UNSTEADY FLOW OF A DUSTY VISCOUS LIQUID THROUGH ELLIPTIC DUCTS

by M. P. PATERIYA, *Department of Mathematics, Government College of Engineering and Technology, Raipur 492002 (M.P.)*

(Communicated by F. C. Auluck, F.N.A.)

(Received 12 September 1974)

The unsteady motion of a dusty viscous liquid with uniform distribution of dust particles through tubes of elliptic sections, under the influence of a time varying pressure gradient, has been investigated by using the technique of integral trasforms. Analytical expressions for fluid velocity profile, the particle 'slip' relative to the fluid and for the particle velocity profile are obtained. Particular cases of results for (i) the harmonic pressure gradient and (ii) the exponential pressure gradient have also been discussed. The results for ordinary viscous flow are deduced by making the mass 'M' of the dust particle tend to zero. The results obtained are found to be in agreement with Gupta (1964).

1. INTRODUCTION

Problems associated with the flow of dusty viscous fluids have been studied by several authors, notably by Saffman (1962), Michael (1965), Michael and Miller (1966), Liu (1966), Michael and Norey (1968). Michael and Norey have considered the motion of a dusty gas contained between two co-axial cylinders which start to rotate impulsively from rest.

The unsteady flow of a dusty viscous liquid through a circular cylinder under the influence of exponential pressure gradient has been discussed by Rao (1969). In a recent paper, Verma and Mathur (1973) have investigated an analogous problem by assuming an expression for the flow velocity.

In the present paper we have investigated the problem of unsteady motion of a dusty viscous liquid through long elliptic ducts under the influence of a pressure gradient varying arbitrarily with time. The flow through a channel of two co-axial elliptic pipes has also been considered. The solution has been furnished by using integral transforms. Particular cases of results for harmonic and exponential pressure gradients have also been discussed. The results for ordinary viscous flow are deduced by making the mass 'M' of the dust particle tend to zero. These results are in agreement with those obtained by Gupta (1964).

In deriving the solution of the above problem we have assumed that

(i) the dust particles are uniform in size and shape and are uniformly distributed, bulk concentration of dust being small enough to be neglected,

(ii) the density of the material of dust is large compared to fluid density so that the mass concentration of dust is an appreciable fraction of unity,

(iii) the fluid and the dust particles are at rest initially.

2. Required Integral Transforms

Gupta (1964) has introduced a finite integral transform defined as

$$\bar{f}(q_{2n,\,m}) = \int_0^{\xi_0} \int_0^{2\pi} f(\xi, \eta) (\cosh 2\xi - \cos 2\eta)\, Ce_{2n}(\xi, q_{2n,\,m})\, ce_{2n}(\eta, q_{2n,\,m})\, d\xi\, d\eta \qquad ...(2.1)$$

for which the inversion theorem is

$$f(\xi, \eta) = \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \frac{\bar{f}(q_{2n,\,m})\, Ce_{2n}(\xi, q_{2n,\,m})\, ce_{2n}(\eta, q_{2n,\,m})}{\pi \int_0^{\xi_0} C^2e_{2n}(\xi, q_{2n,\,m}) [\cosh 2\xi - \Theta_{2n,\,m}]\, d\xi} \qquad ...(2.2)$$

where $q_{2n,\,m}$ is a root of the equation

$$Ce_{2n}(\xi, q) = 0 \qquad ...(2.3)$$

and $\Theta_{2n,\,m}$ is given as [Mclachlan 1947, eqn. (9), pp. 177]

$$\Theta_{2n,\,m} = A_0^{(2n)} A_2^{(2n)} + \sum_{r=0}^{\infty} A_{2r}^{(2n)} A_{2r+2}^{(2n)}. \qquad ...(2.4)$$

The sum (2.2) must be taken over all the positive roots of eqn. (2.3).

