CONJUGATE PROBLEM OF HEAT TRANSFER IN A SWEAT COOLED POROUS HOLLOW CYLINDER THROUGH WHICH HOT FLUID IS FLOWING

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Conjugate problem of a sweat cooled porous hollow cylinder through which hot fluid is flowing along with axis of the cylinder, has been solved with prescribed boundary conditions, The importance of the problem is to study the temperature distribution on the surface of the bodies in contact with hot fluid.

INTRODUCTION

In aeronautical engineering, blading and casing of gas turbines, combustion chamber liners, rocket nozzles and outside skin of high speed flying vchicles are few examples where it is necessary to cool down the hotter surface by means of same cooling arrangements. The author (Marwah 1972) has already studied the problem of a transpirated cooled porous plate in conjugation with fluid flowing upon its surface.

Weinbaum and Wheelar (1949) have solved the non-conjugate problem for a hollow cylinder in which coolant is flowing radially towards the axis of the cylinder. Grootenhius (1959) has also solved the problem for the case of a flat plate considering a third order differential equation to find out the temperature distribution in the porous bodies.

In this paper heat conduction equation in a hollow cylinder has been solved in conjugation with the energy equation of the fluid flowing through the cylinder along the axis of the cylinder. A third order differential equation has been solved to find out the temperature distribution in the porous cylinders after applying Kummer's technique (Poole 1936). Graphical representations have been depicted for various distances coordinates along the axis as well as for different conductivities of the porous cylinder.

NOMENCLATURE

T_1 = temperature of the porous cylinder
T_c = temperature of the collant
T_2 = temperature of the hot fluid
T_∞ = temperature of the hot fluid at $r = 0$
K_s = thermal conductivity of the porous cylinder.
K_f = thermal conductivity of the hot fluid
l = thickness of the porous cylinder
s = porosity of the cylinder
d = diameter of the cylinder
h = co-efficient of heat transfer wherein heat flows from hot fluid to the hollow cylinder
G_c = mass of the collant flowing per unit time
C_c = specific heat of the collant
U_m = velocity of the hot fluid flowing
a_1 = outer radius of the cylinder
b_1 = inner radius of the cylinder
$b = A_2/A_3$
$c = A_2/A_1$
$A_1 = K_s . 2\pi\, l\, (l-s)$
$A_2 = 2\pi^2\ Nd\, l\, h$
$A_3 = G_c\ C_c$
N = number of forces in the porous body
Z = distance coordinate along the axis of the cylinder

PROBLEM

Heat balance equation in the porous cylinder taking into account of collant entering in it is prescribed as

$$A_1 \left(\frac{d\,T_1}{d\,r} + r\,\frac{d^2\,T_1}{dr^2} \right) = A_2\, r\,(T_1 - T_c) = A_3\,\frac{d\,T_c}{dr}\,. \tag{1}$$

The direction of heat flow by conduction in the porous cylinder is taken to be opposite than that of the direction of the collant. Hot fluid is flowing along the axis of the cylinder as shown in Fig. 1 and is supposed to be laminar flow with constant temperature gradient. Therefore, the energy equation of the fluid (Goldstein 1950) is as follows :

$$2\,U_m \left(1-\left(\frac{r}{a_1}\right)^2\right)A = K_f\left(\frac{d^2g}{dr^2}+\frac{1}{r}\,\frac{dg}{dr}\right) \tag{2}$$

where A is the constant temperature gradient along the axis of the cylinder. The temperature T_2 is taken to be connected with $g\,(r)$ with the relation

$$T_2 = Az + g(r) \tag{3}$$

where $g\,(r)$ is the solution of differential equation (2).

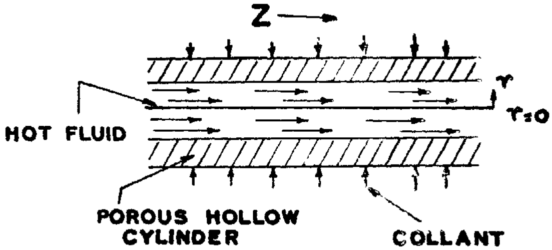

FIG. 1

From 1st and 2nd of eqn. (1)

$$T_c = T_1 - \frac{A_1}{A_2}\left(\frac{1}{r}\,\frac{d\,T_1}{dr}+\frac{d^2\,T_1}{dr^2}\right) \tag{4}$$

