theta=\theta_0} = -2 \tan\theta_0 + \frac{\epsilon}{2} \cot^2\theta_0. \tag{4.11}$$

Now the trajectories belonging to the normal angle are convex or concave towards the pole o according as

$$z^2(\theta_0) + z(\theta_0) \left[\frac{dN(\theta)}{d\theta}\right]_{\theta=\theta_0} \lessgtr 0 \text{ respectively.}$$

For the proof of this result one may refer to Sansons and Conti (1964). Thus we have using (4.10) and (4.11)

$$z^2(\theta_0)+z(\theta_0)\left[\frac{dN(\theta)}{d\theta}\right]_{\theta=\theta_0} = \frac{\epsilon}{2}\cot\theta_0-\tan^2\theta_0 .$$

Substituting for $\frac{\epsilon}{2}$ from (4.8*a*) we get

$$z^2(\theta_0)z(\theta_0)\left[\frac{dN(\theta)}{d\theta}\right]_{\theta=\theta_0} = -\left[2\tan^2\theta_0+\frac{\sigma^2}{4}\right]. \tag{4.12}$$

Since $z^2(\theta_0)+z(\theta_0)\left[\frac{dN(\theta)}{d\theta}\right]_{\theta=\theta_0}$ is negative,

from (4.12), the trajectories are convex towards the equiibrium point. These trajectories are stable or unstable according as ρ' is negative or positive for increasing τ and a theorem in Sansone and Conti (1964. p. 59). states, if A is a normal angle of the system for which

$$z^2(\theta_0)+z(\theta_0)\left[\frac{dN(\theta)}{d\theta}\right]_{\theta=\theta_0} < 0 ,$$

then all the tranjectories in A tend towards the point at infinity on the invariant ray for τ increasing if $z(\theta_0)<0$ and have for its asymptotic the invariant ray. We, therefore, conclude that the trajectories in the normal angle are also stable or unstable according as $\epsilon \gtrless 0$ and these are shown in Figs. 10. and 11.

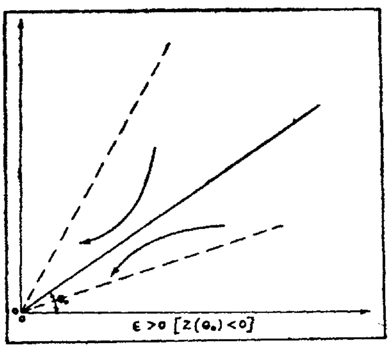


FIG. 10

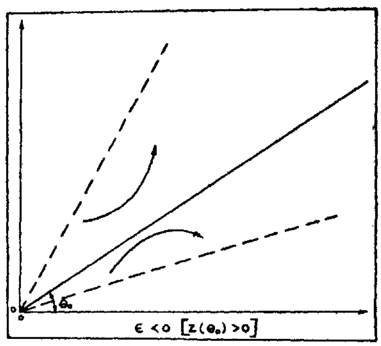


FIG. 11

A similar result with some restriction on the precessional angular velocity however can be established for the complete set of equation (2.8*a*) and (2.9*a*) by second method of Liapunov.

5. Magnus Instability by Second Method of Liapunov

If we perturb the axis of the shell from its normal motion $l=1$, $m=0$, $n=0$ and $P=N$, $q=0$, $r=0$ where p, q, r are the angular velocities of the shell round the body axis kinematically defined along the vectors $\vec{\Lambda}$, $(\vec{\Lambda} \cos\delta - \vec{x})$ $(\vec{x} \times \vec{\Lambda})$, the perturbed state of the shell may be written as

$$\left.\begin{aligned} l &= \cos\delta = 1 + \xi \\ m &= \sin\delta\cos\phi = \bar{m} \\ n &= \sin\delta\,\sin\phi = \bar{n} \end{aligned}\right\} \tag{5.1}$$

and

$$\left.\begin{aligned} p &= N + \alpha \\ q &= \phi'\sin\delta = \bar{q} \\ r &= \delta' = \bar{r}. \end{aligned}\right\} \tag{5.2}$$

It may be noted that $\alpha = 0$ as the axial angular velocity is conserved throughout the motion. Using (5.1) and (5.2) in (2.8a) and (2.9a) we have

$$(\xi + \phi'\sin^2\delta)' + \varepsilon\sin^2\delta = 0 \tag{5.3}$$

and

$$(\xi'^2 + \phi'^2\sin^2\delta)' + \frac{\xi'}{2s} + 2\varepsilon\,\phi'\sin\delta = 0 \tag{5.4}$$

as the perturbation equations.

Also we have the kinematical relation

$$(1+\xi)^2 + \bar{m}^2 + \bar{n}^2 = 1.$$

which gives

$$\xi^2 + 2\xi + \sin^2\delta = 0. \tag{5.5}$$

To construct a Liapunov function for the perturbed system of equations we consider the following functions

$$V_1 = \delta'^2 + \phi'^2\sin^2\delta + \frac{\xi}{2s} \tag{5.6}$$

$$V_2 = \xi + \phi'\sin^2\delta \tag{5.7}$$

and choose an arbitrary function

$$V_3 = \xi^2 + 2\xi + \sin^2\delta. \tag{5.8}$$

The required Liapunov function may now be taken as

$$V = V_1 + 2\lambda V_2 - \left(\lambda + \frac{1}{4s}\right) V_3$$

$$= \delta'^2 + \left[\phi^2 \sin^2 \delta + 2\lambda \phi'^2 \sin \delta - \left(\lambda + \frac{1}{4s}\right) \sin^2 \delta \right] - \left(\lambda + \frac{1}{4s}\right) \xi^2 \tag{5.9}$$

where λ is hitherto arbitrary.