Another transform defined by Gupta is as given below

$$\bar{u}(q_{2n,\,m}) = \int_{\xi_0}^{\xi_1} \int_0^{2\pi} u(\xi, \eta) (\cosh 2\xi - \cos 2\eta)\, B_{2n}(\xi, q_{2n,\,m}) \times Ce_{2n}(\eta, q_{2n,\,m})\, d\xi\, d\eta \qquad ...(2.5)$$

whose inversion theorem is

$$u(\xi, \eta) = \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \frac{\bar{u}(q_{2n,\,m})\, B_{2n}(\xi, q_{2n,\,m})\, Ce_{2n}(\eta, q_{2n,\,m})}{\pi \int_{\xi_0}^{\xi_1} B_{2n}^2(\xi, q_{2n,\,m}) [\cosh 2\xi - \Theta_{2n,\,m}]\, d\xi} \qquad ...(2.6)$$

where $q_{2n,\,m}$ is a root of the equation

$$Ce_{2n}(\xi, q)\, Fey_{2n}(\xi_0, q) - Fey_{2n}(\xi, q)\, Ce_{2n}(\xi_0, q) = 0 \qquad ...(2.7)$$

and

$$B_{2n}(\xi, q_{2n,\,m}) = [\{Fey_{2n}(\xi_0, q_{2n,\,m}) - Fey_{2n}(\xi_1, q_{2n,\,m})\}\, Ce_{2n}(\xi, q_{2n,\,m}) - \{Ce_{2n}(\xi_0, q_{2n,\,m}) - Ce_{2n}(\xi_1\, q_{2n,\,m})\}\, Fey_{2n}(\xi, q_{2n,\,m})] \qquad ...(2.8)$$

The sum (2.6) is to be taken over all the positive roots of (2.7).

The Laplace transform of any function $f(t)$ is defined as

$$\bar{\bar{f}}(p) = \int_0^\infty e^{-pt} f(t)\, dt, \text{Re } p > 0 \qquad ...(2.9)$$

provided the above integral exists.

3. Formulation of the Problem

We shall investigate the laminar flow of an unsteady viscous liquid with uniform distribution of dust particles, through a large elliptic duct, under the influence of a time varying pressure gradient. Since both the dust and fluid particles move along the length of the cylinder, the motion is along the axis Z of the tube and the distribution of dust particles is uniform. The velocity distributions of fluid and dust particles are defined respectively as

$$u_1 = 0, \; u_2 = 0, \; u_3 = u(\xi, \eta, t) \qquad ...(3.1)$$

$$v_1 = 0, \; v_2 = 0, \; v_3 = v(\xi, \eta, t) \qquad ...(3.2)$$

where (u_1, u_2, u_3) and (v_1, v_2, v_3) are velocity components of fluid and dust particles in the ξ, η, z directions respectively.

Further, assuming the number density N of dust particles equal to N_0, a constant, throughout the motion, we obtain the equations of motion for the dusty viscous liquid in elliptic cylindrical coordinates (ξ, η, z) as

$$\frac{\partial p}{\partial \xi} = \frac{\partial p}{\partial \eta} = 0 \qquad ...(3.3)$$

$$\frac{\partial u}{\partial t} = f(t) + \nu \left\{ \frac{2h^{-2}}{\cosh 2\xi - \cos 2\eta} \left(\frac{\partial^2 u}{\partial \xi^2} + \frac{\partial^2 u}{\partial \eta^2} \right) \right\} + \frac{kN_0}{\rho} (v - u) \qquad ...(3.4)$$

$$M \frac{\partial v}{\partial t} = k(u - v) \qquad ...(3.5)$$

where

M = mass of a dust particle,

k = the Stokes resistance coefficient,

ν = the kinematic coefficient of viscosity,

$2h$ = the interfocal length of the cylinder,

$f(t) = -\dfrac{1}{\rho} \dfrac{\partial p}{\partial z}$,

p = the fluid pressure,

and ρ = the fluid density.