Differentiating it w. r. t. r

$$\frac{d\,T_c}{dr} = \frac{d\,T_1}{dr} - \frac{A_1}{A_2}\left(\frac{1}{r}\,\frac{d^2\,T_1}{dr^2}-\frac{1}{r^2}\,\frac{d\,T_1}{dr}+\frac{d^3\,T_1}{dr^3}\right). \tag{5}$$

Again from 2nd and 3rd of eqn. (1)

$$\frac{d\,T_c}{dr} \quad \frac{A_1}{A_2}\,r\left(\frac{d^2\,T_1}{dr^2}+\frac{1}{r}\,\frac{d\,T_1}{dr}\right). \tag{6}$$

Comparing (5) and (6) and rearranging we get

$$r^2\,\frac{d^3\,T_1}{dr^3}+(br^2+1)\,r\,\frac{d^2\,T_1}{dr^2}+((b-c)\,r^2-1)\,\frac{d\,T_1}{dr}=0. \tag{7}$$

The differential equation (7) is reducible to Kummer's first confluent hypergeometric equation of 2nd order of the form

$$r\,\frac{d^2\,T_1}{dr^2}+(\alpha-r)\,\frac{d\,T_1}{dr}-\beta\,T_1=0 \tag{8}$$

The solution of (8) is written as

$${}_1F_1\,(\beta\,;\alpha,r) = 1+\frac{\beta}{1!\,\alpha}\,r+\frac{\beta\,(\beta+1)}{2\,!\,2\,(\alpha+1)}\,r^2+\ldots\ldots \tag{9}$$

Transforming eqn. (7) to the form (8) we get the solution of (7) as

$$\frac{dT_1}{dr} = {}_1F_1\left(\frac{(b-c)}{2b} + \frac{1}{2}; 2; r\right). \tag{10}$$

But we know

$$\frac{d}{du}\,{}_1F_1\,(\beta; \alpha; u) = \frac{\beta}{\alpha}\,{}_1F_1\,(\beta+1; \alpha+1; u). \tag{11}$$

Simplifying and applying the technique of Weinbaum and Wheeler (1949) we get the complete solution of (7) as

$$T_1 = C_1\, e^{-0\cdot 5\, b r^2}\, r^{-\{(c/b)+2\}} + C_2 r^{c/b} + C_3. \tag{12}$$

From eqn. (2) we get

$$g\,(r) = \frac{U_m\, A r^2}{2K_f}\left(1 - \frac{r^2}{4a_1^2}\right) + C. \tag{13}$$

Substituting this value in (3)

$$T_2 = AZ + \frac{U_m\, A r^2}{2K_f}\left(1 - \frac{r^2}{4a_1^2}\right) + C. \tag{14}$$

Boundary conditions applied in the problem are as follows

$$T_1 = 0\,; \qquad r = a_1 \tag{15}$$

$$K_s \frac{dT_1}{dr} = K_f \frac{dT_2}{dr} \quad ; r = b_1 \tag{16}$$

$$T_1 = T_2 \qquad ; r = b_1 \tag{17}$$

$$T_2 = T_\infty \qquad ; r = 0,\ Z = 0. \tag{18}$$

Applying these boundary conditions on (12) and (14) we obtain

$$T_1 = C^{-0\cdot 5\, b r^2}\, r^{-(c/b+2)}\ \Delta_1 + r^{c/b}\ \Delta_2 + \Delta_3 \tag{19}$$

where

$$\Delta_1 = \frac{\left(AZ + \frac{U_m\, A b_1^2}{2k_f}\left(1 - \frac{b_1^2}{4a_1^2}\right) + T_\infty\right) K_s\,(c/b)\, b_1^{\{(c/b)-1\}} - U_m\, A b_1 \left(1 - \frac{b_1^2}{4a_1^2}\right)\left(b_1^{c/b} - a_1^{c/b}\right)}{\left(b + (c/b+2)/b_1^2\right)\left(b_1^{c/b} - a_1^{c/b}\right) K_s e^{-0\cdot 5\, b b_1^2}\, b_1^{-\{(c/b)+1\}} + K_s\, c/b\, b_1^{(c/b-1)}\left(e^{-0\cdot 5\, b b_1^2}\, b_1^{-(c/b+2)} - e^{-0\cdot 5\, b a_1^2}\, a_1^{-(c/b+2)}\right)}$$