V is a positive definite function of the variables δ', $\phi' \sin \delta$, $\sin \delta$ and ξ if and only if the following inequalities are simultaneously satisfied :

$$-\left(\lambda + \frac{1}{4s}\right) > 0 \tag{5.10}$$

$$\begin{vmatrix} 1 & \lambda \\ \lambda & \left(\lambda + \frac{1}{4s}\right) \end{vmatrix} > 0. \tag{5.11}$$

These inequalities result from one stipulation that the quadratic forms inside the square bracket in (5.9) are positive definite.

The inequality (5.11) may be written as

$$\lambda^2 + \lambda + \frac{1}{4s} < 0$$

$$(\lambda + \tfrac{1}{2})^2 - \sigma^2 < 0. \tag{5.11 a}$$

This will be satisfied if and only if

$$\sigma^2 = 1 - \frac{1}{s} > 0 \tag{5.12}$$

and

$$-\frac{1-\sigma}{2} > \lambda > -\frac{1+\sigma}{2}. \tag{5.13}$$

It may be noted that (5.11 a) contains (5.10). Since σ never exceeds unity and λ must always be negetive by (5.13).

If we choose a $\lambda < 0$ such that it is bounded as shown in (5.13) and further assume

$$\sigma^2 > 0. \tag{5.14}$$

The function V is positive definite. Now if we calculate dV/dt along all the perturbed trajectories given by (5.3), (5.4) with (5.5), we have

$$\frac{dV}{d} = -2\epsilon \sin^2 \delta \, (\phi' + \lambda) \tag{5.15}$$

For all unsteady motions where $\phi' + \lambda > 0$, i.e. where the precessional angular velocity of the shell exceeds $|\lambda|_{min}$ and λ is bounded as shown in

(5.13) it is now clear that even with a stability reserve (5.12) $\varepsilon < 0$ will make the motion of the shell unstable, otherwise the motion is asymptotically stable when $\epsilon > 0$.

6. Magnus Effect in Nielsen-Synge Stability Criteria

We shall consider the stability conditions of Nielsen-Synge (1946) in the form

$$K_1 < 0 \tag{6.1}$$

$$K_1^2 K_2 + K_1 K_2 K_4 - K_4^2 > 0. \tag{6.2}$$

These are the only necessary and sufficient conditions so far as the classical linar stability theory is concerned (Rath and Ram 1969). Here (6.2) contains all the magnus effects and therefore we must examine only this condition. For the purpose we consider projectiles whose centre of gravity and centroids coincide so that the effective aerodynamic force system is that of Fowler *et al.* (1921) only (See Nielson and Synge 1946).

In addition we may assume as usual with Murphy (1953) that the cross-Magnus force is weak and contributions to stability comes only from the cross-Magnus torque. Therefore we have

$$K_1 = -(\nu + h) \tag{6.3}$$

$$K_2 = \Omega \tag{6.4}$$

$$K_3 = \nu h - \frac{\Omega^2}{4s} \tag{6.5}$$

$$K_4 = \Omega (\gamma - \nu . \tag{6.6}$$

The aerodynamic parameters ν, h have been defined by Fowler *et al.* (1929) to be associated with the Normal force and a torque that usually damps the cross angular velocity of the shell.

Now substituting (6.3) – (6. 6.) in (6.2) and with straight forward algebraic deduction we have

$$0 < \frac{1}{s} < \frac{4(h+\gamma)(\nu-\gamma)}{(\nu+h)^2} + \frac{4h}{\Omega^2} . \tag{6.7}$$

For common artillery shells the second term in the right hand side of (6.7) is negligible compared to the first and we are left with the final stability conditions as

$$0 < \frac{1}{s} < \frac{4(h+\gamma)(\nu-\gamma)}{(\nu+h)^2} . \tag{6.8}$$

Since $\nu + h$ is non-negative due to (6.1) and (6.3) we must have $h+\gamma$ and $\nu-\gamma$ both non-negative as this is clear from (6.8).* Thus an admissible range of values for γ consistent with gyroscopic stability is

$$-h < \gamma < \nu. \tag{6.9}$$

* This is because $h+\gamma$ and $\nu-\gamma$ must have the same sign and their sum, ie. $h+\nu > 0$.

Since for an artillery shell h and ν are both positive, negative values of γ lies within the limits prescribed by (6.9). But if, however, γ is strongly negative such that $h+\gamma < 0$ gyroscopic stability will not hold. If the damping effect of h is neglected (6.8) reduces to

$$0 < \frac{1}{s} < \gamma\,(\nu - \gamma)/\nu^2 \tag{6.10}$$

and this condition is clearly violated when $\gamma < 0$ and this is precisely the result which we have proved in the previous section. When cross magnus effects due to γ is absent, (6.8) reduces to the well known Nielson-Synge Kent condition.

Concluding Remarks

The stability of a spinning artillery shell is essentially gyroscopic in character. This stability could be lost in the presence of dissipative air forces only. In particular the Magnus torque and torque due to yawing provide the total dissipative effect and this fact has been very clearly brought out in section 6, where we have observed that gyroscopic stability is lost when $h+\gamma < 0$. Obviously when the Magnus torque is strongly negative so that it out-weighs the damping effects provided by the torque due to yawing, instability would result, otherwise it would be erroneous to expect instability just because the Magnus torque is negative.

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