The elliptic coordinates (ξ, η), defined in terms of the cartesian coordinates (x, y), are given by

$$x + iy = h \cosh(\xi + i\eta)$$

The boundary and initial conditions are

(i) $u = v = 0$ at $\xi = \xi_0$, $0 \leqslant \eta \leqslant 2\pi$; $t > 0$ ...(3.6)

(ii) $u = v = 0$ at $t = 0$, $0 < \xi < \xi_0, 0 < \eta < 2\pi$ ...(3.7)

4. Solution of the Problem

Employing the integral transform (2.1) to eqns. (3.4), (3.5) and using (3.6) we obtain

$$\frac{d\bar{u}}{dt} = A f(t) - \nu\lambda^2_{2n,\,m}\, \bar{u} + \frac{l}{\tau}(\bar{v} - \bar{u}) \qquad \text{...(4.1)}$$

and

$$\tau \frac{d\bar{v}}{dt} = \bar{u} - \bar{v} \qquad \text{...(4.2)}$$

where $4q_{2n,\,m}/h^2 = \lambda^2_{2n,\,m}$, $MN_0/\rho = l$, the mass concentration of dust, $M/K = \tau$, the relaxation time and

$$A = \int_0^{\xi_0}\int_0^{2\pi} (\cosh 2\xi - \cos 2\eta)\; Ce_{2n}(\xi, q_{2n,\,m})\, Ce_{2n}(\eta, q_{2n,\,m})\; d\xi\, d\eta.$$

Further, the initial condition (3.7) is transformed to

$$\bar{u} = \bar{v} = 0 \text{ when } t = 0. \qquad \text{...(4.3)}$$

Now applying the Laplace transform (2.9) to eqns. (4.1), (4.2) and using (4.3), we get

$$p\bar{\bar{u}} = A\bar{\bar{f}}(p) - \nu\lambda^2_{2n,\,m}\,\bar{\bar{u}} + \frac{l}{\tau}(\bar{\bar{v}} - \bar{\bar{u}}) \qquad \text{...(4.4)}$$

and

$$\tau p\bar{\bar{v}} = \bar{\bar{u}} - \bar{\bar{v}}. \qquad \text{...(4.5)}$$

Now eliminating $\bar{\bar{v}}$ between (4.4), (4.5) and rearranging the resulting equation, we obtain

$$\bar{\bar{u}} = \frac{A(\tau p + 1)\bar{\bar{f}}(p)}{\tau p^2 + (1 + l + \tau\nu\lambda^2_{2n,\,m})\,p + \nu\lambda^2_{2n,\,m}} \qquad \text{...(4.6)}$$

i.e.

$$\bar{\bar{u}} = \frac{A\bar{\bar{f}}(p)}{\tau(\alpha - \beta)}\left[\frac{1 + \tau\alpha}{p - \alpha} - \frac{1 + \tau\beta}{p - \beta}\right] \qquad \text{...(4.7)}$$

where α and β are the roots of the equation

$$\tau p^2 + (1 + l + \tau\nu\lambda^2_{2n,\,m})\,p + \nu\lambda^2_{2n,\,m} = 0$$

Now Laplace inversion of (4.7) yields

$$\bar{u} = \frac{A}{\tau(\alpha - \beta)}\Big[(1 + \tau\alpha) \int_0^t e^{\alpha\lambda} f(t - \lambda)\, d\lambda - (1 + \tau\beta) \int_0^t e^{\beta\lambda} f(t - \lambda)\, d\lambda\Big].$$

Finally, by the application of the inversion theorem (2.2) to the above equation, we obtain the expression for the fluid velocity profile as

$$u(\xi, \eta, t) = \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \frac{A'}{\tau(\alpha - \beta)}\Big[(1 + \tau\alpha) \int_0^t e^{\alpha\lambda} f(t - \lambda)\, d\lambda - (1 + \tau\beta) \int_0^t e^{\beta\lambda} f(t - \lambda)\, d\lambda\Big] Ce_{2n}(\xi, q_{2n,m})\, ce_{2n}(\eta, q_{2n,m}) \quad \ldots(4.8)$$

where

$$A' = A/\pi \int_0^{\xi_0} C^2e_{2n}(\xi, q_{2n,m})\, [\cosh 2\xi - \Theta_{2n,m}]\, d\xi \quad \ldots(4.9)$$

and the sum is taken over all the positive roots of (2.3). Further, from eqns. (4.5) and (4.6), we have

$$\bar{\bar{v}} = \frac{A\bar{\bar{f}}(p)}{\tau p^2 + (1 + l + \tau\nu\lambda^2_{2n,m})\, p + \nu\lambda^2_{2n,m}} = \frac{A\bar{\bar{f}}(p)}{\tau(\alpha - \beta)}\Big[\frac{1}{p - \alpha} - \frac{1}{p - \beta}\Big]. \quad \ldots(4.10)$$