$$\Delta_2 = \frac{\left(AZ + \frac{U_m\, A b_1^2}{2k_f}\left(1 - \frac{b_1^2}{4a_1^2}\right) + T_\infty\right) - \left(e^{-0.5\, b b_1^2}\, b_1^{-(c/b+2)} - e^{-0.5\, b a_1^2}\, a_1^{-(c/b+2)}\right)\Delta_1}{b_1^{c/b} - a_1^{c/b}}$$

$$\Delta_3 = -\left(e^{-0.5\, a_1^2}\, a_1^{-(c/b+2)}\ \Delta_1 + a_1^{c/b}\ \Delta_2\right).$$

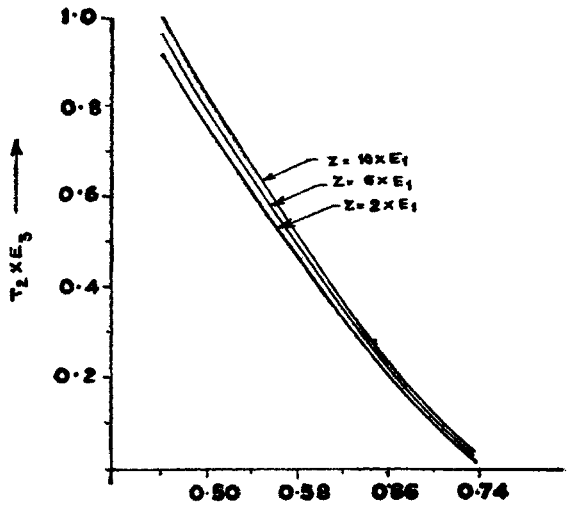

FIG. 2 Temperature distribution of the porous cylinder against different radii for different values of Z.

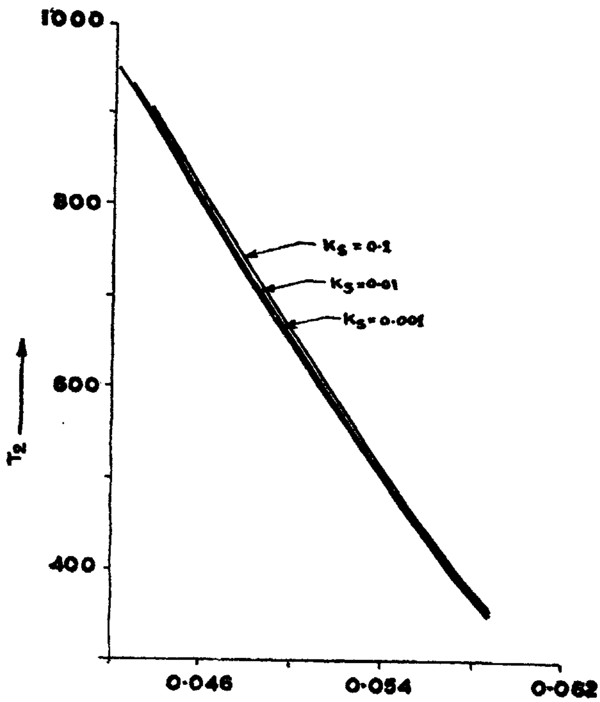

FIG. 3 Temperature distribution of the porous cylinder against radii for different values of thermal conductivity for fixed Z.

Also applying the boundary condition (18) on (14) we get

$$T_2 = AZ + \frac{U_m Ar^2}{2k_f}\left(1 - \frac{r^2}{4a_1^2}\right) + T_\infty . \qquad (10)$$

Results and Discussions

Fig. 2. shows the temperature distribution in the porous cylinder plotted against different radii for various values of distance coordinate Z. It is evident from the same that temperature of the cylinder decreases with increase in radius.

In Fig. 3 the effect of thermal conductivity on the temperature distribution in the porous cylinder has been exhibited. It is also clear as per the result that the effect of coolant fluid is increasad with the increase in the values of thermal conducting, the other values of physical contents taken for numerical values as $A = 1.0$, $U_m = 1E02$, $K_f = 0.01$, $a_1 = 0.0742$, $b_1 = 0.0417$, $T = 1E03$, $b = 13.564E3$, $C = 3.6$.

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