Now taking the Laplace inversion of (4.10) and making the use of inversion theorem (2.2) in succession, we obtain the particle velocity profile as

$$v(\xi, \eta, t) = \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \frac{A'}{\tau(\alpha - \beta)}\Big[\int_0^t e^{\alpha\lambda} f(t - \lambda)\, d\gamma - \int_0^t e^{\beta\lambda} f(t - \lambda)\, d\lambda\Big] Ce_{2n}(\xi, q_{2n,m})\, ce_{2n}(\eta, q_{2n,m}) \quad \ldots(4.11)$$

where the sum is taken over all the positive roots of (2.3). From the relations (4.8) and (4.11), we find an expression for the particle fluid 'slip' velocity as

$$u(\xi, \eta, t) - v(\xi, \eta, t)$$
$$= \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \frac{A'}{\alpha - \beta}\Big[\alpha \int_0^t e^{\alpha\lambda} f(t-\lambda)\, d\lambda$$
$$- \beta \int_0^t e^{\beta t} f(t-\lambda)\, d\lambda \Big] Ce_{2n}(\xi, q_{2n,m})\, ce_{2n}(\eta, q_{2n,m}) \qquad \ldots(4.12)$$

5. Flow Under Harmonically Oscillating Pressure Gradient

Let

$$-\frac{1}{\rho}\frac{\partial p}{\partial z} = f(t) = k_0 \cos \omega t.$$

Substituting it in (4.8), (4.11) and (4.12), we obtain

$$u(\xi, \eta, t) = \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \frac{A'K_0}{\tau(\alpha - \beta)}\left[\frac{1+\tau\alpha}{\alpha^2+\omega^2}\gamma_1 - \frac{1+\tau\beta}{\beta^2+\omega^2}\gamma_2\right]$$
$$\times Ce_{2n}(\xi, q_{2n,m})\, ce_{2n}(\eta, q_{2n,m}) \qquad \ldots(5.1)$$

$$v(\xi, \eta, t) = \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \frac{A'K_0}{\tau(\alpha - \beta)}\left[\frac{\gamma_1}{\alpha^2+\omega^2} - \frac{\gamma_2}{\beta^2+\omega^2}\right]$$
$$\times Ce_{2n}(\xi, q_{2n,m})\, ce_{2n}(\eta, q_{2n,m}) \qquad \ldots(5.2)$$

and

$$u(\xi, \eta, t) - v(\xi, \eta, t)$$
$$= \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \frac{A'K_0}{\alpha - \beta}\left[\frac{\alpha\gamma_1}{\alpha^2+\omega^2} - \frac{\beta\gamma_2}{\beta^2+\omega^2}\right]$$
$$\times Ce_{2n}(\xi, q_{2n,m})\, ce_{2n}(\eta, q_{2n,m}) \qquad \ldots(5.3)$$

where

$$\gamma_1 = \alpha e^{\alpha t} - \alpha \cos \omega t + \omega \sin \omega t$$

and

$$\gamma_2 = \beta e^{\beta t} - \beta \cos \omega t + \omega \sin \omega t.$$

6. Flow under the Pressure Gradient Decreasing Exponentially with Time

Let

$$-\frac{1}{\rho}\frac{\partial p}{\partial z} = f(t) = k_0 e^{-\mu t}$$

where k_0 and μ are positive constants.

Then, from (4.8), (4.11) and (4.12), we have

$$u(\xi, \eta, t) = \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \frac{A'k_0}{\tau(\alpha-\beta)} \left[\frac{1+\tau\alpha}{\alpha+\mu}\phi_1 - \frac{1+\tau\beta}{\beta+\mu}\phi_2 \right] \times Ce_{2n}(\xi, q_{2n,m})\, ce_{2n}(\eta, q_{2n,m}) \quad \ldots(6.1)$$

$$v(\xi, \eta, t) = \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \frac{A'k_0}{\tau(\alpha-\beta)} \left[\frac{\phi_1}{\alpha+\mu} - \frac{\phi^2}{\beta+\mu} \right] \times Ce_{2n}(\xi, q_{2n,m})\, ce_{2n}(\eta, q_{2n,m}) \quad \ldots(6.2)$$

and

$$u(\xi, \eta, t) - v(\xi, \eta, t) = \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \frac{A'k_0}{\alpha-\beta} \left[\frac{\alpha\phi_2}{\alpha+\mu} - \frac{\beta\phi_2}{\beta+\mu} \right] \times Ce_{2n}(\xi, q_{2n,m})\, ce_{2n}(\eta, q_{2n,m}) \quad \ldots(6.3)$$

where

$$\phi_1 = e^{\alpha t} - e^{-\mu t}$$

and

$$\phi_2 = e^{\beta t} - e^{-\mu t}.$$

Putting $\mu = 0$, we get the results corresponding to constant pressure gradient as

$$u(\xi, \eta, t) = \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \frac{A'k_0}{\tau(\alpha-\beta)} \left[\frac{e^{\alpha t}}{\alpha} - \frac{e^{\beta t}}{\beta} + \frac{\alpha-\beta}{\alpha\beta} + \tau(e^{\alpha t} - e^{\beta t}) \right] \times Ce_{2n}(\xi, q_{2n,m})\, ce_{2n}(\eta, q_{2n,m}) \quad \ldots(6.4)$$

$$v(\xi, \eta, t) = \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \frac{A'k_0}{\tau(\alpha-\beta)} \left[\frac{e^{\alpha t}}{\alpha} - \frac{e^{\beta t}}{\beta} + \frac{\alpha-\beta}{\alpha\beta} \right] \times Ce_{2n}(\xi, q_{2n,m})\, ce_{2n}(\eta, q_{2n,m}) \quad \ldots(6.5)$$

and

$$u(\xi, \eta, t) - v(\xi, \eta, t) = \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \frac{A'k_0}{\alpha-\beta} [e^{\alpha t} - e^{\beta t}] \times Ce_{2n}(\xi, q_{2n,m})\, ce_{2n}(\eta, q_{2n,m}). \quad \ldots(6.6)$$

When the dust is very fine, the relaxation time of dust particles decreases and ultimately as $\tau \to 0$, the fluid velocity profile u and the particle velocity v become identical.

If the masses of the dust particles are small, their influence and the fluid flow is reduced and in the limit as $M \to 0$, the fluid becomes ordinary viscous.

Thus, in the limit when $M \to 0$, eqns. (6.1) and (6.4) furnish the expressions of the flow velocity for ordinary viscous fluid under the influence of exponential and constant pressure gradients respectively, as

$$u(\xi, \eta, t) = \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \frac{A' k_0}{\nu \lambda_{2n,m}^2 - \mu} [e^{-\mu t} - e^{-\mu \lambda_{2n,m}^2 t}] \times Ce_{2n}(\xi, q_{2n,m}) \, ce_{2n}(\eta, q_{2n,m}) \quad \ldots(6.7)$$

and

$$u(\xi, \eta, t) = \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \frac{A' k_0}{\nu \lambda_{2n,m}^2} [1 - e^{-\mu \lambda_{2n,m}^2 t}] \, Ce_{2n}(\xi, q_{2n,m}) \, ce_{2n}(\eta, q_{2n,m}). \quad \ldots(6.8)$$

These expressions are in agreement with the results obtained by Gupta (1964) for an ordinary viscous flow.

7. Flow through Long Co-axial Elliptic Ducts

We shall now consider the problem of unsteady flow of the dusty viscous liquid through two co-axial elliptic ducts. Let ξ_0, ξ_1, be the radii of cross-section where $\xi_1 > \xi_0$. The equations of motion (3.3)–(3.5) still hold in the region $\xi_1 \leqslant \xi \leqslant \xi_0$, $0 \leqslant \eta \leqslant 2\pi$; $t \geqslant 0$.

The boundary and initial conditions are

$$u = v = 0$$

when

$$\xi = \xi_0, 0 \leqslant \eta \leqslant 2\pi; \; t > 0 \quad \ldots(7.1)$$

$$u = v = 0 \text{ when } \xi = \xi_1, \; 0 \leqslant \eta \leqslant 2\pi; \; t > 0 \quad \ldots(7.2)$$

$$u = v = 0 \quad \text{when } t = 0, \; \xi_1 < \xi < \xi_0, 0 < \eta < 2\pi \quad \ldots(7.3)$$

Employing the transform (2.5) to eqns. (3.4), (3.5) in the light of (7.1), (7.2) and treating the resulting equations in a similar way as in section 4, we obtain

$$u(\xi, \eta, t) = \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \frac{B'}{\tau(\alpha - \beta)} \Big[(1 + \tau\alpha) \int_0^t e^{\alpha\lambda} f(t-\lambda)\, d\lambda - (1 + \tau\beta) \int_0^t e^{\beta\lambda} f(t-\lambda)\, d\lambda \Big] B_{2n}(\xi, q_{2n,m})\, ce_{2n}(\eta, q_{2n,m}) \quad \ldots(7.4)$$

$$v(\xi, \eta, t) = \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \frac{B'}{\tau(\alpha - \beta)} \Big[\int_0^t e^{\alpha\lambda} f(t-\lambda)\, d\lambda - \int_0^t e^{\beta\lambda} f(t-\lambda)\, d\lambda \Big] B_{2n}(\xi, q_{2n,m})\, ce_{2n}(\eta, q_{2n,m}) \quad \ldots(7.5)$$

and

$$u(\xi, \eta, t) - v(\xi, \eta, t) = \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \frac{B'}{\alpha - \beta} \Big[\alpha \int_0^t e^{\alpha\lambda} f(t-\lambda)\, d\lambda - \beta \int_0^t e^{\beta\lambda} f(t-\lambda)\, d\lambda \Big] \times B_{2n}(\xi, q_{2n,m})\, ce_{2n}(\eta, q_{2n,m}) \quad \ldots(7.6)$$

where

$$B' = B/\pi \int_{\xi_0}^{\xi_1} B_{2n}^2(\xi, q_{2n,m}) [\cosh 2\xi - \Theta_{2n,m}]\, d\xi$$

$$B = \int_{\xi_0}^{\xi_1} \int_0^{2\pi} (\cosh 2\xi - \cos 2\eta)\; B_{2n}(\xi, q_{2n,m})\, ce_{2n}(\eta, q_{2n,m})\; d\xi\, d\eta$$

and the sum is taken over all the positive roots of (2.7).

8. Flow Under Harmonically Oscillating Pressure Gradient

Let

$$-\frac{1}{\rho}\frac{\partial p}{\partial z} = f(t) = k_0 \cos \omega t$$

Then, from (7.4), (7.5) and (7.6), we get

$$u(\xi, \eta, t) = \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \frac{B' k_0}{\tau(\alpha - \beta)} \Big[\frac{1 + \tau\alpha}{\alpha^2 + \omega^2} \gamma_1 - \frac{1 + \tau\beta}{\beta^2 + \omega^2} \gamma_2 \Big] \times B_{2n}(\xi, q_{2n,m})\, ce_{2n}(\eta, q_{2n,m}) \quad \ldots(8.1)$$

$$v(\xi, \eta, t) = \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \frac{B'k_0}{\tau(\alpha - \beta)} \left[\frac{\gamma_1}{\alpha^2 + \omega^2} - \frac{\gamma_2}{\beta^2 + \omega^2} \right] \times B_{2n}(\xi, q_{2n,m}) \, ce_{2n}(\eta, q_{2n,m}) \quad \ldots(8.2)$$

and

$$u(\xi, \eta, t) - v(\xi, \eta, t) = \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \frac{B'k_0}{\alpha - \beta} \left[\frac{\alpha\gamma_1}{\alpha^2 + \omega^2} - \frac{\beta\gamma_2}{\beta^2 + \omega^2} \right] \times B_{2n}(\xi, q_{2n,m}) \, ce_{2n}(\eta, q_{2n,m}) \quad \ldots(8.3)$$

where γ_1 and γ_2 are as given in section 5.

9. Flow under the Pressure Gradient Decreasing Exponentially with Time

Let

$$-\frac{1}{\rho}\frac{\partial p}{\partial z} = f(t) = k_0 e^{-\mu t}$$

Then, eqns. (7.4), (7.5) and (7.6) reduce after simplification to

$$u(\xi, \eta, t) = \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \frac{B'k_0}{\tau(\alpha - \beta)} \left[\frac{1 + \tau\alpha}{\alpha + \mu} \phi_1 - \frac{1 + \tau\beta}{\beta + \mu} \phi_2 \right] \times B_{2n}(\xi, q_{2n,m}) \, ce_{2n}(\eta, q_{2n,m}) \quad \ldots(9.1)$$

$$v(\xi, \eta, t) = \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \frac{B'k_0}{\tau(\alpha - \beta)} \left[\frac{\phi_1}{\alpha + \mu} - \frac{\phi_2}{\beta + \mu} \right] \times B_{2n}(\xi, q_{2n,m}) \, ce_{2n}(\eta, q_{2n,m}) \quad \ldots(9.2)$$

and

$$u(\xi, \eta, t) - v(\xi, \eta, t) = \sum_{n=0}^{\infty} \sum_{m=1}^{\infty} \frac{B'k_0}{\alpha - \beta} \left[\frac{\alpha\phi_1}{\alpha + \mu} - \frac{\beta\phi_2}{\beta + \mu} \right] \times B_{2n}(\xi, q_{2n,m}) \, ce_{2n}(\eta, q_{2n,m}) \quad \ldots(9.3)$$

where ϕ_1 and ϕ_2 are as defined in section 6.

The results corresponding to constant pressure gradient may be obtained by putting $\mu = 0$.

As mentioned earlier in section 6, the liquid will behave as ordinary viscous liquid when $M \to 0$ in the limit and if $\tau \to 0$, the velocities of the fluid and dust particles become identical.

Thus, if $M \to 0$, we get the corresponding results for ordinary viscous flow as discussed by Gupta (1964).

Acknowledgements

The author is indebted to Prof. R. S. Sharma and Dr. N. K. Choubey for constant encouragement in the preparation of this paper.

References

Gupta, R. K. (1964). A finite transform involving Mathieu Functions and its application. *Proc. natn. Inst. Sci., India, A* **30**, 779–95.

Liu, J. T. C. (1966). Flow induced by an oscillating infinite flat plate in a dusty gas. *Phys. Fluids*, **9**, 1716.

Mclachlan, N. W. (1947). Theory and Applications of Mathieu Functions. Oxford University Press, London.

Michael, D. H. (1965). Kelvin-Helmholtz instability of a dusty gas. *Proc. Camb. phil. Soc.*, **61**, 659–71.

Michael, D. H., and Miller, D. A. (1966). Plane parallel flow of a dusty gas. *Mathematika*, **13**, 97–109.

Michael, D. H., and Norey, P. W. (1968). The laminar flow of a dusty gas between rotating cylinders. *Q. Jl Mech. appl. Math.*, **21**, 375–88.

Rao, P. S. S. (1969). Unsteady flow of dusty viscous liquid through circular cylinder. *Defence Sci. J.*, **19**, 135–38.

Saffman, P. G. (1962). On the stability of laminar flow of a dusty gas. *J. Fluid Mech.*, **13**, 120–28.

Verma, P. D., and Mathur, A. K. (1973). Unsteady flow of a dusty viscous liquid in a circular tube. *Indian J. pure appl. Math.*, **4**, 133–38.